Introduction to Airbreathing Propulsion

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Lecture – 45
Centrifugal Compressor (Contd.,)

Let us continue the discussion on turbo machinery and we will continue the centrifugal compression and then follow with the other compressor.

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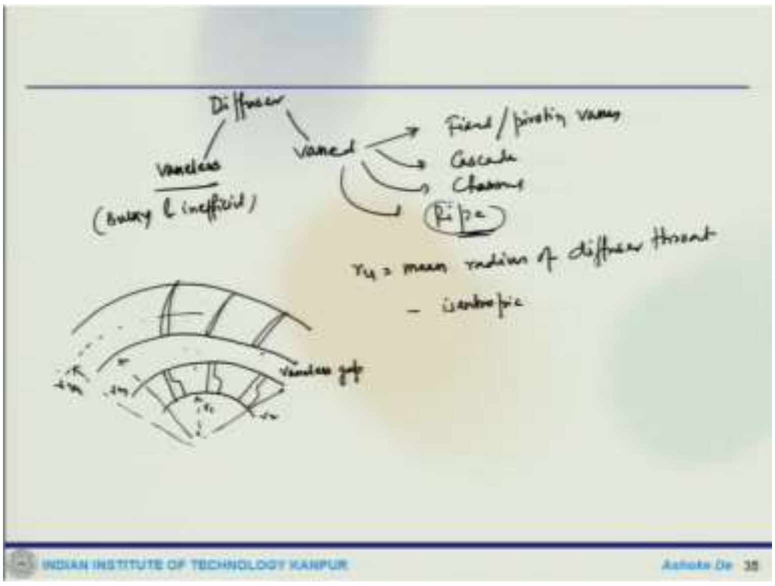

So this is where we actually stopped in the discussion in the last lecture so we are talking about the diffuser and vaneless diffuser and vane diffuser and this is the schematic of the geometry where we are talking about the gap which is a vaneless gap and the whole idea is that these are the different radius and radius and r_4 is the mean radius of the diffuser throat. So what now we are trying to look at is that it will pass I mean pass through these passages.

And finally we get the proper pressure rise so that is how it happens if the air does not enter smoothly that is one of the issue here if air does not enter smoothly then what will happen? There would be flow separation and because of the flow separation there would be performance degradation in the compressor. So, what is important is that design of the diffuser become important so this is an important task and out of that obviously vane is one component second is the throat.

So these are the two components so there could be vaneless space between the impeller under diffuser now because the air leaving the impeller has to traverse through this gap it direction may change because it may not remain same leaving the impeller tip. So, to prove the correct inlet angle of the diffuser vane so you need to consider these vaneless passes here in the design consideration.

Now since no further energy is supplied to the air once it is leaving the impeller that angular momentum must be conserved in this vaneless passage and what is that you can neglect frictional losses and the process is isentropic.

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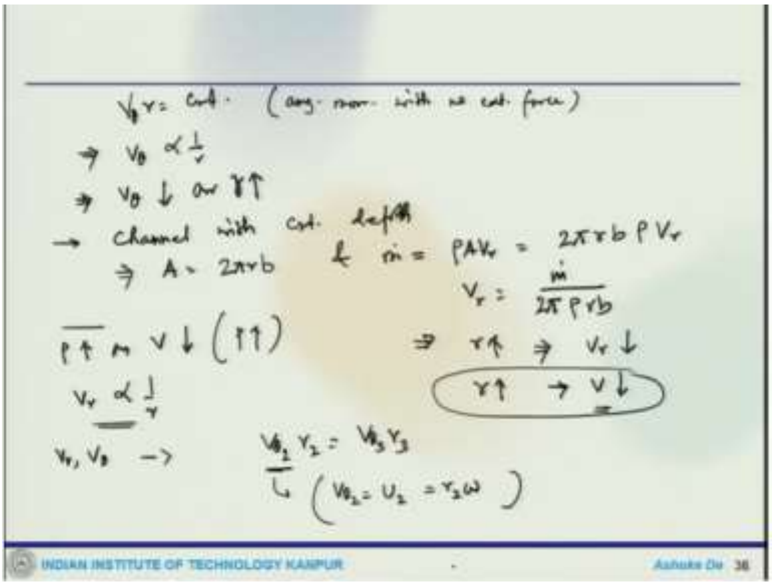

So how will I go about it? So theoretically there should

$$V_\theta r = constant$$

so this is your angular momentum with no external force okay? So, this gives you an idea how the

$$V_\theta \propto \frac{1}{r}$$

and one can see the V_θ will decrease as r goes up so the tangential compressor velocity decreases for impeller tip to diffuser. Now the flow passage in the channel of constant depth if we consider.

