**Introduction to Airbreathing Propulsion**

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**Lecture – 49**
**Axial Compressor (Contd.,)**

Okay, so let us continue the discussion on axial flow compressor and what we are in the middle of the discussion is that we have started looking at the stage dynamics and how you need to design the axial flow compressor and rather the need of the axial flow compressor in the modern gas turbine engine and then how the things actually changes the flow passages when it passes through rotor and stator.

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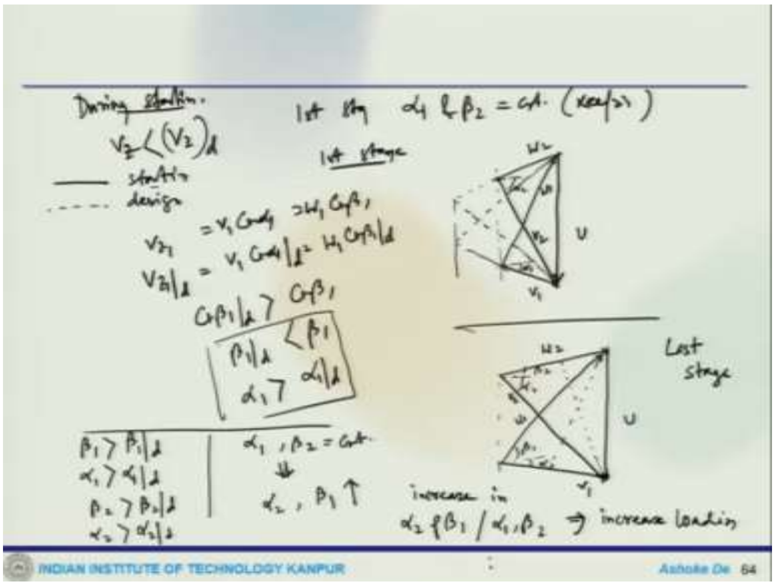


So, we will continue the discussion where we stopped in the last lecture that during starting the first stage and the last stage design condition, so this is what we looked at the first stage and the and we said this dark line will represent the starting and the dotted line will represent the design condition and first stage, so this is how your velocity triangle would look like and then these would be the conditions for design purposes.

Now at the second stage or the last stage, the triangle would look like slightly different, so this will go like this and so you can see how it actually looks like and this is U, so this is $W_2$, this would be $W_1$ and as usual this is $\alpha_1$, this is $\beta_1$, $\alpha_2$, $\beta_2$ and this is my $V_1$ and this would be $V_2$ and the design condition may be like this and the other case it would be like this.

So, now this case, so this is the case where it is last stage velocity triangle, so what happens if you look at it that

$$\beta_1 > \beta_{1_{design}}$$

$$\alpha_1 > \alpha_{1_{design}}$$

$$\beta_2 > \beta_{2_{design}}$$

$$\alpha_2 > \alpha_{2_{design}}$$

So, essentially $\alpha_1$ and $\beta_2$ are kept constant, so which means $\alpha_2$ and $\beta_1$ actually increases.

So, now the increase in $\alpha_2$ and $\beta_1$ or $\alpha_1$ and $\beta_2$, so essentially increase the loading, so whatever it happens this happens, so higher pressure rise plus and possible positive incident separation may cause stall and surge.

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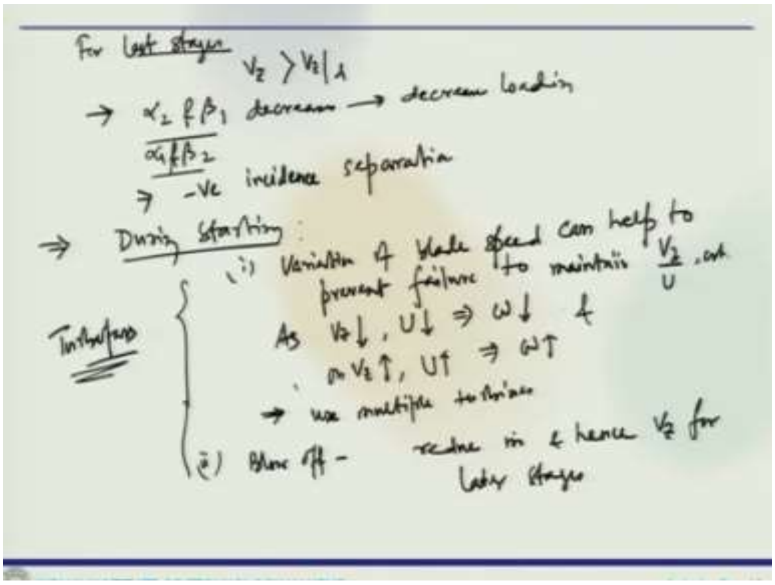


