Introduction to Airbreathing Propulsion

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Lecture – 05
Review of Fluid Mechanics, Thermodynamics

Discussion on the fluid mechanics and thermodynamics review. So, what we have been talking about that control mass and control volume system and what how Reynold transport theorem can be applied to derive all the basic conservation laws and if you just recall from the last lecture.

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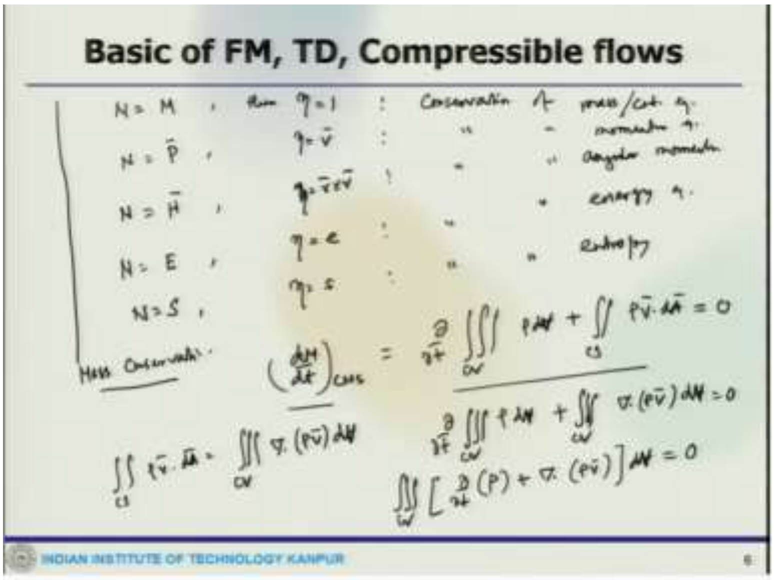

Basic of FM, TD, Compressible flows

$N = M$, then $\eta = 1$: Conservation of mass/cont. eq.

$N = \bar{P}$, $\eta = \bar{V}$: " – momentum eq.

" angular momentum

$N = \bar{H}$, $\eta = \bar{r} \times \bar{V}$: " " energy eq.

$N = E$, $\eta = e$: " " entropy

$N = S$, $\eta = s$:

Mass Conservation:

$$\left(\frac{dM}{dt}\right)_{sys} = \frac{\partial}{\partial t}\iiint_{CV} \rho\, d\forall + \iint_{CS} \rho \bar{V} \cdot d\bar{A} = 0$$

$$\iint_{CS} \rho \bar{V} \cdot d\bar{A} = \iiint_{CV} \nabla \cdot (\rho \bar{V})\, d\forall$$

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\, d\forall + \iiint_{CV} \nabla \cdot (\rho \bar{V})\, d\forall = 0$$

$$\iiint_{CV} \left[\frac{\partial}{\partial t}(\rho) + \nabla \cdot (\rho \bar{V})\right] d\forall = 0$$

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This is what we have shown that for a different property and how you can derive all these different conservation equations.

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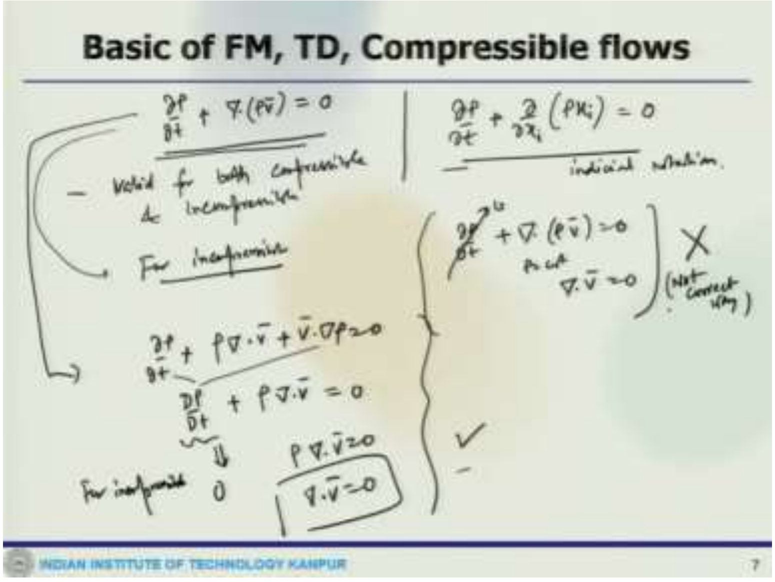

