Introduction to Airbreathing Propulsion

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Lecture – 04
Review of Fluid Mechanics

So, let us continue the discussion, now we have already seen the overview of the basic jet propulsion or aircraft engine they are different operation and all these and how one can very simplistic way, can calculate the thrust. Now, we will start or continue our discussion on the some of the basic understanding of fluid mechanics and then followed by some basic laws of thermodynamics and some of the compressible flows.

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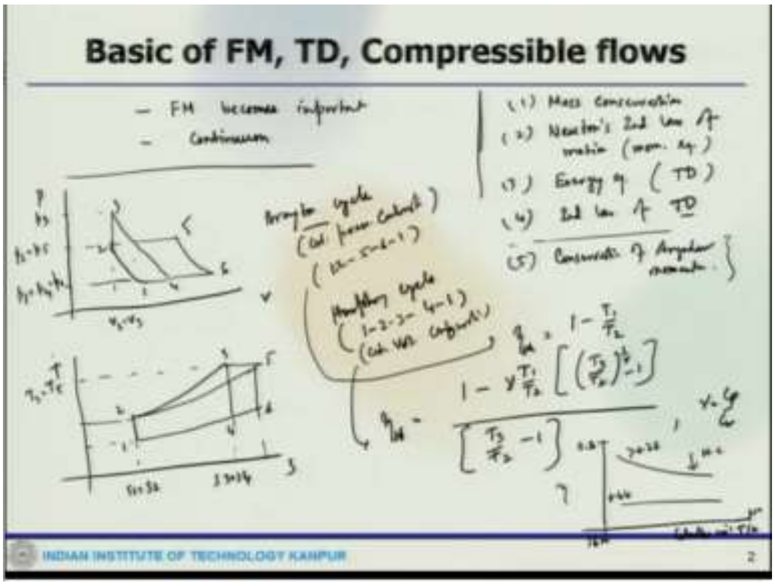

Because these are going to be important later on when we move to the thermo dynamical analysis of the jet engine, so let us start with that, so the basic discussion on the; now when you talk about basic fluid, so what you have already looked at that the thrust is developed by importing the momentum on the working fluid, so fluid mechanics becomes important and then in propulsion, so we consider the working fluid to be in continuum.

So, the continuum hypothesis would be valid and there are 4 basic conservation laws which one has to satisfy obviously, there will be mass conservation okay, these are basic governing laws, then Newton's second law of motion or momentum equation, Newton's second law of motion or one can think about momentum equation, third this is energy equation or conservation of energy which comes from thermodynamics.

And then we also need second law of thermodynamics, okay and now one can write these different form of equations in different ways, now apart from this we need one more conservation laws which is the conservation of angular momentum, so this would be required in addition of basic 4 conservation laws, while we will be talking about the turbo machinery part like for turbine or compressor, okay.

Now, there are 2 thermodynamical cycle where things can operate; one is the; if you look at that one is the Brayton cycle, another is the Humphrey cycle. So, first let us see the Brayton cycle, so if I draw the PV diagram for a Brayton cycle, so this could be 1 to 2 then it goes to 3, comes to 4, so this can 2, 3, 1, 4, there could be 5, 6, so P2 is P5 here, this is P3, P1, P4 and equals to P6.

So, this is the volume V2, V3, now this is a Brayton cycle where you have constant pressure combustion okay, now other one if I draw the T-S diagram, so 1 to 2, then it goes to 3, then it comes to 4; 1, 2, 3, 4, 5, 6, so now this guy is 1-2- 5- 6- 1 so that is in the PV diagram, now when we talk about the Humphrey cycle, it is 1- 2- 3- 4- 1, so this portion is Humphrey cycle in PV diagram, so this is the T-S diagram.

