Introduction to Airbreathing Propulsion
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Lecture – 41
Centrifugal Compressor (Contd.,)

Okay so let us continue the discussion on centrifugal compressor. So what we started up is the difference between different turbo machines or why they call the dynamical system and then we start off with the compressor cum centrifugal compressor, so where we stopped here.

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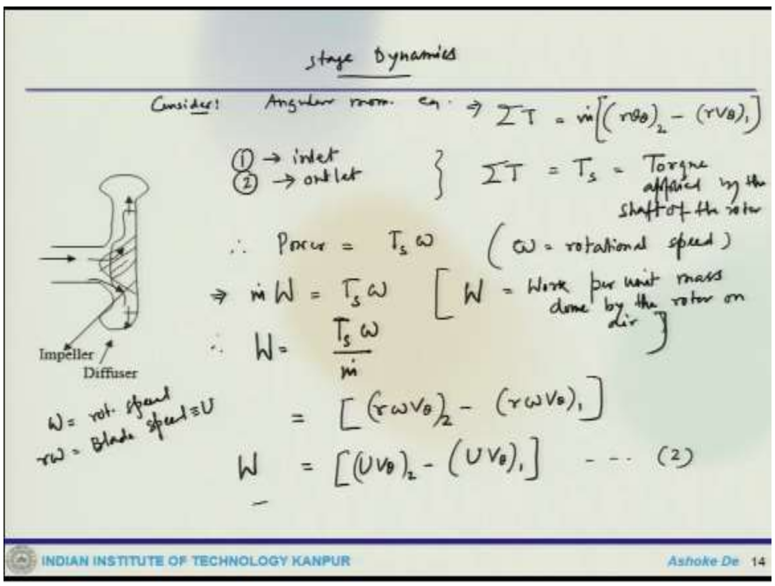

And last; these things with discussing about the stage dynamics and just to get you on board this is where we actually got the equation that can be applied depending on the inlet and outlet station, so this is where we did.

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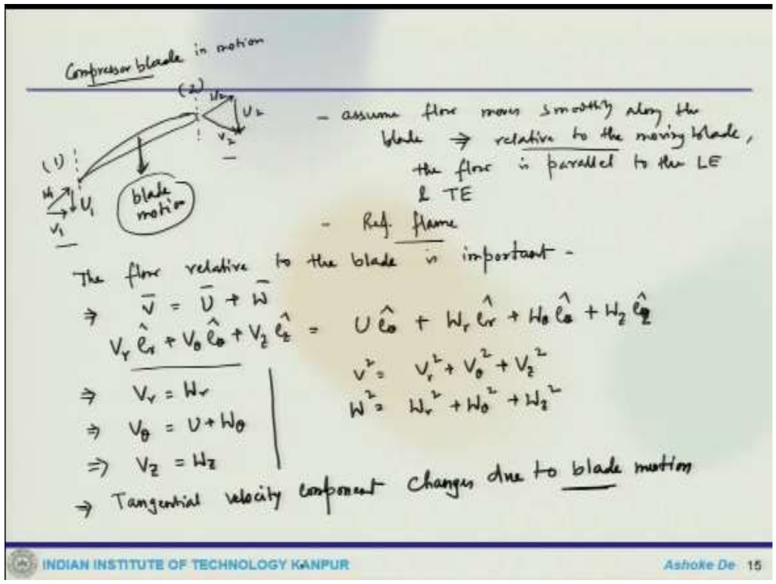

Now let us consider, so let us say this is a compressor blade so consider this as a compressor blade which is in motion. And this is the blade direction of the blade motion and this section is the inlet which is the; and the; going out at station 2, so these are all your; at the trailing edge and the leading edge and these are your typical velocity triangle. So if we assume the flow moves smoothly along the blade which is essentially relative to the moving blade so; and the flow is parallel to the leading edge and trailing edge.