Basic of FM, TD, Compressible flows

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\bar{v}) = 0$$

- Valid for both compressible & incompressible

For incompressible

$$\frac{\partial\rho}{\partial t} + \rho\nabla\cdot\bar{v} + \bar{v}\cdot\nabla\rho = 0$$

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{v} = 0$$

For incompressible $\Downarrow$ 0 $\quad \rho\nabla\cdot\bar{v} = 0$

$$\boxed{\nabla\cdot\bar{v} = 0}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

indicial notation.

$$\cancel{\frac{\partial\rho}{\partial t}}^{0} + \nabla\cdot(\rho\bar{v}) = 0$$

hence $\nabla\cdot\bar{v} = 0$ ✗ (Not correct way)

✓

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And we have with this particular one we have talked about the mass conservation equation or rather continuity equation and how to reach the incompressible limit.

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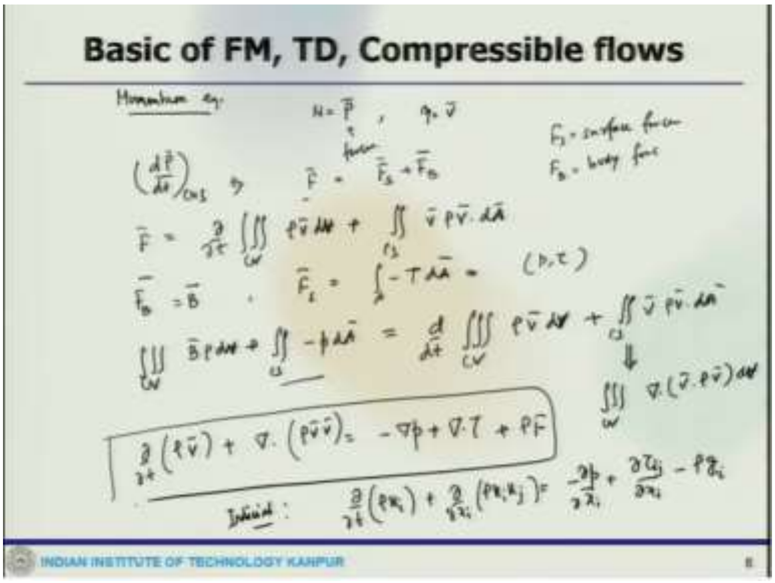

Basic of FM, TD, Compressible flows

Momentum eq.

$N = \bar{P}$ (force), $\eta = \bar{v}$

F_s = surface force
F_B = body force

$$\left(\frac{d\bar{P}}{dt}\right)_{CMS} \Rightarrow \bar{F} = \bar{F}_s + \bar{F}_B$$

$$\bar{F} = \frac{\partial}{\partial t}\iiint_{CV} \rho\bar{v}\, d\forall + \iint_{CS} \bar{v}\rho\bar{v}\cdot d\bar{A}$$

$$\bar{F}_B = \bar{B}, \quad \bar{F}_s = \int -T\, d\bar{A} = (p, \tau)$$

$$\iiint_{CV} \bar{B}\rho\, d\forall + \iint_{CS} -p\, d\bar{A} = \frac{d}{dt}\iiint_{CV} \rho\bar{v}\, d\forall + \iint_{CS} \bar{v}\rho\bar{v}\cdot d\bar{A}$$

$$\Downarrow \quad \iiint_{CV} \nabla\cdot(\bar{v}\rho\bar{v})\, d\forall$$

$$\boxed{\frac{\partial}{\partial t}(\rho\bar{v}) + \nabla\cdot(\rho\bar{v}\bar{v}) = -\nabla p + \nabla\cdot\tau + \rho F}$$