So the channel with constant depth so what would be the area then area would be

$$A = 2\pi rb$$

and the mass flow rate would be

$$\dot{m} = \rho A V_r$$

So, this one can write

$$\dot{m} = 2\pi r b \rho V_r$$

$$V_r = \frac{\dot{m}}{2\pi r b \rho}$$

So now here again you can see if r increases this will get you that V_r decreases. So, the radial component velocity also decreases from impeller tip to diffuser inlet and also when r actually increases so over all v also decreases.

So, some diffusion takes place in the vaneless space also okay so this is what is quite important that even in that vaneless space that you get some sort of a diffuser or the diffusion process to take place which means the flow field velocity that also decreases. However, one can see when the density actually increases as V decreases so that because your pressure actually increases and hence

$$V_r \propto \frac{1}{r}$$

so these variations depends on the variation of density also one has to calculate this by using that continuity equation.

So now if you have V_r and V_θ at the leading of the so V_r and V_θ at the leading is of the diffuser vanes are calculated the vane angle can be also estimated from the conservation of angular momentum how let us see so

$$V_{\theta 2} r_2 = V_{\theta 3} r_3$$

So $V_{\theta 2}$ is the tangential velocity at the impeller blade tip. So, already we can obtain that $V_{\theta 2} = U_2 = r_2 \omega$ so this one can get from the analysis of the impeller.

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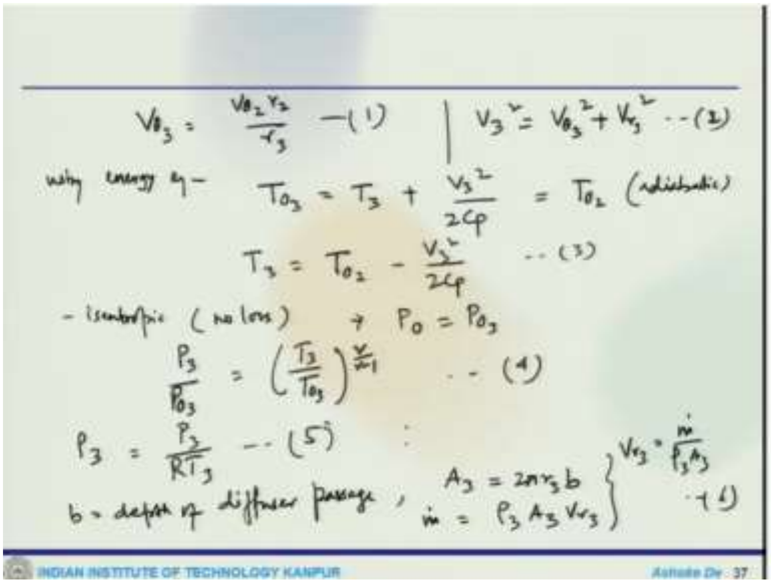

Now what we can get

$$V_{\theta 3} = \frac{V_{\theta 2} r_2}{r_3}$$

which is let us say equation 1 and one has

$$V_3^2 = V_{\theta 3}^2 + V_{r3}^2$$

so that is let us say 2. Now energy equation using energy equation one can write

$$T_{03} = T_3 + \frac{V_3^2}{2C_p} = T_{02}$$

Now

$$T_3 = T_{02} - \frac{V_3^2}{2C_p}$$

so that is let us say get this. Now we have said this process is isentropic which means no loss so we will get P_0 would be P_{03}.

So, one can write

$$\frac{p_3}{p_{03}} = \left(\frac{T_3}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}}$$

which is equation 4. And we have one more equation which is coming from density which is

$$\rho_3 = \frac{p_3}{RT_3}$$

now b is the depth of the diffuser passage so what will happen

$$A_3 = 2\pi r_3 b$$

and mass flow rate would be

$$\dot{m} = \rho_3 A_3 V_{r3}$$

$$\dot{m} = 2\pi r_3 b \rho_3 V_{r3}$$

$$V_{r3} = \frac{\dot{m}}{2\pi r_3 b \rho_3}$$

that is let us say equation 6.