So this is a very important statement to understand because when you say that the flow goes smoothly along the blade but this is again relative to the moving blade. Here is a situation which is slightly different compared to our stationary component. Here the blade is in motion, so this is an very important, so there is a rotation which is associated with that and so what makes the things difference is that what kind of reference frame that we used for analyzing this kind of system because one can use the relative frame of reference, one can convert things to a different frame of reference and look at the things. So this statement is quite important.

Now what you can have the flow relative to the blade is also important. This is also quite important. And what we have from the velocity triangle, you can have

$$\bar{V} = \bar{U} + \bar{W}$$

Now if you expand this, this is

$$\bar{V} = V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{e}_z$$

so these are the different component of velocity triangle or the velocity vector so these are the component. Then U is in the theta direction only because this is the blade motion. Similarly, W will have 3 component like

$$\bar{W} = W_r \hat{e}_r + W_\theta \hat{e}_\theta + W_z \hat{e}_z$$

$$V_r \hat{e}_r + V_\theta \hat{e}_\theta + V_z \hat{e}_z = U \hat{e}_\theta + W_r \hat{e}_r + W_\theta \hat{e}_\theta + W_z \hat{e}_z$$

So which straightaway give you

$$V_r = W_r$$

and also

$$V_\theta = U + W_\theta$$

and similarly

$$V_z = W_z$$

So these are the things what one can get out of this velocity vector of the velocity vector triangle. And what about V square, this is nothing but

$$V^2 = V_r^2 + V_\theta^2 + V_z^2$$

$$W^2 = W_r^2 + W_\theta^2 + W_z^2$$

so all the; so there is; now we are going back to the sort of an vector or the calculus here.

What is important here to note is that the tangential velocity component that changes due to changes due to blade motion. So this is a very, very important statement to note here. So now we have an situation where we get this relationship between the velocity vector and all this. Now moving ahead, we look at the energy equation.

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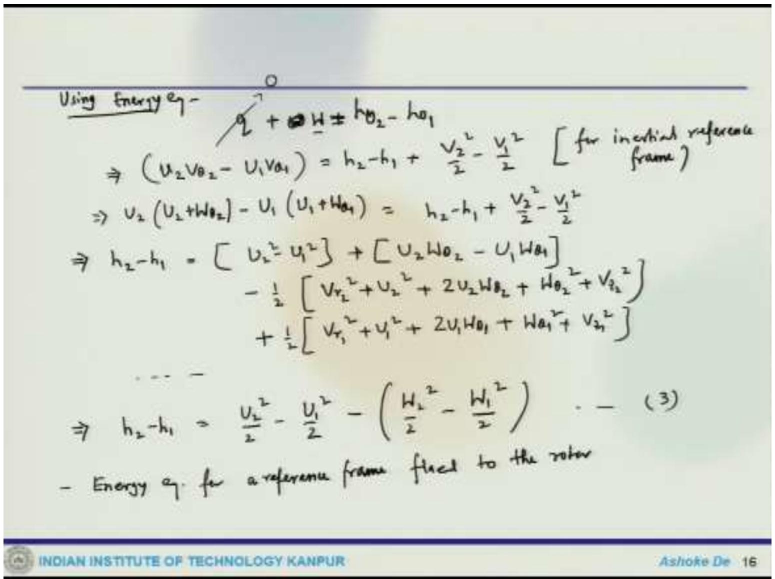

So we using the energy equation what we get, so we write down that

$$q + W = h_{02} - h_{01}$$

So there is no heat transfer takes place so that goes off then what we get

$$(U_2 V_{\theta 2} - U_1 V_{\theta 1}) = h_2 - h_1 + \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

So one can note here this is what we are writing for inertial frame of reference or reference frame whatever you call it.

So that is quite important to note what we are writing and this work, this is exactly what we have obtained per unit mass that is what we are writing there. So it just expanding this term; now if you do little bit of algebra finally what you can do you can do it yourself these few more lines and finally after rearrangement what you can get

$$h_2 - h_1 = \frac{U_2^2}{2} - \frac{U_1^2}{2} - \left(\frac{W_2^2}{2} - \frac{W_1^2}{2}\right)$$

So this is what you get and this is let us say equation number 3, okay. So that is what you get when you apply the energy equation. So now one can do that; so this is what you get for example this is your energy equation for a reference frame fixed to the rotor, okay. So this is what you get for that thing, so that means you are applying to a streamline observed in that coordinate system.