Indicial: $\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_i} - \rho g_i$

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So now we will look at the similarly momentum equation. Now one can always derive this like an what is typically done in any fluid mechanics course or any fluid mechanics book that taking an elemental control volume and you can derive because it is nothing but the Newton's law of motion f is equals to m a from there one can derive the same set of equation. But the idea here is not to derived in that fashion rather looking at the things from point of view of the RTT.

Now so here N would be ρ and η would be velocity vector for momentum equation. So, what one can write that d rho this is N is if you see N is, this is the forces let us say vector. So this would be forces so d rho by dt for control mass system which is nothing but the all the forces that is summation of all FS+ FB. So, if FS is surface force and so these are acted at the boundary of the control volume are like for example reaction forces, pressure forces, shear forces these are all surface forces and FB is the body force.

So, these forces are acted on the material inside the control volume and this could be proportional to the mass or the volume of the fluid. For example, gravitational forces, electrostatic forces, electromagnetic forces like that and this F is the sum of all the external forces which are acted on that particular now this

$$F = \frac{\partial}{\partial t} \iiint_{CV} \rho \bar{V} d\forall + \iint_{CS} \bar{V} \rho \bar{V} \cdot d\bar{A}$$

Now if we say let us say the body forces are B and all surface forces we can like TdA these could be pressure shear stress something like that then what we get is that

$$\iiint_{CV} B\rho d\forall + \iint_{CS} -Pd\bar{A} = \frac{\partial}{\partial t} \iiint_{CV} \rho \bar{V} d\forall + \iint_{CS} \bar{V} \rho \bar{V} \cdot d\bar{A}$$

Now this one particular one we can write like control volume delta dot v rho v dV then what we get finally clubbing this term and this also we can use and then finally what we can write

$$\frac{\partial}{\partial t} (\rho \bar{V}) + \nabla(\rho \bar{V} \bar{V}) = -\nabla P + \nabla \cdot \tau + \rho F$$

So, this is what you get from that. Now similarly this is in vector form one can write this equation in also indicial notation and that would look like just for your reference I am just providing that thing but you can look at any of these fluid mechanics book to review this derivation or the different notation for the same so these are.

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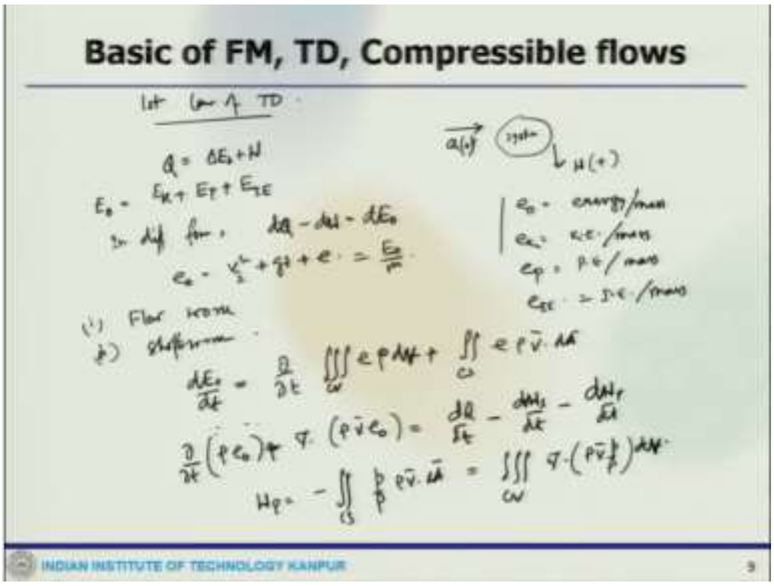

