Introduction to Airbreathing Propulsion
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology – Kanpur

Lecture – 43
Centrifugal Compressor (Contd.,)

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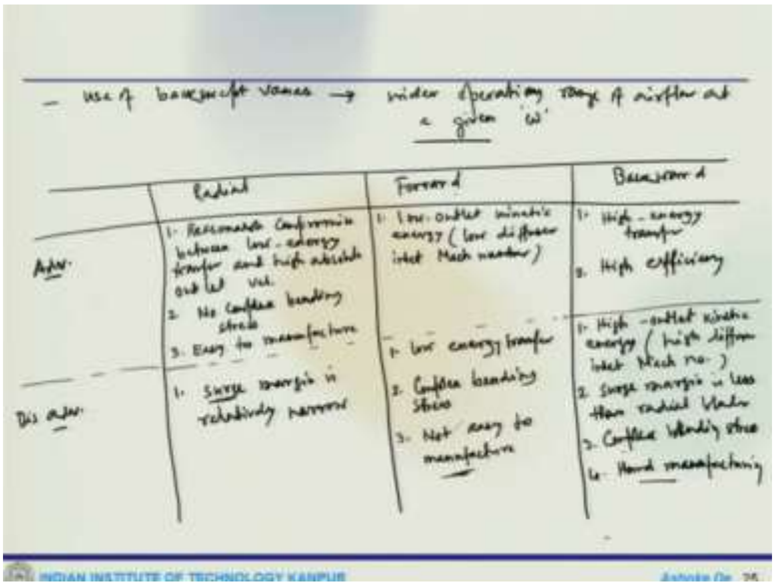

The compressor impeller and now we consider both together so this is where we have stopped.

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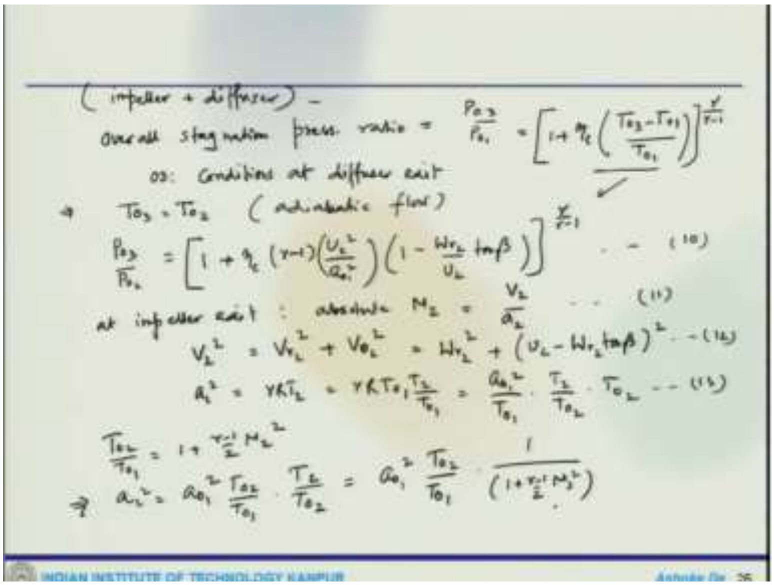

Now what we can do just we can consider both the impeller and the diffuser together and then analyse the system. So the overall stagnation pressure ratio so that would be

$$= \frac{p_{03}}{p_{01}} = \left[1 + \eta_c \left(\frac{T_{03} - T_{01}}{T_{01}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

So, this is η_c is the efficiency of the compressor where we have already 3, 1, 2 this 3 or is your condition at diffuser exit.

So they and 1 or 2 I have already known now we can at the same time we can say that

$$T_{03} = T_{02}$$

because this is adiabatic flow. So, once you have that claim that $T_{03} = T_{02}$ you can write this

$$\frac{p_{03}}{p_{02}} = \left[1 + \eta_c(\gamma - 1)\frac{U_2^2}{a_{01}^2}\left(1 - \frac{W_{r2}}{U_2}\tan\beta\right)\right]^{\frac{\gamma}{\gamma-1}}$$

So already you so this is the expression and now $T_{03} = T_{02}$ then the ratio of the pressure temperature rises which we have already derived so that is replaced here.