So, you have $T_3 = T_5$, this is 1, 2, so you have $S_1 = S_2$, $S_3 = S_4$, now in Brayton cycle, one can write the efficiency is $1 - \frac{T_1}{T_2}$, this is again ideal and Humphrey cycle is constant volume combustion process, so there one can derive the thermal efficiency like

$$\frac{1 - \gamma \frac{T_1}{T_2}\left[\frac{T_3}{T_2}^{\frac{1}{\gamma}} - 1\right]}{\left[\frac{T_3}{T_2} - 1\right]}$$

where gamma is Cp by Cv.

So, you can see the cycle efficiency in Humphrey cycle depends on also T_1 by T_2 and the temperature $\frac{T_3}{T_2}$; $\frac{T_3}{T_2}$ in effect of pressure. So, if you plot a curve for these 2 for just to compare, this is the combustor exit temperature in Kelvin, so let us say it starts with somewhere 1600 to goes to and this is the eta efficiency, let us say this is 0.8 and the cycle efficiencies are different.

Now, one goes like this, this is your Humphrey cycle which is 0.78 and somewhere Brayton cycle efficiency would be around 0.64 something like that, so there is a difference but obviously, there is another issue which is the process of combustion; one is constant volume, one is constant pressure, so there is a big difference between these 2.

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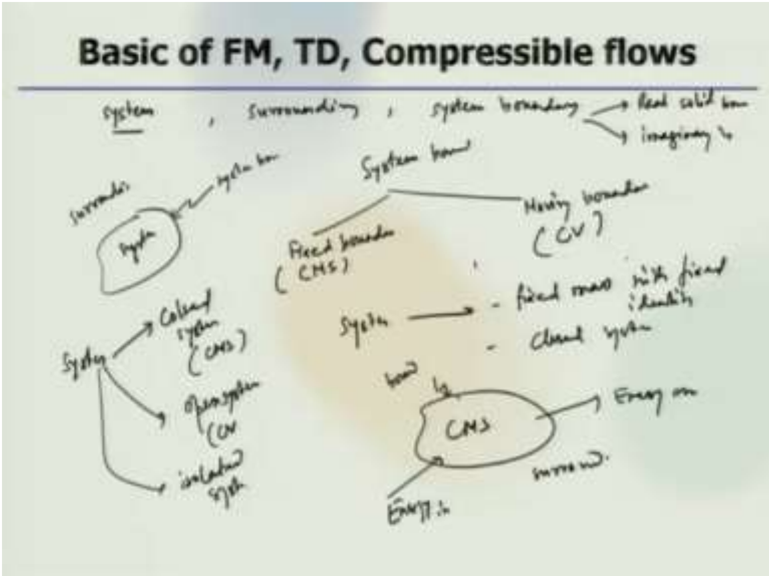

So, this is the basic thermo dynamical cycle that one; now some definitions; one definition could be system, this is a quantity of matter in space which is analysed during a problem, then you have surroundings that means this is everything external to the system, then you have system boundary, so the separation present between system and surroundings this is; so there could be solid boundary, real solid boundary or there could be imaginary boundary.

So, just like if you have a; so system like this, this is all surroundings and this is your system boundary, now the system boundary can be classified into 2 different categories; one is the fixed boundary which is called the control mass system or it could be moving boundary which is called the control volume system. Now, which boundary is to choose that depends on the problem to be analysed or another one, so any not the system boundary rather we put the classification for a particular system or the third one could be isolated system.

So, one is closed system, so let us say okay, we will let us put it the; so we will now define based on the system, there could be closed system which is control mass system, there could be open system which is control volume system, then there could be isolated system, okay. Now, when you talk about system, what is system; system is a collection of matter of fixed one of fixed identity, it may change say position and thermal condition but always have the same matter and hence the mass.

So, system has its system of fixed mass with fixed identity, then the type of system is usually referred to as; so when the system has fixed mass with fixed identity this is called the closed system and if there is a closed system or control mass system, there is no mass transfer across the system boundary, okay but there is no mass transfer across the system boundary which takes place but the energy transfer can take place across the system boundary. Then there is control, so if this is a control mass system, so one can have energy in, energy out, this is the boundary and this is the surrounding.