Now, for last stages or

$$V_z > V_{z_{design}}$$

so when $\alpha_2$ and $\beta_1$ decreases, it actually at the same time decreases the loading on blade passages, so this could be $\alpha_2$ and $\beta_1$ or $\alpha_1$ and $\beta_2$, so either of them actually decreases it decreases the loading so, which means there is a negative incidence separation. Now, what happens during starting front and back stages behave in opposite manner, so this is what actually happened during starting.

So, one can see that like for that is what for turbofans, what you can do the since the front stage and the back stage they behave in a different manner or rather opposite manner, you have variation of blade speed can help to prevent failure to maintain that ratio of $\frac{V_z}{U}$ which is sort of constant. So, what happens; as $V_z$ decreases U decreases, which means rotational speed decreases.

And as $V_z$ increases U increases, which means rotational speed also increases, so just to accommodate this, one has to use multiple turbines, so just to avoid the situation or second situation could be blow up which means take or rather bleed off, take some air at middle stages and pass it through the bypass, thus so when you do that, so you reduce m dot and hence $V_z$ for later stages, okay. So, this is what typically done in turbofan, so these are the some of the issues,

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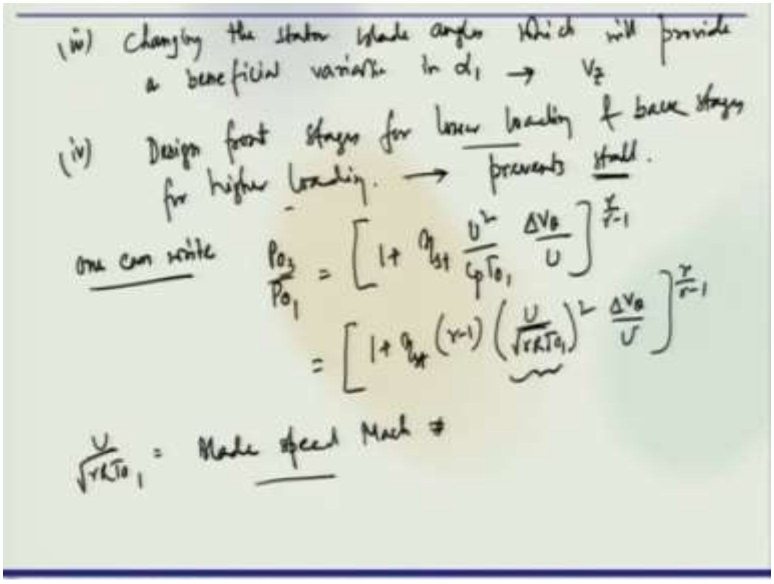


Now, another possibility during starting is that you can change the stator blade angles which will provide a beneficial variation in $\alpha_1$, so that causes the right change in $V_z$ or fourth it could be, you could design front stages for lower loading and back stages for higher loading. So, you can distribute the loading pattern, so the front stages you design such that it actually the loading is low and the back stages could be the loading could be high.

And that is the way if you design probably, you would be able to get a desired pressure rise without having too much of difficulties. Now, this you can think about from a design point of view, so that is in actual operation they may operate with similar characteristics and this can prevent stall, so that is one of the important situation what one would be. So, one can write like the pressure rise between

$$\frac{p_{03}}{p_{01}} = \left(1 + \eta_{st}\frac{U^2}{C_p T_{01}}\frac{\Delta V_\theta}{U}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{03}}{p_{01}} = \left[1 + \eta_{st}(\gamma - 1)\left(\frac{U}{\sqrt{\gamma R T_{01}}}\right)^2 \frac{\Delta V_\theta}{U}\right]^{\frac{\gamma}{\gamma-1}}$$

So, here this quantity that $\frac{U}{\sqrt{\gamma R T_{01}}}$ is blade speed Mach number, so this is another important parameter while looking at the design.