Now at impeller exit so that is the condition 2 so the absolute Mach number is M_2 which will be

$$M_2 = \frac{V_2}{a_2}$$

and we can write all the other information related to the velocity triangle for example

$$V_2^2 = V_{r2}^2 + V_{\theta 2}^2 = W_{r2}^2 + (U_2 - W_{r2}\tan\beta)^2$$

So what this so one has to be careful I mean in the sense you have to be kind of well aware of the velocity triangle at different location.

Because all these what we are writing from the vector calculus nothing else and

$$a_2^2 = \gamma R T_2 = \gamma R T_{01}\frac{T_2}{T_{01}} = \frac{a_{01}^2}{T_{01}}\frac{T_2}{T_{02}}T_{02}$$

so that is 13 and we have

$$\frac{T_{02}}{T_{01}} = 1 + \frac{\gamma - 1}{2}M_2^2$$

so that gives me

$$a_2^2 = a_{01}^2\frac{T_{02}}{T_{01}}\frac{T_2}{T_{02}} = a_{01}^2\frac{T_{02}}{T_{01}}\frac{1}{1 + \frac{\gamma - 1}{2}M_2^2}$$

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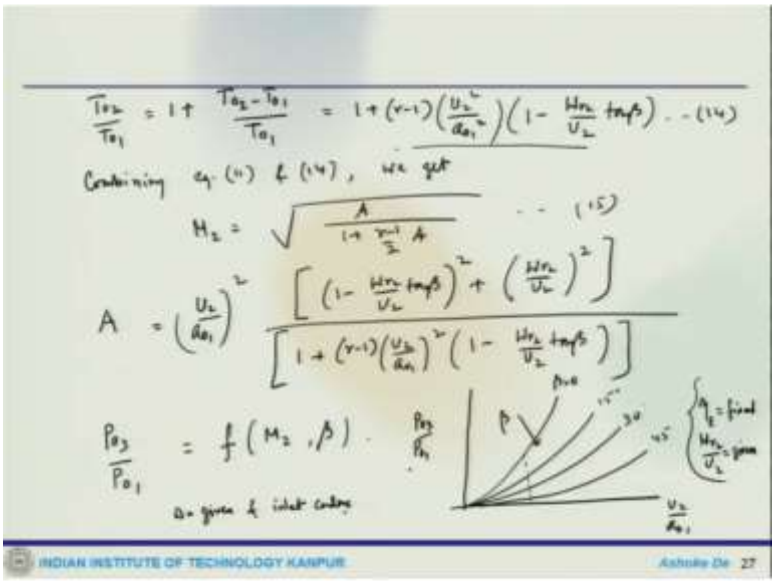

So if we rather write in a slightly in a different way so

$$\frac{T_{02}}{T_{01}} = 1 + \frac{T_{02} - T_{01}}{T_{01}} = 1 + (\gamma - 1)\frac{U_2^2}{a_{01}^2}\left(1 - \frac{W_{r2}}{U_2}\tan\beta\right)$$

Now what you can do whatever we have obtained so far now if you combine equation 11 and 14 what you get is that

$$M_2 = \sqrt{\frac{A}{1 + \frac{\gamma - 1}{2}A}}$$

So where a is the expression which involves say all this term that we have derived.

So this would involve

$$A = \left(\frac{U_2}{a_{01}}\right)^2 \frac{\left[\left(1 - \frac{W_{r2}}{U_2}\tan\beta\right)^2 + \left(\frac{W_{r2}}{U_2}\right)^2\right]}{\left[1 + (\gamma - 1)\frac{U_2^2}{a_{01}^2}\left(1 - \frac{W_{r2}}{U_2}\tan\beta\right)\right]}$$

So that is what you have now one can find out this dependence of now

$$\frac{p_{03}}{p_{01}} = f(M_2, \beta)$$

So different change of angle you will have different situation so one can look at the plot how this would look like for example let us say if I have to plot here $\frac{p_{03}}{p_{01}}$ and this side is the $\frac{U_2}{a_{01}}$.

So the plots will go like this. So this is how the things would change so that is how you are increasing beta this might be β 0 this could be 15 this could be 30 this could be 45 I mean for particularly let us say η_c is fixed or given and also for the ratio of the $\frac{W_{r2}}{U_2}$ also something given.

So when these two parameter are given then you can see this and this is could be somewhere the line belongs.

Now for same blade and inlet condition so $\beta = 0$ which this curve actually tells you that for a given blade speed where the RPM is given and Inlet conditions are known then $\beta = 0$ which is the radial blade is the maximum pressure ratio that it gives so for a particular ratio of the $\frac{U_2}{a_{01}}$ this guy is giving you the maximum one. So, this is what it tells.