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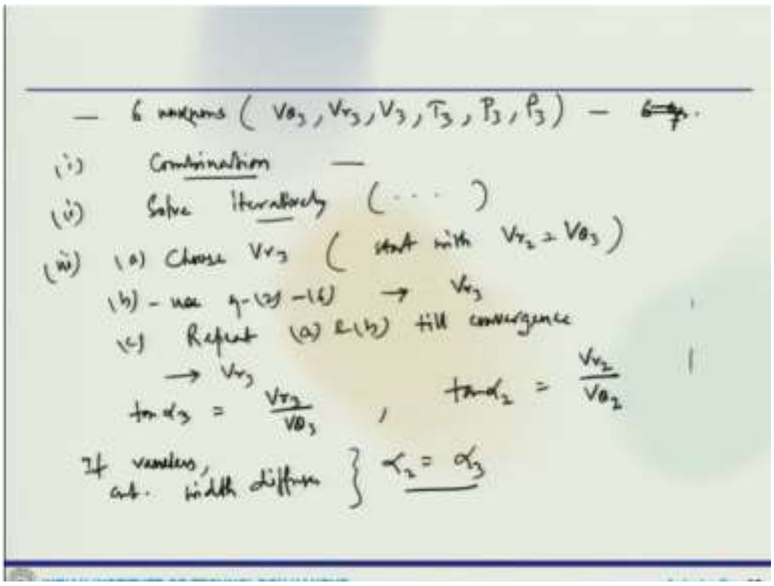

So here we get if you look at these equations there are 6 unknown which are $V_{\theta 3}, V_{r3}, V_3, T_3, p_3, \rho_3$ and we got 6 equations too. So, but equation is not independent so there are two ways one can solve it so number 1 one can combine the equation to form a single algebraic equation of one variable solve that equation so basically it is a combination process. So, one can combine that and in a single algebraic equation and to obtain one variable and then from there you back calculate the others.

Or one can do you can solve iteratively so if someone wants to check this out he may or she may be used to do that you can take this one as a task of some sort of homework and try to see that how it can be done or third approach one can do let us say we can first some guess value or choose some V_{r3}. So, you start with $V_{r2} = V_{\theta 3}$ then use equation 2 to 6 and you obtain V_{r3}. So, let us say this is step a, step b and this is step c.

We can repeat this a and b till convergence and then to get V_{r3} so once you have that that you can find out

$$tan\alpha_3 = \frac{V_{r3}}{V_{\theta 3}}$$

and

$$tan\alpha_2 = \frac{V_{r2}}{V_{\theta 2}}$$

So, if it is vaneless or constant with diffuser then α_2 will be α_3 so that is what you can get going ahead with the design process.

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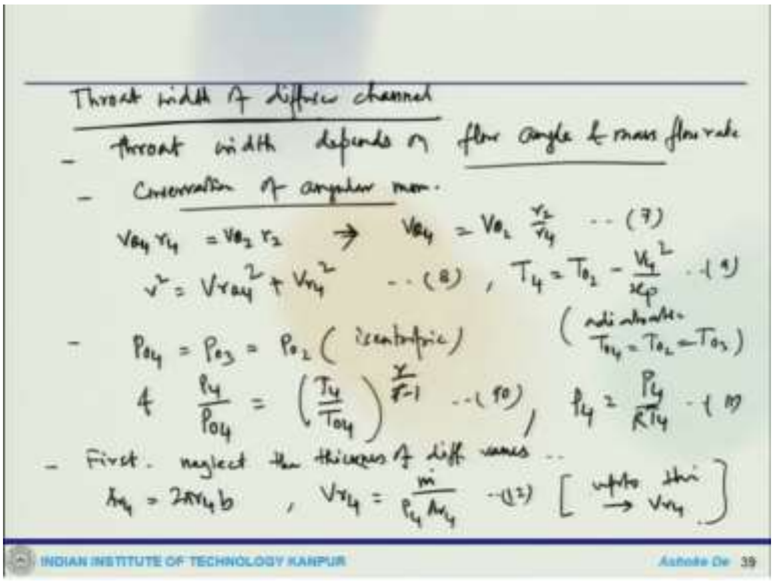