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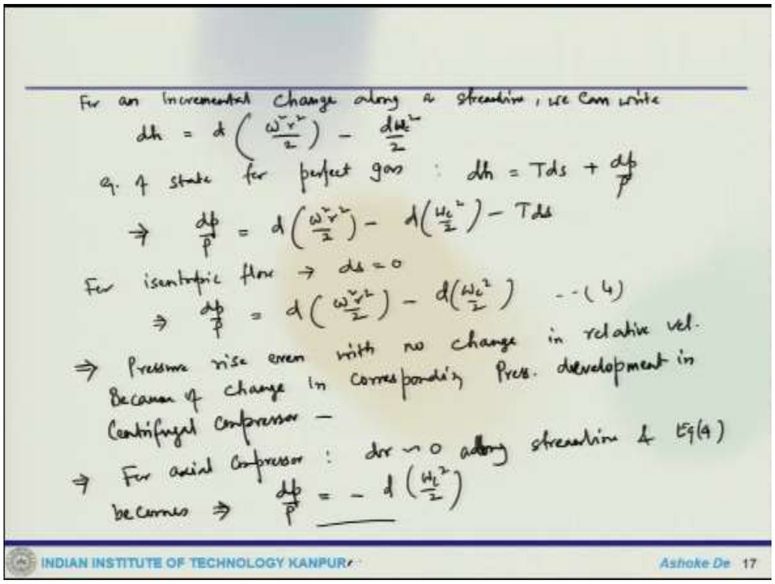

Now what I can write that let us say, for an incremental change along a streamline, we can write

$$dh = d\left(\frac{\omega^2 r^2}{2}\right) - \frac{dW_c^2}{2}$$

or this W is basically work, so you can put W_c just to avoid any confusion because W_c that will be the work of the compressor. Now if you use equation of state for perfect gas so equation of state for perfect gas what I can write

$$dh = Tds + \frac{dp}{\rho}$$

so which in turn you can write

$$\frac{dp}{\rho} = d\left(\frac{\omega^2 r^2}{2}\right) - \frac{dW_c^2}{2} - Tds$$

Now if you assume or for isentropic flow, so this is going to be $ds = 0$, so this system of equation becomes

$$\frac{dp}{\rho} = d\left(\frac{\omega^2 r^2}{2}\right) - \frac{dW_c^2}{2}$$

so that is equation 4. So which tells you there is an pressure rise even with no change in relative velocity. So this happens because of change in corresponding pressure development in the centrifugal compressor. So this is what you can always conclude that there is an pressure rise even there is no change in relative velocity.

Now at the same time one can have for axial compressor, this dr is essentially 0 along streamline and this equation 4 that becomes

$$\frac{dp}{\rho} = -\frac{dW_c^2}{2}$$

So when you look at this one can clearly see for axial compressor rotor one can obtain a pressure rise.

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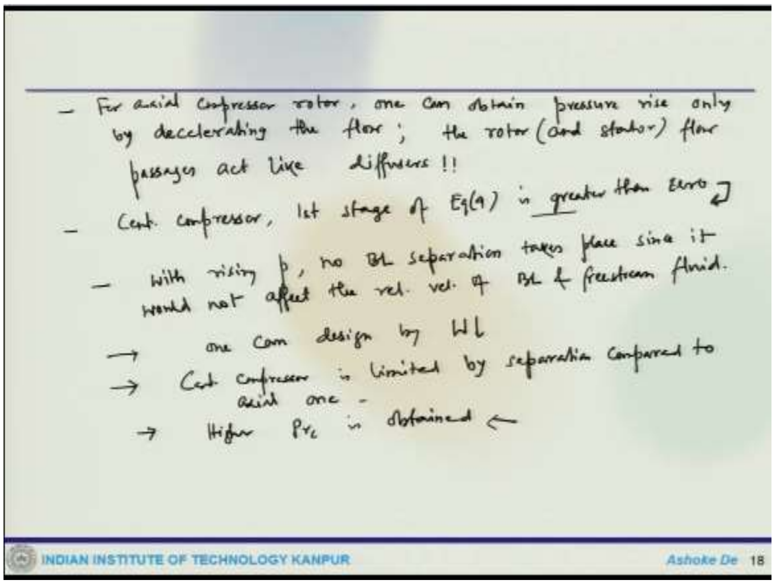