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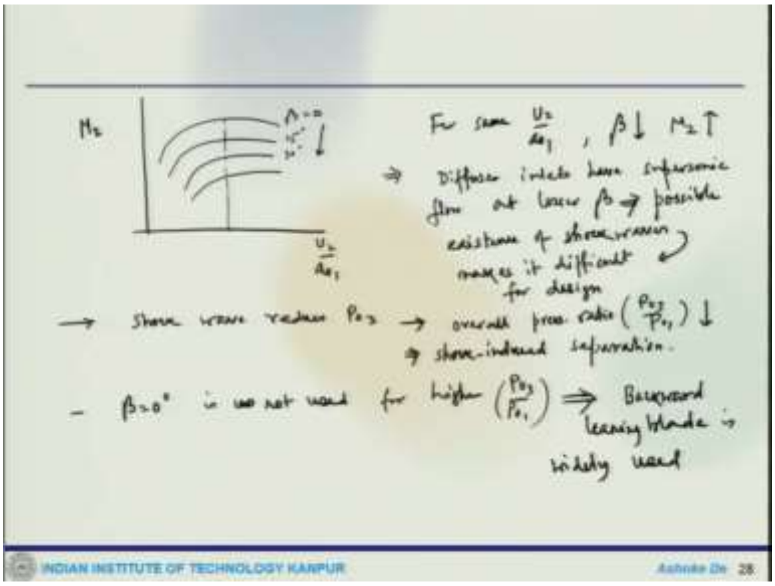

Now I can or one can look at the variation of Mach number M_2 with the $\frac{U_2}{a_{01}}$ so that also varies in a particular way. So like this this varies so again you can see the situation where this is again β 0 and that is the increasing β so this could be 15, 30 so this how the M_2 varies and all these so one can say for same $\frac{U_2}{a_{01}}$ which is let us say this is a particular point.

So, when beta decreases M_2 increases so that is how it happens I mean 0 is the maximum Mach number that you have at the exit location. So that means the diffuser inlets have supersonic flow at lower β. So, once you say that when the diffuser Inlet have supersonic flow at lower beta. So, this will lead to immediately another problem there could be the occurrence or the existence of possible existence of shockwaves.

So that in turn makes the design difficult makes it difficult for design. So, these are interlinked if you look at the particular blade configuration or what a kind of blade configuration one should opt for these are sort of related to the flow conditions and what you have so when you have shockwaves there would be other consequences as well. So, shockwaves obviously reduces p_{03} so which in turn the overall pressure ratio that is $\frac{p_{03}}{p_{01}}$ which also decreases.

So, when the shock wave would be there because of these different conditions so p03 actually reduces and once p_{03} reduces it will reduce the overall pressure ratio. So, this is called the shock induced separation okay. So, this particular phenomena often known as so one can say for beta 0 is not preferred or used for higher $\frac{p_{03}}{p_{01}}$ that means if you are mean for higher pressure ratio then one should not use the radial blade okay.

So, what it allows or rather paves the way for using the most preferred one is the backward leaning blade. So that is what backward leading blade is widely used or preferred or whatever you can think. So, all these are having issues with different kind of blades now as we have already discussed these particular components and all these things they are exposed to the higher angle of rotation.

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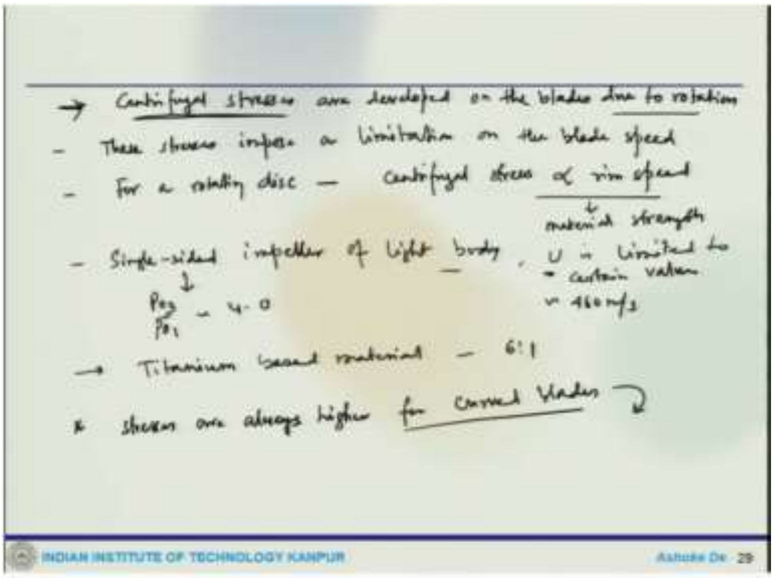