Now then we can look at the throat width of diffuser channel. So, this is another aspect of it that one can do so air entering the diffuser at its throat is also at an angle and hence the throat region also has to be designed accordingly so that is why this is important. Now the throat width actually that depends on flow angle and mass flow rate. So that is what it is again so one can do that using the conservation of angular momentum we can write that

$$V_{\theta 4}r_4 = V_{\theta 2}r_2$$

from here we will get

$$V_{\theta 4} = \frac{V_{\theta 2}r_2}{r_4}$$

Now again

$$V^2 = V_{\theta 4}^2 + V_{r4}^2$$

and we can write

$$T_4 = T_{02} - \frac{V_4^2}{2C_p}$$

that let us say equation. So, this is what we are writing because again it is an adiabatic so

$$T_{04} = T_{02} = T_{03}$$

so that we could write that kind of situation. Now again it is an isentropic flow so

$$p_{04} = p_{02} = p_{03}$$

because it is isentropic.

Now the same thing

$$\frac{p_4}{p_{04}} = \left(\frac{T_4}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

and you have

$$\rho_4 = \frac{p_4}{RT_4}$$

okay. So, one can start at the first case or first approximation one can say one can neglect the thickness of diffuser vanes. So, once you do that then it becomes quite simplified and one can write

$$A_{r4} = 2\pi r_4 b$$

and from there we will write

$$V_{r4} = \frac{\dot{m}}{A_{r4}\rho_4}$$

so one can do iteration to this to get V_{r4} so iteration can continue up to this.

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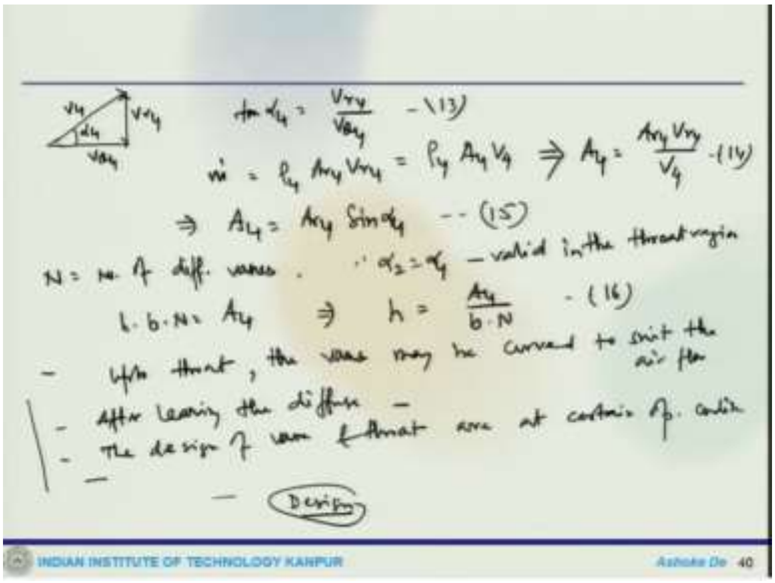

And to get that now if you look at that velocity triangle so this is how it will look like so this is V_4 this is α_4 this is $V_{\theta 4}$ this is V_{r4} so my

$$tan\alpha_4 = \frac{V_{r4}}{V_{\theta 4}}$$

$$\dot{m} = \rho_4 A_{r4} V_{r4} = \rho_4 A_4 V_4$$

$$A_4 = \frac{A_{r4} V_{r4}}{V_4}$$

that is 14. Now

$$\frac{V_{r4}}{V_4} = \sin \alpha_4$$

so we will write that

$$A_4 = A_{r4} \sin \alpha_4$$

Now let us say if n is the number of diffuser vanes so $\alpha_2 = \alpha_4$ is valid in the throat region okay so I can write h*b *N is A_4 from where I will get

$$h = \frac{A_4}{b * N}$$

So, from you can see the throat area is calculated okay now also one can note here the length of the diffuser passage also depends on the maximum possible diverging angle under required pressure rise. Now up to throat the vanes may be curved to swift the air flow. Okay and after the throat air is along the passage and control the walls of the passage could be straight again when it leaves the diffuser or after leaving the diffuser the air can be combined and put to a single combustor. Now all different diffuser bands maybe fit into diffuser combustor which is common to gas turbines and the design of vane and throat are at certain operating condition.

So other conditions apart from this the other conditions the flow will not be smooth. So, one can to find out all these details basically how much flow will deviate what kind of deviation that could be there so one can use detailed CFD analysis to find out that thing okay. So, this is how the design portion of this component to be looked at.