So you can see that for axial compressor rotor one can obtain pressure rise only by decelerating the flow which means the rotor as well as the stator or and stator flow passages act like and diffuser, flow passages act like diffusers. So this is quite natural once you look at these particular equations and what happened I mean this also brings the difference between the axial flow compressor and the centrifugal compressor.

Now on the other hand in centrifugal compressor first stage of equation 4 is greater than 0, so which in turn means that the pressure rise could develop in the rotor even there were no change in the relative velocity W, so this is what it means. That means with rising p, no boundary layer separation takes place since it would not affect the relative velocity of boundary layer and free stream fluid.

So the design while doing the design one can do the design so one can design the system by reducing W. And also it is important to note there here is that centrifugal compressor is limited by separation compared to axial one. So centrifugal compressor is limited by separation compared to axial one which means one can get higher Prc is; so that straight to a explain why a single-stage centrifugal compressor can provide or can get you higher pressure rise compared to axial one. Now let us move to the rotor and see.

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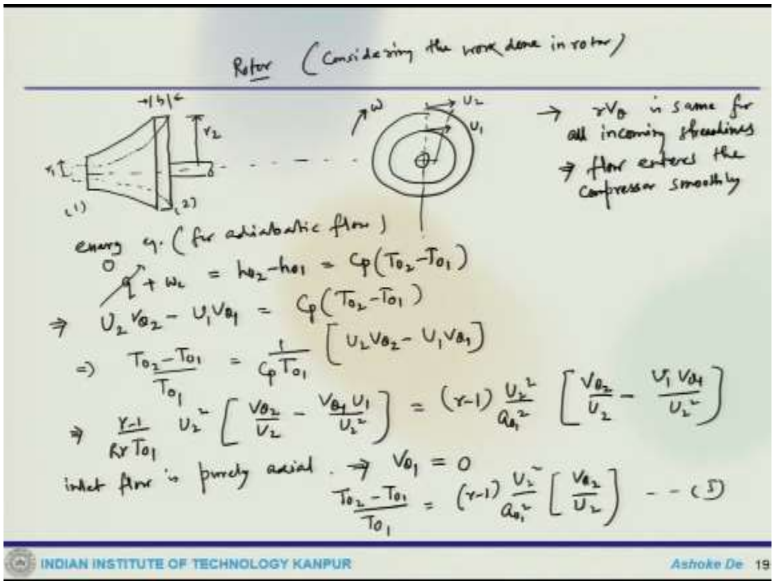

So let me draw the picture first. So we have these then; so that is how it goes, so you have this one here, this one here so let us say this is U_2 this is U_1 and this is how rotation is there, so I can have an system like this; here the shaft is sitting there and this would be; this is the schematic of the section. So that 2, this is 1, so this width you can think about B then this radius is r_2 then this would be r_1 so, so this is the rotor, so we are considering the work done in rotor, okay.

So what you; there is an assumption that the rV_θ is same for all incoming stream lines, so which says that the flow enters the compressor smoothly. So this is what we get, so that means the flow enters the compressor smoothly. Now we write the energy equation for adiabatic flow

$$q + w_c = h_{02} - h_{01} = C_p(T_{02} - T_{01})$$

So what we have is that

$$U_2 V_{\theta 2} - U_1 V_{\theta 1} = C_p(T_{02} - T_{01})$$

what we get

$$\frac{T_{02} - T_{01}}{T_{01}} = \frac{1}{C_p T_{01}}(U_2 V_{\theta 2} - U_1 V_{\theta 1})$$