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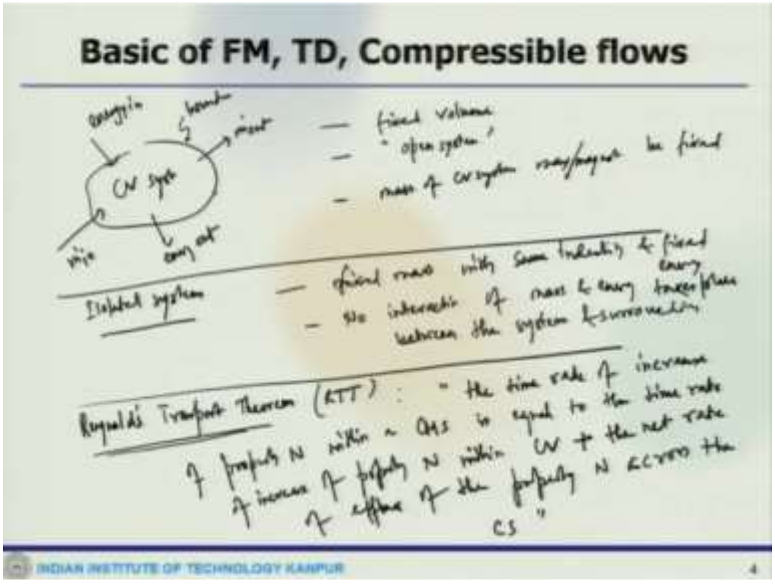

Now, when you talk about the control volume system, then this is the boundary, so you can mass in and mass out, you could have energy in and energy out, okay. So, the example of such control volume is a definite volume designated in space and matter can flow in and out of it, so this is the boundary of this volume is also called control surface, since fluid consists of infinitesimal packets of small system control volume approach is more appropriate.

Like you can think about the example of heat exchanger pump and so certain things it has a fixed volume that is one thing, so this is referred as open system or control volume system and we said mass transfer can take place, energy transfer also can take place across the system boundary, so this is more appropriate, so this one can see this control volume can be as a fixed region across which mass and energy transfer are studied.

So, boundary of the control volume, where this mass energy transfer takes place is called the control surface, mass of the control volume or the open system may or may not be fixed, so mass of CV system may or may not be fixed that is another important thing when the net influx of the mass across the control surface equals to the zero of the mass of the system is fixed or vice versa.

So, the identity of the mass in a control volume always changes like the case of control mass system and most of the; as I said engineering devices in general represent an open system or control system, control volume system. Now, other one is the isolated system, so the isolated system it is a system of fixed mass with same identity and fixed energy, so this is a fixed mass with same identity and fixed energy.

So, there is no interaction of mass and energy takes place between the system and the surroundings, between the system and surroundings, in more informal words, an isolated system like a clothes shop amidst a busy market, okay. Now, these are the different definition, so now we will go to that one important thing is the Reynolds transport theorem or called RTT, so a study of fluid in Eulerian approach always require some mathematical modelling of the control volume either in differential form or integral form.

So, the statements of principle conservation of mass momentum energy with reference to a control volume is quite necessary, so this can be derived by invoking, so what it states that the RTT says the time rate of increase of property in within a control mass system is equal to the time rate of increase of property N within control volume plus the net rate of efflux of the property N across the control surface.