Basic of FM, TD, Compressible flows

1st law of TD

$\overrightarrow{Q(+)}$ system ↳ $W(+)$

$Q = \Delta E_0 + W$

$E_0 = E_K + E_P + E_{IE}$

In diff form, $dQ - dW = dE_0$

$e_0 = \frac{V^2}{2} + gz + e = \frac{E_0}{m}$

e_0 = energy/mass

e_K = K.E./mass

e_P = P.E./mass

e_{IE} = I.E./mass

(i) Flow work

(ii) shaft work

$$\frac{dE_0}{dt} = \frac{\partial}{\partial t}\iiint_{CV} e\rho d\forall + \iint_{CS} e\rho\bar{V}\cdot d\bar{A}$$

$$= \frac{dQ}{dt} - \frac{dW_s}{dt} - \frac{dW_f}{dt}$$

$$\frac{\partial}{\partial t}(\rho e_0) + \nabla\cdot(\rho\bar{V}e_0) = \frac{dQ}{dt} - \frac{dW_s}{dt} - \frac{dW_f}{dt}$$

$$W_f = -\iint_{CS}\frac{p}{\rho}\rho\bar{V}\cdot d\bar{A} = \iiint_{CV}\nabla\cdot\left(\rho\bar{V}\frac{p}{\rho}\right)d\forall$$

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Now we look at the energy equation and for energy equation also you can derive similarly. But for that we just look at the first law of thermodynamics so it accounts for the rate of change of energy or either a system or a control volume. So, what it states that the net heat added to a system should be equal to the sum of the change in stored energy of the system and the work done by the system of the surroundings.

So typically, if you let us say have a system like this then the heat added here this is the work output of the system. So, when heat is added this should be positive when the work is output that should be also positive. So we can write that $\nabla E_0 + W$ is the stored energy, which is E_K, E_P, E_{IE} and in differential form so you write $dQ - dW = dE_0$ and per unit mass E_0 would be energy per unit mass.

This is kinetic energy per unit mass this is potential energy per unit mass and this is internal energy per unit mass. So we have all these terminology and also one can write that this E_0 should be $\frac{V^2}{2} + gz + e$ which is nothing but E_0/m. Now there are two kind of work one is the flow work which is work done on the surroundings and second would be shaft work.

So the shaft work is there any other form of work for example work transfer through the control surface by shaft electric current etc. Now similarly if we use the RTT so we write

$$\frac{dE_0}{dt} = \frac{\partial}{\partial t}\iiint_{CV} e\rho d\forall + \iint_{CS} e\rho\bar{V}\cdot d\bar{A}$$

So you put things back there and so this once you take the divergence and all these so what you can look at those couple of steps further for detailing and you get back

$$\frac{\partial}{\partial t}(\rho e_0) + \nabla \cdot (\rho \bar{V} e_0) = \frac{dQ}{dt} - \frac{dW_s}{dt} - \frac{dW_p}{dt}$$

So, here Wp is

$$W_p = -\iint_{CS} \frac{P}{\rho} e\bar{V} \cdot d\bar{A} = \iiint_{CV} \nabla \cdot (\frac{P}{\rho} e\bar{V})\, d\forall$$