Now you can expand this term which one can write

$$\frac{\gamma-1}{R\gamma T_{01}}U_2^2\left(\frac{V_{\theta2}}{U_2}-\frac{U_1V_{\theta1}}{U_2^2}\right)=(\gamma-1)\frac{U_2^2}{a_{01}^2}\left(\frac{V_{\theta2}}{U_2}-\frac{U_1V_{\theta1}}{U_2^2}\right)$$

Now typically the inlet flow is purely axial so that means $V_{\theta1}=0$. So when the inlet flow is axial this; so that get me to this equation

$$\frac{T_{02}-T_{01}}{T_{01}}=(\gamma-1)\frac{U_2^2}{a_{01}^2}\left(\frac{V_{\theta2}}{U_2}\right)$$

so that is our equation 5.

So the whole thing when we assume the inlet flow is axial this is what we get for the rotor. Now it depends what kind of blade arrangement you have. So there could be possibly three different kind of blade arrangement.

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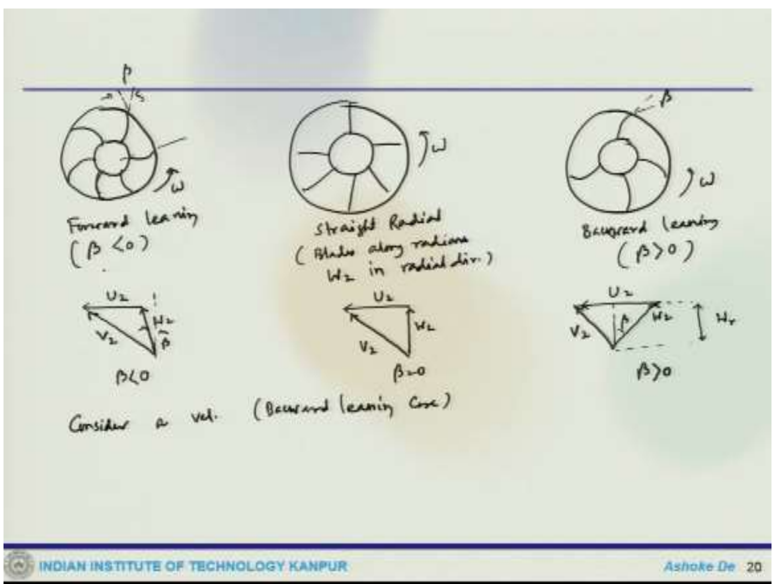

One possible is that you can have like this kind of blade arrangement like this, so this goes like this, so this is the angle called β and you can have; then this called forward-leaning where $\beta < 0$. So the other case one can have is that straight radial kind of blades. So this is the rotation, so this is straight radial or third option could be the backward-leading that means the blade could be arranged in this direction still the rotation in this direction.

So this could be the angle beta. Here this is called backward-leaning that means $\beta > 0$. So here if you look at this for the forward-leaning the blade curvature is in the or the rather the blade curved in the direction of rotation, so this is where the blades along radians W_2 in radial direction and the backward-leading when $\beta > 0$ blades curve in the direction opposite to the rotation.

So if you look at the velocity triangle for this forward-leaning you can have; this is the rotational direction, so what you can have is this, this and; so this would be U_2 V_2 and W_2. So then this case it would be straight radial, so this is what it is so your V_2 W_2 V_2. So this case $\beta = 0$ and so this is the angle beta so here the $\beta < 0$ and this case you will have W_2 and then so this is U_2 V_2 W_2 and this is my β and this should be my W_r component and this case $\beta > 0$.

So when you have different kind of setup of the blade or the different configuration or arrangement of the blades so you will have different velocity triangle and forward-leaning or the straight or the backward-leaning they will have different kind of velocity triangle to represent the flow field when it passes through the rotor. Now depending on this kind of different blade configuration not only the flow physics will change also the dynamics of the system is going to change and what kind of work you get out of this system.

So let us consider an a velocity triangle or a backward-leaning case, okay and we will look at the detail analysis what happens when you take that kind of blade. So we will stop here and continue the discussion in the next lecture.