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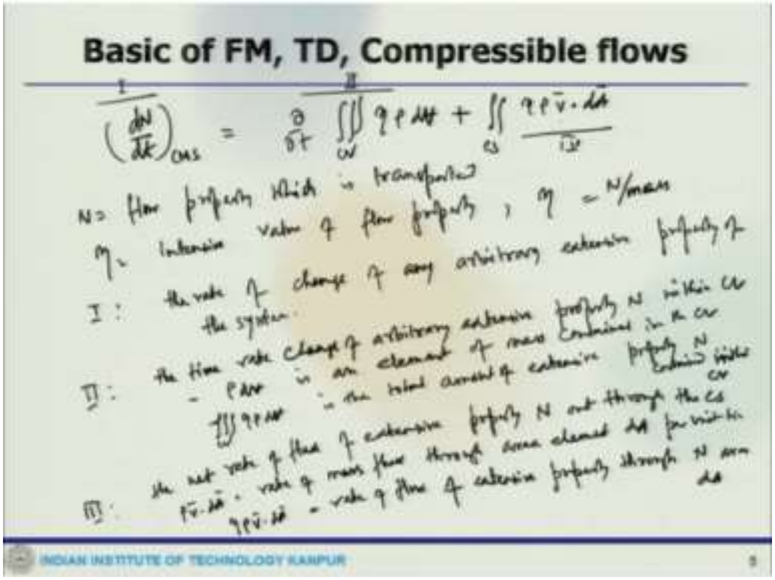

So, this is the statement it says, so if we write that if; so once you derive that this will looks like

$$\left(\frac{dN}{dt}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{CV} \eta\rho d\forall + \iint_{CS} \eta\rho\bar{V}\cdot d\bar{A}$$

so this V is velocity, this ∀ is volume, so N is the flow property which is transported, η is the intensive value of flow property and so which is N/mass.

And then this is the first term, this is second term, this is third term, so the first term is the rate of change of any arbitrary extensive property of the system, term 2 which is the time rate change of arbitrary extensive property N within control volume, so where $\rho d\forall$ is an element of mass contained in the CV and $\eta\rho d\forall$ which is $\eta\rho d\forall$ is the total amount of extensive property N contained within CV.

Third term, which is the net rate of flux of extensive property N out through the control surface, here $\rho\bar{V}\cdot d\bar{A}$ is the rate of mass flux through area element dA per unit time and $\eta\rho\bar{V}\cdot d\bar{A}$ is the rate of flux of extensive property N through area dA.

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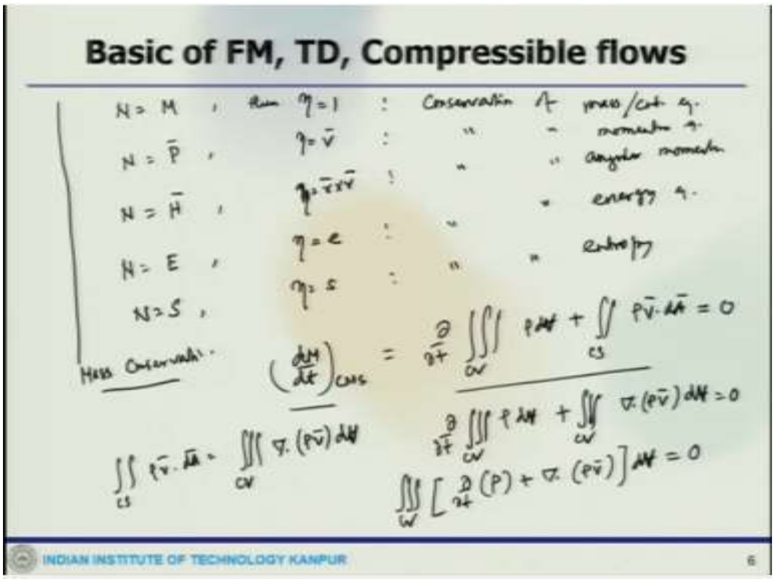

Now, once you see that; so this gets you so

when N=M then η =1, which leads to the conservation of mass or continuity equation,

if N =P then $\eta = \bar{V}$, this is conservation of momentum equation,

if N=H, $\eta = \bar{r} \times \bar{V}$, this is conservation of angular momentum,

if N=E, $\eta = e$, this is conservation of energy equation,

if N=S, $\eta = s$, conservation of entropy.