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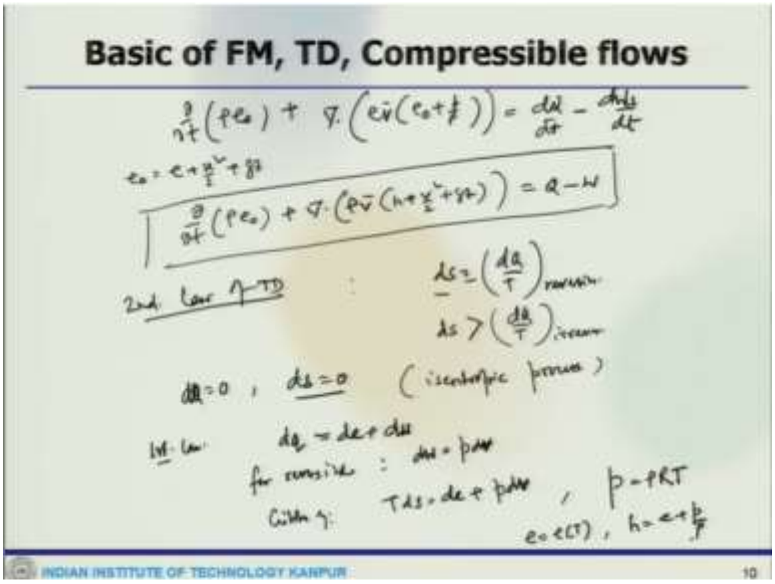

Basic of FM, TD, Compressible flows

$$\frac{\partial}{\partial t}(\rho e_0) + \nabla \cdot \left(e\bar{v}(e_0 + \frac{p}{\rho})\right) = \frac{dQ}{dt} - \frac{dW_s}{dt}$$

$$e_0 = e + \frac{v^2}{2} + gz$$

$$\boxed{\frac{\partial}{\partial t}(\rho e_0) + \nabla \cdot \left(\rho\bar{v}\left(h + \frac{v^2}{2} + gz\right)\right) = Q - W}$$

2nd law of TD : $ds = \left(\frac{dQ}{T}\right)_{reversible}$

$ds > \left(\frac{dQ}{T}\right)_{irreversible}$

$dQ = 0$, $ds = 0$ (isentropic process)

1st law: $dq = de + dw$

for reversible : $dw = p\,dv$

Gibbs eq: $T\,ds = de + p\,dv$, $p = \rho RT$

$e = e(T)$, $h = e + \frac{p}{\rho}$

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So the equation transform back to

$$\frac{\partial}{\partial t}(\rho e_0) + \nabla \cdot \left(e\bar{V}(e_0 + \frac{P}{\rho})\right) = \frac{dQ}{dt} - \frac{dW_s}{dt}$$

this is subtract on heat transfer. So since

$$e_0 = e + \frac{u^2}{2} + gz$$

one can write it back

$$\nabla \cdot \left(\rho\bar{V}\left(h + \frac{v^2}{2} + gz\right)\right) = Q - W$$

so that is what you get. Now if you look at so this derivation also one can do in a conventional approach. Now quickly what second law of thermodynamics states it is talk about the entropy and what if your process is reversible then this can be written as for reversible process and the ds is the change in entropy in a reversible heat exchange of dQ and a temperature t.

Now if it is an irreversible process then ds would be always dQ/t for irreversible process. Now if the process is reversible and adiabatic then dQ is 0 which means ds is 0. So that means there is no change in entropy. So this process is called the I mean isentropic process okay and the second law is used to basically describe the ideal operation of an engine for the calculation of efficiencies and other parameters.

Now some other thermo dynamical property of the gases like again if we look at from first law what we can write dq=de+dw. Now for reversible process we can write dw=pdV and then the Gibbs equation becomes Tds=de+pdV and the perfect gas equation states that p= ρ RT okay and we can write the so if these are thermally perfect gas then e would be only function of temperature and

$$h = e + \frac{P}{\rho}$$

so which follows that it is function of temperature now the definition says specific