So there are issues of the centrifugal stresses. So centrifugal stresses are developed and that because of the rotation so this on the blades due to rotation. So this is one of the ongoing problem or always being a problem for this kind of rotating machines because of the rotation the blades or the any structure which is there they are exposed to this rotation they facing I mean they face this kind of high stresses so that is one aspect of it that one should know there are centrifugal stresses which are developed on the blades due to the rotation.

So, what that leads to because this has other bottlenecks like these stresses which are developed, they impose a limitation on the blade speed. So once you have lower blade speed you will have lower level of stresses if you have higher blade space I mean this is again we are talking about in terms of let us say keeping the RPM is fixed in that case if you reduce blade speed stress would be less if you increase the blade speed your stress level would go high.

Secondly for let us say the rotating disc so that what happens this centrifugal stress are proportional to the rim speed. So that also kind of lead to the design for the material strength. So that is another important or the choice of material or the what kind of material should be chosen for these kind of things that also kind of dictated by this kind of things. Now other thing is that if you have let us say single sided impeller of light body then obviously u is limited to certain values limited to certain values.

And if you have certain values that will also give you a certain overall pressure ratio and typically single sided impeller this is somewhere around 460 meter per second which will limit these things to around 4 and now if you use some kind of a let us say titanium based material. So that will also lead to these stresses are always higher on for curved blades so that is another important I mean the stresses are always higher for curved blades. So that is another issue so once you have that in turn so that poses a limitation on pressure. So, these are the certain things.

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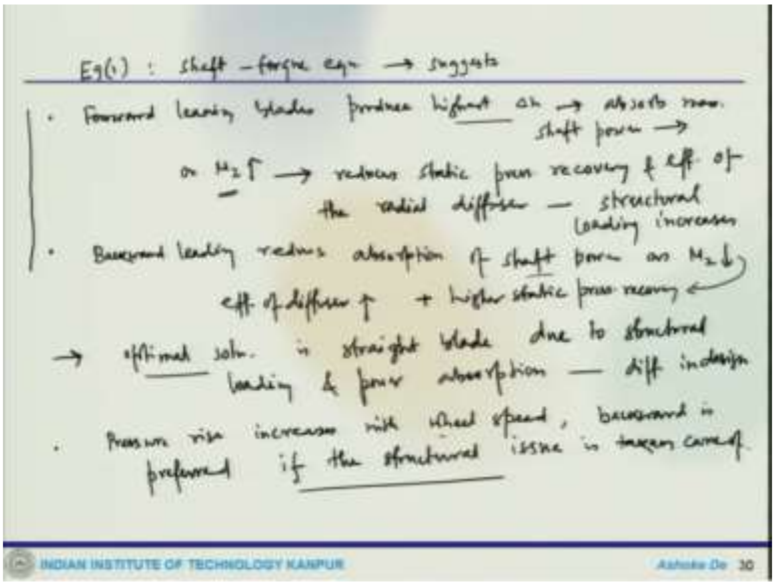

Now if you have some slightly more discussion on the other things like what you look at the from equation 1 which is our shaft torque equation which clearly suggests few thing that number 1 the forward-leaning blades produce highest delta h okay so which means it can absorb maximum shaft power and that because as M_2 increases which reduces static pressure recovery and efficiency of the radial diffuser.

So that means the forward-leaning blades if you use that produce the highest delta h so that it is able to absorb the maximum shaft power and that happens because they M_2 increases. So once the M_2 increases that reduces static pressure recovery and efficiency of the radial diffuser which also means that the structural loading also increases. So structural loading increases at the same time okay.

Now when you compare with the backward leading case so the backward leading blades that reduces observers reduces absorption of shaft power as M_2 decreases. So, which will give us higher static pressure recovery plus efficiency of diffuser also goes up. So there is a one-to-one comparison what happens with the forward leading blade and what happens with the backward leading blade.