So, you can see for different N and η, you can derive all the conservation laws that we have talked at the beginning that means, this RTT is quite handy to define this kind of governing laws. for example, so we can write for example, let us say the mass conservation equation, we write

$$\left(\frac{dM}{dt}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{CV} \rho d\forall + \iint_{CS} \rho\bar{V}\cdot d\bar{A} = 0$$

Because the total mass in a control mass system is constant, so this is 0, so this is our mass conservation equation, now using the Gauss divergence theorem one can write

$$\iint_{CS} \rho\bar{V}\cdot d\bar{A} = \iiint_{CV} \nabla\cdot(\rho\bar{V}) d\forall$$

then this will become

$$\frac{\partial}{\partial t}\iiint_{CV} \rho d\forall + \iiint_{CV} \nabla\cdot(\rho\bar{V}) d\forall = 0$$

, so you can write.

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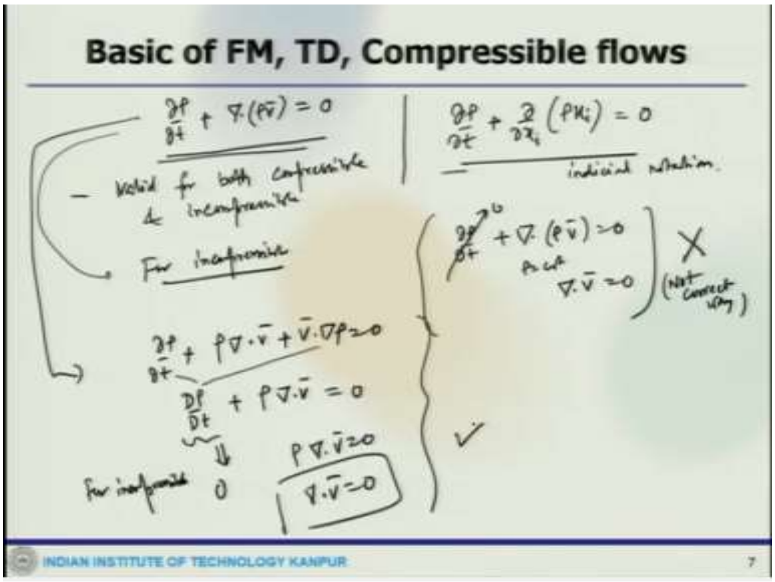

So, if you take this, then one can write

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0$$

this holds good for any volume and it vanishes these things, so what we can write;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0$$

okay, so this is what one can write in terms of vector notation and if somebody writes in indicial notation, this would be

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho \overline{x_i}) = 0$$

so this is in indicial notation.

So, one can write indicial notation also, now this is an equation of mass conservation or continuity equation which is valid for any flows, like this is valid for both compressible and incompressible. Now, for incompressible we can derive the special situation for incompressible flow but do not make this mistake which is very common is that okay, incompressible flow density does not change that is not the way it is derived.

Now, you can see what; so density does not vary too much but so people usually make this mistake straight away okay, for incompressible flow you just say you write down this and you say this goes 0, then density constant, so this becomes

$$\nabla \cdot \bar{V} = 0$$

, so these thing is not correct, so this is not correct way of doing or derive that equation rather one has to do for incompressible limit that expand that the second part which will get you

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \bar{V} + \bar{V} \nabla \cdot \rho = 0$$

Now, this term would give me $\frac{\partial \rho}{\partial t} + \bar{V} \nabla \cdot \rho$, so this term and this term collectively, so this is called the total derivative or material derivative, for incompressible limit this goes to 0, so this becomes $\rho \nabla \cdot \bar{V} = 0$, since rho is not equals to 0, this is what you get for the incompressible flow, so this is the right way of doing it these things. So, we will stop here and we will continue the discussion for the other conservation laws and how we can derive in the next lecture.