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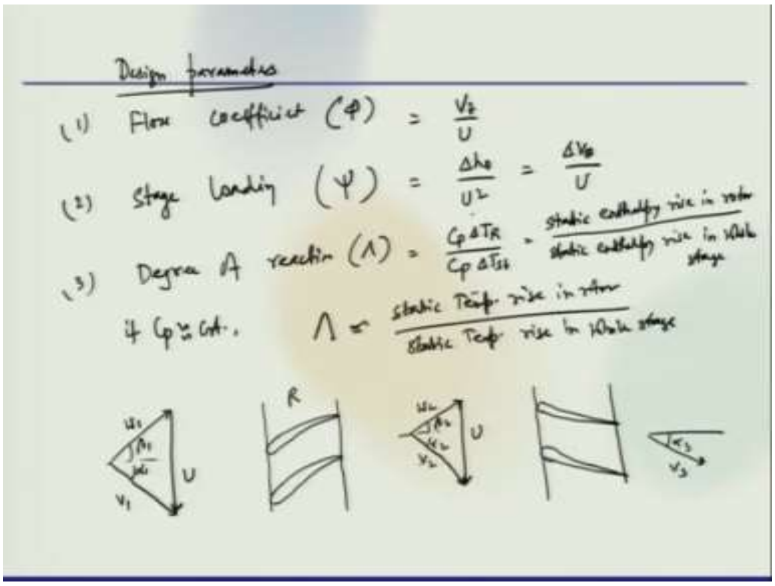


Now, when you talk about that what are the basic design parameters, so there are few design parameters or primarily 3 design parameters which are frequently used for parametric study for axial compressor, one is the flow coefficient, so which is represented usually ϕ, so one can write that it will $\frac{V_z}{U}$, so this is the ratio between axial and the rotational speed. Second could be stage loading, so that is typically represented at $\psi$.

$$\psi = \frac{\Delta h_0}{U^2} = \frac{\Delta V_\theta}{U}$$

so enthalpy rise that is the ratio of total enthalpy rise to the blade kinetic energy and third could be degree of reaction which is like this, now that so this is given as

$$\Lambda = \frac{C_p \Delta T_R}{C_p \Delta T_{st}} = \frac{static\ enthalpy\ rise\ in\ rotor}{static\ enthalpy\ rise\ in\ whole\ stage}$$

okay.

So, this is where the if you look at that diffusion pressure or the pressure rise that takes place both in stator and rotor and but it increases in static, I mean that is essentially increases the static pressure, so the total pressure rise could be attributed to the both the blade rows, rotor does the dynamic work, hence pressure rise in the rotor is an important design factor because this does the dynamic, they exposed to the dynamic loading.

And that is why the degree of reaction is sort of represented like this, now if $C_p$ is assumed to be constant, then $\Lambda$ become essentially we can write, it is a static temperature rise in rotor to static temperature rise in whole stage, okay. So, if you consider a blade state, so this how the velocity triangle would look like, so that is $U, \beta_1, \alpha_1, V_1, W_1$ and so these are my blade rows, so that is let us say rotor.

Then there would be the velocity triangle, so this is again $W_2, \beta_2, \alpha_2, V_2$ then there is a set up, so finally it goes out like this, so this is you can think about a blade state like that, that you have rotor and stator and that now, we can derive the expression for degree of reaction.

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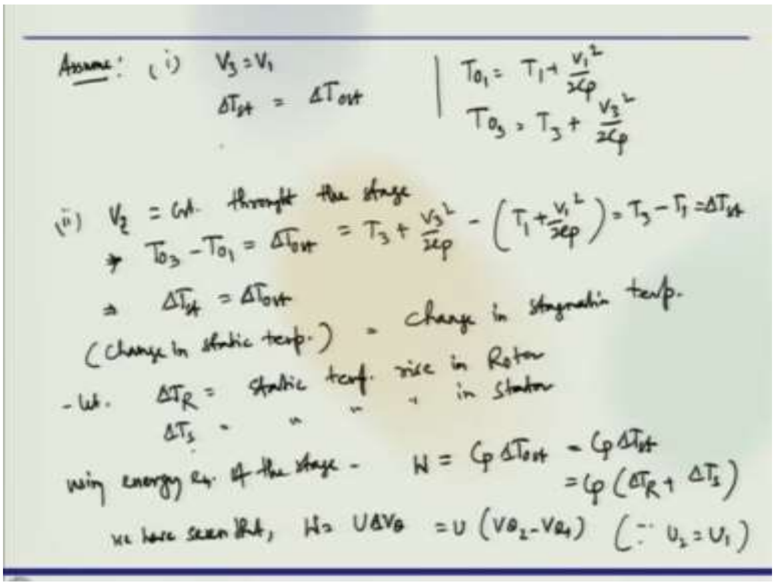