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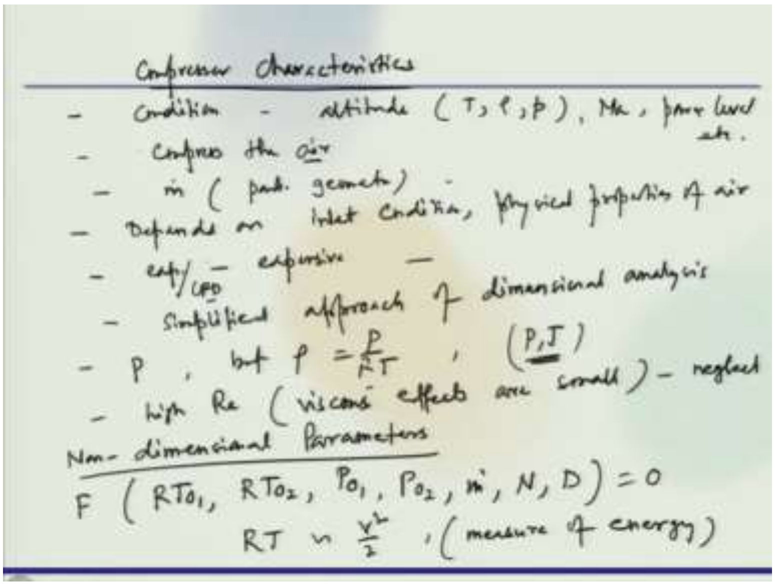

Now once we have that then the next important parameter is the compressor characteristics why this is important because end of the day these characteristics curve would determine what would be the operational range or what is the region where these any particular combustors should work or operate? So, beyond that point so the reason is that you have different conditions or operating conditions for example you have altitude and then you have temperature density pressure, Mach number you have power level.

So, all these are conditions which are coming from the real system or the real engine where the engine in build with the compressor is going to operate. So, as we have already talked about several times that the main idea behind the compressor is to compress the air or incoming air. So, it essentially pressurized them coming air dynamically and the pressure rise depends on mass flow rate for a particular it is a particular geometry okay.

Now performance of any compressor maybe specified by curves of delivery pressure and the temperature and also the rotational speed. So, these are all depends all these things depends on inlet conditions physical properties of air. So, if one has to look at the complete detailed analysis either it has to be done through proper experimental analysis in a rig with a proper or detailed instrumentation or one can go for the very high-fidelity computational approach and to look at that.

But any experimental procedure these are expensive with kind of detailed equipment or instruments or which is required to get this kind of curve. So at the same time any computational analysis also could be expensive quite a bit because these are all dynamic machines with rotating components at certain RPM and rotating. So the best approach which could be there is a simplified approach of dimensional analysis.

So, this is what one can do and find out these things so using dimensional analysis the complete characteristics of a compressor can be presented using two curves and with a variations of the other parameters. So, P is an important variable and also but density is connected with P so if we choose P and T as independent variable then density can be specified.

Now for the highly turbulent flow of a compressor also viscous effects are very small so that means high Reynolds number case where the viscous effects are small so we can neglect that so the viscous effects would be neglected. So, the non-dimensional parameters would be the a non-dimensional parameter space can be defined based on that. Now the function would be RT_{01}, RT_{02}, P_{01}, P_{02} mass flow rate these are 0. So D is the RT one can look at $\frac{V^2}{2}$ which is sort of an measure of energy.

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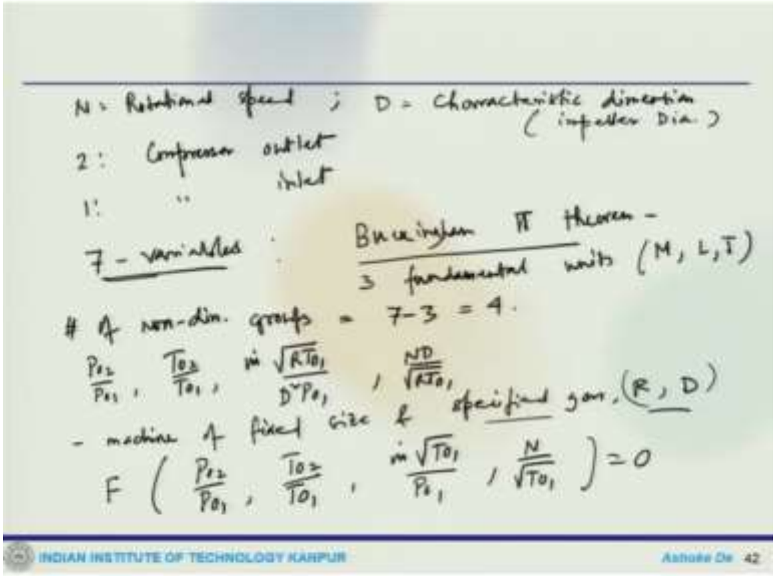