$C_v = \frac{de}{dt}$ and $C_p = \frac{dh}{dt}$

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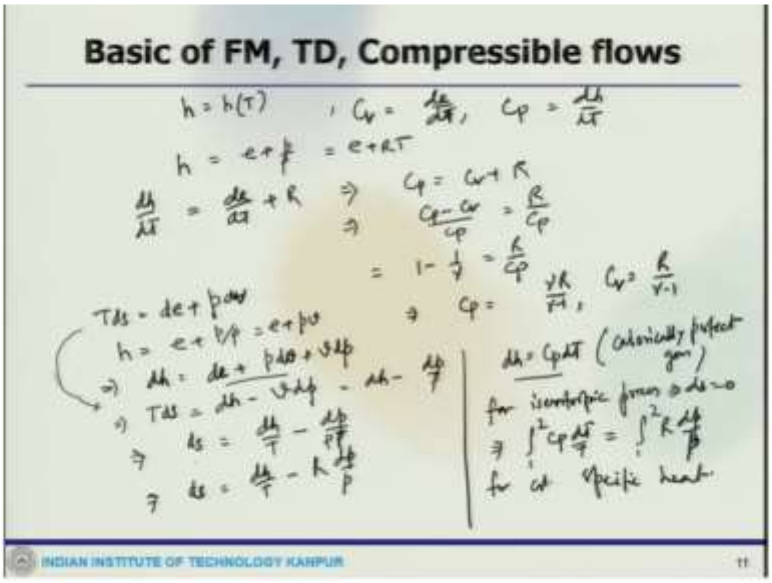

Basic of FM, TD, Compressible flows

$h = h(T)$, $C_v = \frac{de}{dT}$, $C_p = \frac{dh}{dT}$

$h = e + \frac{p}{\rho} = e + RT$

$\frac{dh}{dT} = \frac{de}{dT} + R \Rightarrow C_p = C_v + R$

$\Rightarrow \frac{C_p - C_v}{C_p} = \frac{R}{C_p}$

$= 1 - \frac{1}{\gamma} = \frac{R}{C_p}$

$\Rightarrow C_p = \frac{\gamma R}{\gamma - 1}$, $C_v = \frac{R}{\gamma - 1}$

$Tds = de + pdv$

$h = e + p/\rho = e + pv$

$\Rightarrow dh = de + pdv + vdp$

$\Rightarrow Tds = dh - vdp = dh - \frac{dp}{\rho}$

$\Rightarrow ds = \frac{dh}{T} - \frac{dp}{\rho T}$

$\Rightarrow ds = \frac{dh}{T} - R\frac{dp}{p}$

$dh = C_p dT$ (calorically perfect gas)

for isentropic process $ds = 0$

$\Rightarrow \int_1^2 C_p \frac{dT}{T} = \int_1^2 R \frac{dp}{p}$

for cst specific heat.

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So, which follows that h is function of temperature. Now the definition is specific $C_v = \frac{de}{dt}$ and $C_p = \frac{dh}{dt}$. So, what we get $h = e + \frac{P}{\rho}$ which is e+RT. Now if I take dh/dt this is de/dt+R which means Cp = Cv + R. So, if you take both sides with the so

$$\frac{C_P - C_V}{C_P} = \frac{R}{C_P}$$

which would be

$$1 - \frac{1}{\gamma} = \frac{R}{C_P}$$

So, what we can get

$C_P = \frac{R\gamma}{\gamma-1}$ and $C_v = \frac{R}{\gamma-1}$

Now some other equation what we can write like from the Tds= de + pdV so that gives me that

$$h = e + \frac{P}{\rho}$$

so or rather $h = e + PV$ then we can write dh=de+pdv+vdp. So, which is Tds=dh-vdp. So here de+pdv this one can use this relationship here and so this is dh-dp/rho. So, if you divide both sides by t this would become

$$ds = \frac{dh}{dt} - \frac{dp}{\rho T}$$

So you write $ds = \frac{dh}{T} - \frac{dp}{P}$

So we can write like dh=C_pdT this is for calorically perfect gas. So there are two types of situations one is calorically perfect and it is the thermally perfect once you use this relationship. So for isentropic process this should be ds = 0 so what we get $\frac{C_P dT}{T} = R\frac{dp}{P}$. Now we can do the integration on both sides for constant specific heat what we get.