So, let us assume that $V_3 = V_1$, that means air leaves the stage with same axial velocity with which enters, so that gives you back the $\Delta T_{st} = \Delta T_{ost}$, so I can write

$$T_{01} = T_1 + \frac{V_1^2}{2C_p}$$

and

$$T_{03} = T_3 + \frac{V_3^2}{2C_p}$$

second; $V_z$ is constant throughout the stage, so that gets me back

$$T_{03} - T_{01} = T_3 + \frac{V_3^2}{2C_p} - \left(T_1 + \frac{V_1^2}{2C_p}\right) = T_3 - T_1 + T_{st}$$

so which one can write, so change in static temperature, change in the stagnation temperature of the stage, so this is change in static temperature which is the change in stagnation temperature. Let $\Delta T_R$ is assumed to be static temperature rise in rotor and $\Delta T_R$ is static temperature rise in stator, so just careful with this subscript.

Because when I use, s that is stator but when I use st, that means the whole stage which is represented, now we use the energy equation of the stage. So,

$$W = C_p \Delta T_{ost} = C_p \Delta T_{st} = C_p(\Delta T_R + \Delta T_S)$$

Now, what we have seen that, we have seen that

$$W = U \Delta V_\theta = U[V_{\theta 2} - V_{\theta 1}]$$

$$U_2 = U_1$$

okay.

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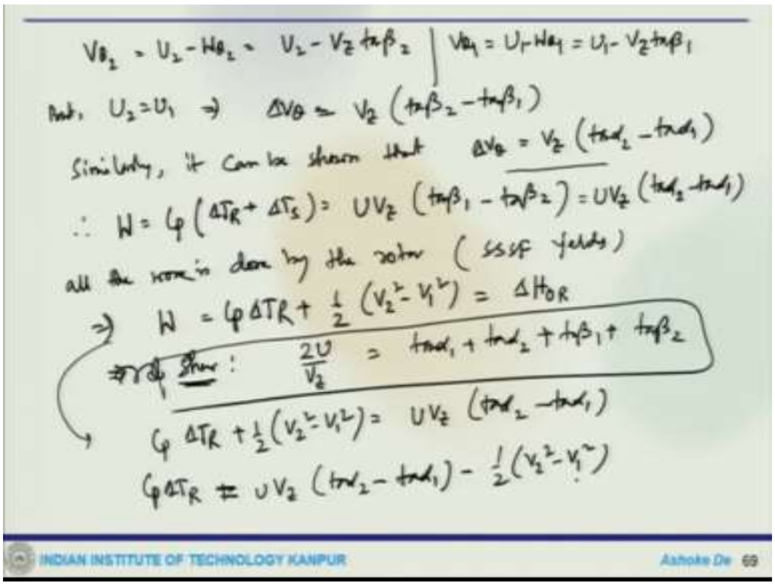


Now, one can write that

$$V_{\theta 2} = U_2 - W_{\theta 2} = U_2 - V_z \tan \beta_2$$

And

$$V_{\theta 1} = U_1 - W_{\theta 1} = U_1 - V_z \tan \beta_1$$

$$U_2 = U_1$$

so that gives me

$$\Delta V_\theta = V_z(\tan\beta_2 - \tan\beta_1)$$

So, similarly it can be shown that

$$\Delta V_\theta = V_z(\tan\alpha_2 - \tan\alpha_1)$$

so using the velocity triangle one can also, so this is there. So,

$$W = C_p(\Delta T_R + \Delta T_S) = UV_z(\tan\beta_2 - \tan\beta_1) = UV_z(\tan\alpha_2 - \tan\alpha_1)$$

But all the work is done by the rotor, so all the work is done by the rotor, so steady state, steady flow analysis yields that

$$W = C_p\Delta T_R + \frac{1}{2}(V_2^2 - V_1^2) = \Delta H_0 R$$

So, one can show that like

$$\frac{2U}{V_z} = \tan\alpha_1 + \tan\alpha_2 + \tan\beta_1 + \tan\beta_2$$

so this can be shown, so that is what we can use this and write

$$C_p\Delta T_R + \frac{1}{2}(V_2^2 - V_1^2) = UV_z(\tan\alpha_2 - \tan\alpha_1)$$

So,

$$C_p\Delta T_R = UV_z(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}(V_2^2 - V_1^2)$$

it is just an rearrangement of the thing.