So, once you increase the M_2 so one case for forward leading it absorb maximum shaft power but the backward leaning reduces the absorption of the shaft power. So once that happens this gives you the higher static pressure recovery and the efficiency of the so I mean if you see there are issues with this so that way one can always claim that the optimal solution is sort of an using it straight blade due to structural loading and power absorption.

So that is there so that is what the optimal solution but this will have difficulties in design because there are issues that already we have discussed that could be the shockwave. Now the last one more point which one can note about the pressure rise so the pressure rise increases with full speed okay so that way if you look at it so obviously the backward leaning blade would be backward is always preferred if the structural issue is taken care of.

So if you can take care of the structural part or issues which are related to the structural part then one can always go for backward because with this rotational speed you can have a higher pressure region and that is what essentially you want because the final objective of this is that you want to get higher pressure rise.

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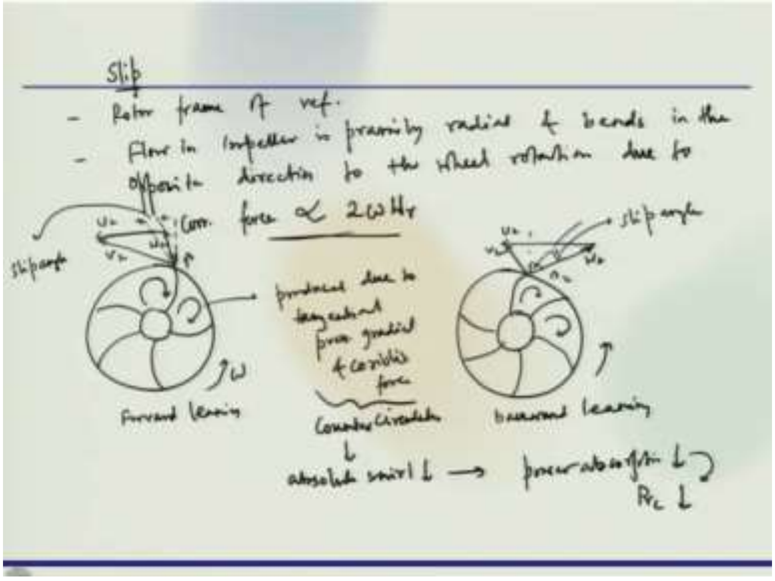

Now we introduce another thing which is important here is called slip. So, this is pretty much since we are in the this happens in the rotor frame of reference okay now what happens is the flow in the impeller is primarily radial and it bends in the opposite direction to the wheel rotation due to Coriolis forces which will be proportional to $2\omega W_r$ So that is another now new thing which is coming into the picture the existence of the Coriolis forces and that would happen things are rotating frame of reference.

So, the counter circulation in the impeller passage toward trailing edge is produced. So, one can I mean we can draw this picture and then explain is a little bit better let us say we have a blade like this so we go like this so these are the so this is the rotation omega so this is obviously forward-leaning. So, if I draw the triangle this is what it goes out then this would be the another vector component and finally this so this u_2 v_2 w_2 and the so this is the angle β.

So, this would be the angle β and there could be a situation which is so let us draw this little bit better way so you get okay this is my w_2 and I can have as per the angle. So, this is another portion which is called the so this guy is the slip angle so that angle is the slip angle so similar thing one can look at for a backward leaning blade. So we can draw like this and this should go like this you will have that is our velocity vector so this is u_2 v_2 w_2 then you have this so this one is β_2 and this angle is the essentially this angle is called the slip angle okay so what happens so that when the rotation is in this direction.

So, this is your backward leaning so what happens is there that when there is a rotation and this is let us say the forward so you have a circulation like this counter circulation. So, these are the counter existence of this case also like this so there is a counter circulation which exists there. Now this counter circulation in the impeller passage towards the trailing edge so this is leading and this is trailing towards the tailing edge is produced due to the presence of tangential pressure gradient and Coriolis force.

So, these are produced due to tangential pressure gradient and Coriolis force okay now what it does so when you have this counter circulation so this is what the counter circulation. Now this counter circulation because when there are counter circulation this also causes in reaction in absolute swirl. So absolute swirl there is a reduction therefore reduction in the power absorption.

So, power absorption and in turn P_{rc} also goes down so what essentially it happens is that you have this existence of this counter circulation which is causing this kind of problem. So, you have these blade passages and that because of the rotation you one you have the Coriolis forces and then there would be slip so we will stop here and finish this discussion on the slip in the next lecture.