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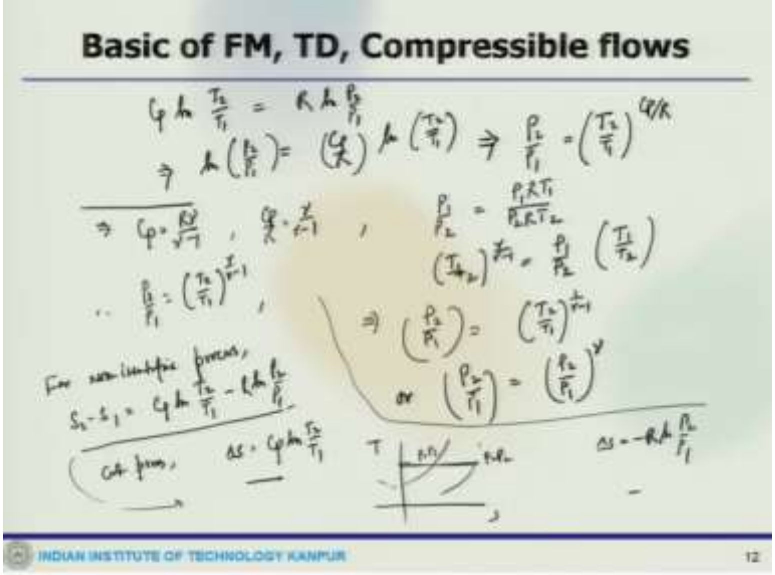

Basic of FM, TD, Compressible flows

$C_p \ln\frac{T_2}{T_1} = R\ln\frac{p_2}{p_1}$

$\Rightarrow \ln\left(\frac{p_2}{p_1}\right) = \left(\frac{C_p}{R}\right)\ln\left(\frac{T_2}{T_1}\right) \Rightarrow \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{C_p/R}$

$\Rightarrow C_p = \frac{R\gamma}{\gamma-1}, \quad \frac{C_p}{R} = \frac{\gamma}{\gamma-1},$

$\therefore \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}},$

$\Rightarrow \left(\frac{\rho_2}{\rho_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

or $\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$

For non-isentropic process,

$s_2 - s_1 = C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$

Const. pres., $\Delta s = C_p\ln\frac{T_2}{T_1}$

$\Delta s = -R\ln\frac{p_2}{p_1}$

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$$C_P ln\frac{T_2}{T_1} = R \cdot ln\frac{P_2}{P_1}$$

So that gives us back

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{C_P}{R}}$$

. So these are the sort of relationship that one get from this now we have already seen some other relationship like what we have seen $C_P = \frac{R\gamma}{\gamma-1}$ and then this ratio of P1 by P2 we can write

$$\frac{P_1}{P_2} = \frac{\rho_1 RT_1}{\rho_2 RT_2}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \frac{\rho_1}{\rho_2} * \frac{T_1}{T_2}$$

Now since we have

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Or

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

So, this is what you get the relationship for all the isentropic process where there is no change in entropy. So, keep that in mind these are only for isentropic process. Now for if we have for non-isentropic process what we can write

$$S_2 - S_1 = C_P ln\frac{T_2}{T_1} - R \cdot ln\frac{P_2}{P_1}$$

So that you get from this equation and getting the integration between two points you get that relationship for non-isentropic process. So, if it is a constant pressure for constant pressure this $S_2 - S_1 = C_P ln\frac{T_2}{T_1}$. So that either you can show it from here using this equation for constant pressure or you can draw a sort of TS diagram like this so this is let us say P equals P1 here P equals P2 here.

Now this is on the constant temperature line so if the temperature line is constant this

$$S_2 - S_1 = -R \cdot ln\frac{P_2}{P_1}$$

okay so these are some of this relationship that is commonly used for different kind of processes and we should be aware of that. Now with that brief idea I mean now the whole idea is that you can review all these details these are just to sort of refresh our memory about all these. So, you can look at any other textbook and all these just to go into the detail.