And the other parameters like N is rotational speed where D is the characteristics dimension. So, characteristics dimension so typically one can take the impeller Dia. So, that is what one can consider 2 is the compressor outlet one is compressor inlet so we have essentially if we look at this non dimensional parameter space we have 1, 2, 3, 4, 5, 6, 7. So we have 7 variables now we invoke the Buckingham pi theorem if we invoke that we got 7 variables and we have 3 fundamental units which are M, L and T.

So we need number of nondenominational groups would be 7 – 3 which is 4 and this nondenominational groups could be like $\frac{p_{02}}{p_{01}}, \frac{T_{02}}{T_{01}}, \frac{\dot{m}\sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}}$ Now when you consider a machine of fixed size so the machine of fixed size and consider the specified gas or so once you say that you take a fixed size machine and specify the gas then R and D these are invariable.

So, these R and D can be omitted from this non dimensional groups because we have considered it or chosen a particular gas. So, this will become then once we omitted that thing this will become

$$F\left(\frac{p_{02}}{p_{01}}, \frac{T_{02}}{T_{01}}, \frac{\dot{m}\sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}}\right) = 0$$

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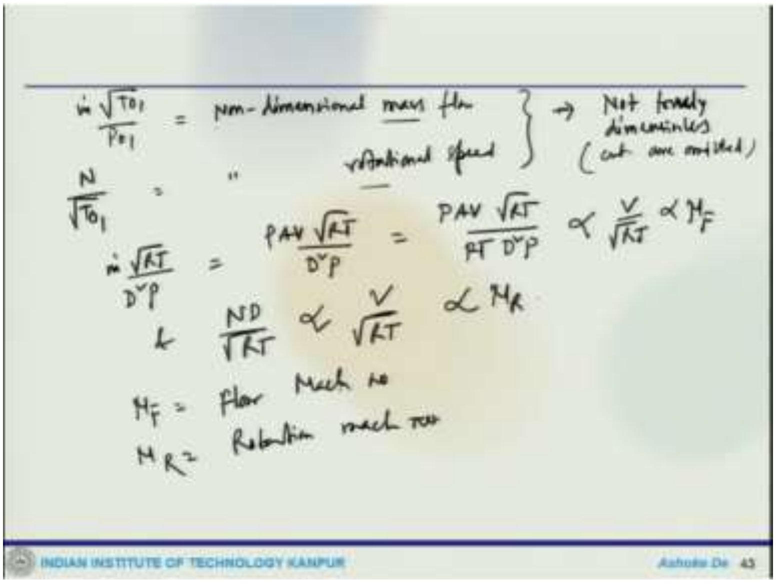

You can see in this nondenominational group one is that $\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$ which is non dimensional mass flow and then this is $\frac{N}{\sqrt{T_{01}}}$ which is non dimensional rotational speed. So, these are in this non-dimensional group these are the non-dimensional two parameters that would be important for further analysis one is the non-dimensional mass flow rate another is the non-dimensional rotational speed.

But if one look at it they are this is just one note they are not truly dimensionless since the constants are omitted okay. So, one can get the characteristics plot by plotting each of these group with respect to another while keeping the other parameters constant. Now just to not here this

$$\frac{\dot{m}\sqrt{RT}}{D^2 p} = \frac{\rho AV\sqrt{RT}}{D^2 p} = \frac{pAV\sqrt{RT}}{RTD^2 p} \propto \frac{V}{\sqrt{RT}} \propto M_F$$

$$\frac{ND}{\sqrt{RT}} \propto \frac{V}{\sqrt{RT}} \propto M_R$$

So, M_F is the flow Mach number and M_R is the rotational Mach number. So, these are the two different parameters that we get. So, we will stop it here and continue in the next session.