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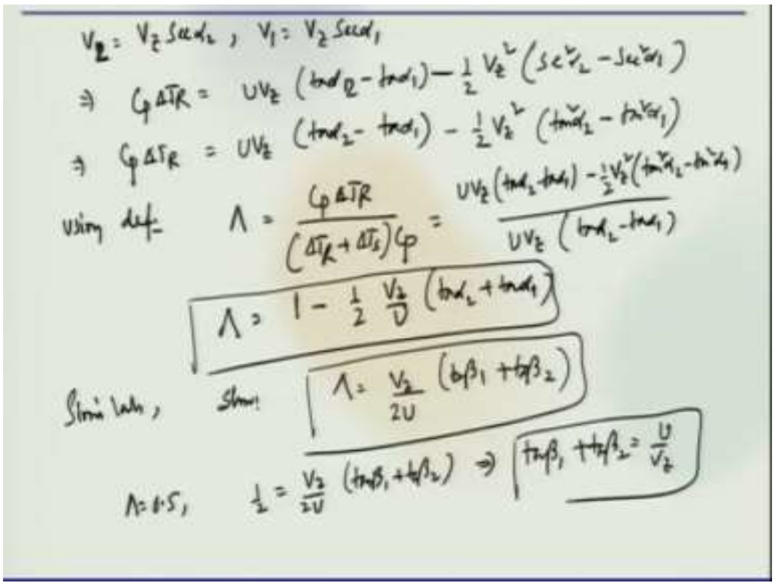


Now, we can replace

$$V_2 = V_z \sec\alpha_2$$

$$V_1 = V_z \sec\alpha_1$$

$$C_p \Delta T_R = UV_z(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}(V_z \sec\alpha_2{}^2 - V_z \sec\alpha_1{}^2)$$

so further simplifications it will get us

$$\Lambda = \frac{C_p \Delta T_R}{C_p(\Delta T_R + \Delta T_S)} = \frac{UV_z(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_z^2(\tan\alpha_2{}^2 - \tan\alpha_1{}^2)}{UV_z(\tan\alpha_2 - \tan\alpha_1)}$$

$$\Lambda = 1 - \frac{1}{2}\frac{V_z}{U}(\tan\alpha_2 + \tan\alpha_1)$$

so this is what one would get. So, similarly one can show that this would be

$$\Lambda = \frac{V_z}{2U}(\tan\beta_1 + \tan\beta_2)$$

so that is also possibly can be seen or done.

So, if you consider a special case where let us say 50 percent enthalpy rise in rotor and 50 percent in the stage term, then this would be 0.5, so which means

$$\frac{1}{2} = \frac{V_z}{2U}(\tan\beta_1 + \tan\beta_2)$$

which gives me back

$$\tan\beta_1 + \tan\beta_2 = \frac{U}{V_z}$$

okay.

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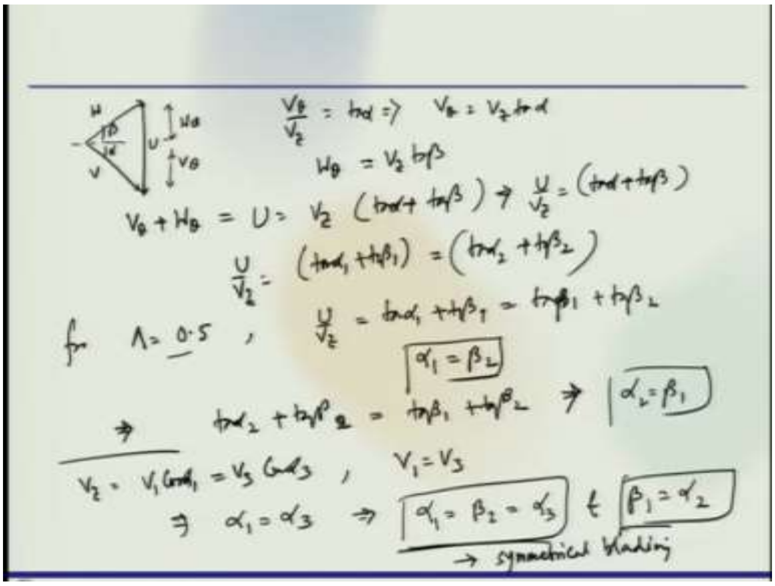