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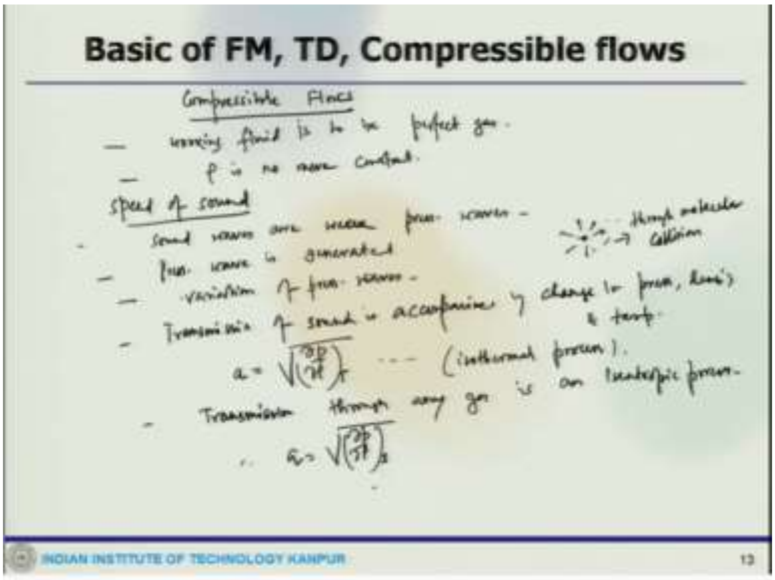

Basic of FM, TD, Compressible flows

Compressible Flows

- working fluid is to be perfect gas.
- ρ is no more constant.

Speed of sound

- Sound waves are weak pressure waves - through molecular collision
- pressure wave is generated
- variation of pressure waves.
- Transmission of sound is accompanied by change in pressure, density & temp.

$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_T}$ --- (isothermal process).

- Transmission through any gas is an isentropic process.

$\therefore a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$

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So, you move to that with the discussion on compressible flows. So now what important thing for the compressible flow discussion here what we will be discussing here the working fluid is so working fluid is to be perfect gas so that is one of the thing that. So whatever thermodynamical relationship that we have just looked at they will also be true and can be used.

Second important things for compressible flow is that density is no more constant. So this is a variable density situation and it changes quite drastically it is not like that it remains constant. Now the first thing which comes is speed of sound. How do we define speed of sound? It just like air is composed of molecules they are moving in random motion but they have a mean velocity and average energy which is quite known.

Now sound waves are sort of weak pressure wave so sound waves are weak pressure waves and they are carried by moving air molecules. Now when there is a source of sound it releases some energy which is usually picked up by the neighbouring molecules and causing an increase in their kinetic energy. So obviously if there is a source which emits the sound waves or the noise.

So, these are picked up by this particle surrounding it and once they pick up that so there would be increase in the kinetic energy. Now this kinetic energy which is picked up by the neighbouring immediately mining model molecules they are transmitted from one molecule to another molecule and these transmit a process of this transmission happens through molecular collision.

So, this sound energy which is from here it gets transmitted through the molecular collision. So, the pressure wave is generated due to that. So, this pressure wave is generated by the source is transmitted through the molecular collision. So, when that happens our eardrums actually pick the variation of pressure wave. So, they are these variation of pressure waves these are picked up by our eardrums and it passes by and our brain sense it thus we actually hear the sound.

This is how actually we are able to hear some sound which is generated somewhere else. Now this transmission of sound is accompanied by change in pressure, density and temperature. So, Newton provided an expression for estimating the sound wave which is $\sqrt{\left(\frac{\partial P}{\partial \rho}\right)_T}$. So, this is where it was considered to be an isothermal process but later on it was found that this transmission of sound waves through here or any gas is an isentropic process which means that definition would be $\sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S}$. So, this is what the definition which was kind of given later on so and then since it is a isentropic process we can see how one can estimate the sound wave from here. So, we will just stop the discussion here and continue in the next lecture.