So, now going back to the velocity triangle; so if you go back to the velocity triangle, this is how it looks, so this is W, this is $\beta$, this is $\alpha$, V, this is U, so this portion is $W_\theta$, this portion is $V_\theta$, so what we get,

$$\frac{V_\theta}{V_z} = \tan\alpha$$

so that means

$$V_\theta = V_z \tan\alpha$$

and similarly,

$$W_\theta = V_z \tan\beta$$

So,

$$V_\theta + W_\theta = U = V_z(\tan\alpha + \tan\beta)$$

So, from where one can write

$$\frac{U}{V_z} = \tan\alpha_1 + \tan\beta_1 = \tan\beta_1 + \tan\beta_2$$

$$\tan\alpha_2 + \tan\beta_2 = \tan\beta_1 + \tan\beta_2$$

$$\alpha_2 = \beta_1$$

$$\alpha_1 = \beta_2$$

so these are the conditions when you have 50 percent of the degree of reaction. So, $V_z$ is constant throughout the stage, so since that is there, so one can write

$$V_z = V_1 \cos\alpha_1 = V_3 \cos\alpha_3$$

$$V_1 = V_3$$

so which will get

$$\alpha_1 = \alpha_3$$

So, that means

$$\alpha_1 = \beta_2 = \alpha_3$$

and

$$\alpha_2 = \beta_1$$

so this kind of design when we get these angles like this, so these are called this kind of design called symmetrical blading, okay, so the blade design will look like that.

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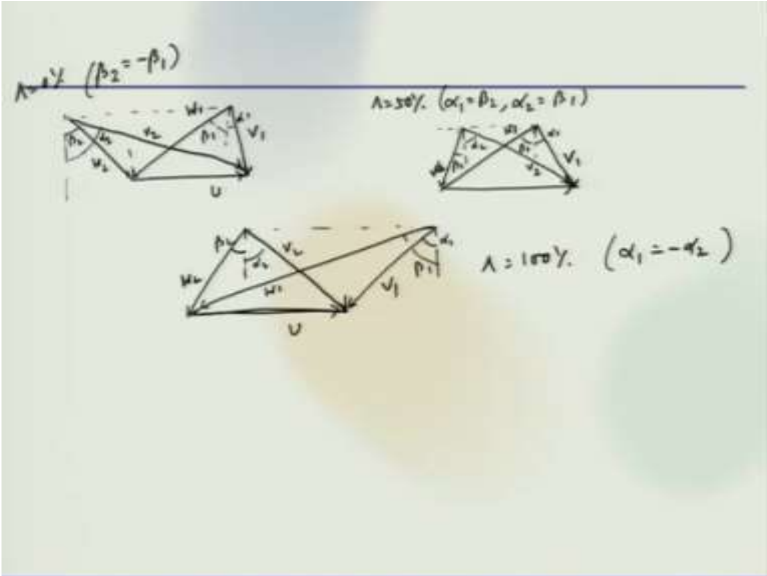


So, if you look at the different scenario, then this is how it is going to look like, let us say we have these, so this okay, so this is U, this is $V_1$, this is $W_1$, so this is $W_2$ and this component is $V_2$, so this is where, this is the case where is 0 percent and $\beta_2 - \beta_1$, so this is my $\beta_2$, this is going to be my $\alpha_2$ this is $\alpha_1$, this is $\beta_1$ so that is the situation where this is the case.

Now, when this is 50 percent that is the case

$$\alpha_2 = \beta_1$$

$$\alpha_1 = \beta_2$$

so my profile will look like different, so this will go like that so that is my $V_1$, $W_1$, so that is $W_2$, this is $V_2$ $\alpha_1$ $\beta_1$ $\alpha_2$ $\beta_2$, okay. So and the last situation which could possibly happen is that like this and I can have this, this is U, so this is $W_2$ $\beta_2$ $\alpha_2$ $V_2$ $W_1$ $V_1$, so this one $\alpha_1$, this one $\beta_1$ and this is the case where Λ is 100 percent, so $\alpha_1 = -\alpha_2$, so these are different velocity triangles for different degrees of reaction, so we will continue the discussion in the next lecture.