Introduction to Airbreathing Propulsion

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Lecture - 34
Performance/Cycle Analysis: Turboprop

So continue the discussion on the prop turbofan and turboshaft kind of engines. So this would be again I mean by this time you have enough idea about this turbojet or turbofan kind of cycle analysis. So this would be pretty quick and we will touch upon all the important aspect of this turbofan, some kind of engine. So let us start with some example here like.

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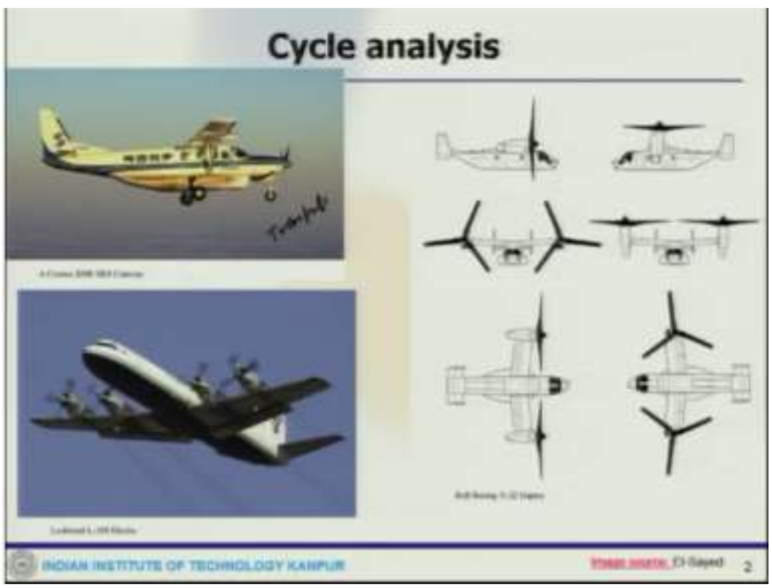

The example where top left you see, that is an example of Cessna aircraft, where this is again a turboprop engine, which is sitting there and then this one is also Lockheed. This is also turboprop based engine and also you can see these are some of these other turboprop kind of applications, which are there. So now the turboprop engines could be single, double, or triple spool. The single spool engine, the only one turbine drives one compressor and propeller.

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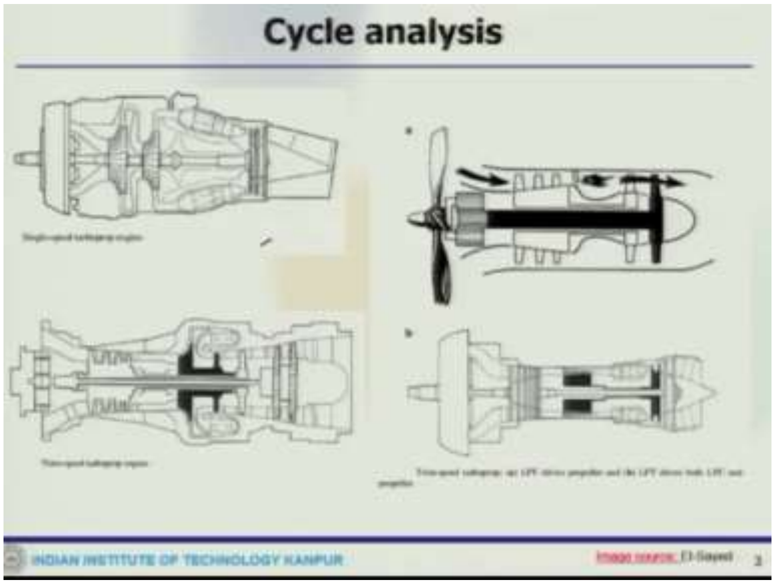

Like that would be like this is a single spool one, while it will have one single propeller or these things, then the other could be the low-pressure turbine drives either the single compressor or these and then you can have; this is a twin spool configuration. Here you can see the twin spool configuration. Here the LPT turbine drives the propeller. So here the LPT driver and drives both the LPC and propeller. This is another example.

Then you can have a three spool turboprop engine also, which is there and you can see there, as I said, there could be single spool, two spools, three spools, different kind of engines and there is a quite a bit of history of this kind of engine which are there that it started with, I mean still again from I think I would say 1940s onward and different kind of development and now this turboprop engines are also in use depending on the application, because at the low speed applications, they are quite efficient. So this can be used now.

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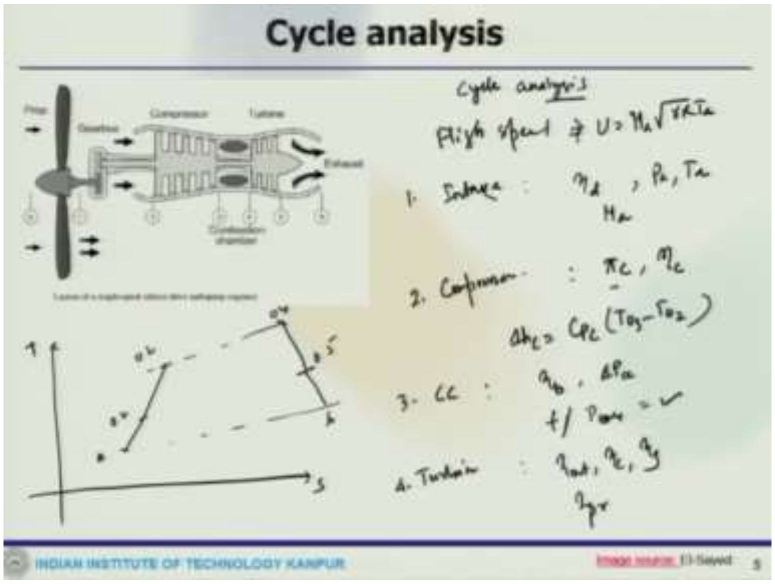

Now this is a single spool layout of a turboprop engine and we can look at the cycle analysis for this one. So what we have, let us say the flight speed here. So I can draw the quickly the TS diagram. So the TS diagram if I draw, this is a this goes to 2. So compared to turbojet or turbofan, this would be quite simple and that way I mean in point of view of the analysis, because you have less components. System is not that complex compared to those engines.

Now this flight speed U, which is would be

$$U = M_a\sqrt{\gamma R T_a}$$

Now first intake that we see; the intake has an isentropic efficiency of η_d and ambient conditions of pressure and temperature and the flight Mach number is M_a the temperature and the pressure are the intake and outlets are T_{02}, P_{02} which can be calculated that we have already seen. Then, it comes to compressor, where again the pressure ratio $\pi_c$pi c, isentropic efficiency η_c.

Then using those, the compressor things can be calculated and the specific power for the compressor which can be calculated like

$$\Delta h_c = C_{Pc}(T_{03} - T_{02})$$

Then combustion chamber, again when you come to the combustion chamber, you know the combustor efficiency and also the pressure drop across the combustion chamber, then we can find out fuel-air ratio, outlet pressure P after the combustion chamber. So all these can be again calculated.

Fourth turbine, so since the turbine drives the compressor as well as the propeller, so the portion of the power transfer to each is known in advance. So then it is easy to do the energy balancing. Now the portion of the turbine power is also delivered to propeller. This propeller power is independent on other several efficiencies, like mechanical efficiency of the turbine, compressor, gearbox and then also propeller efficiency.

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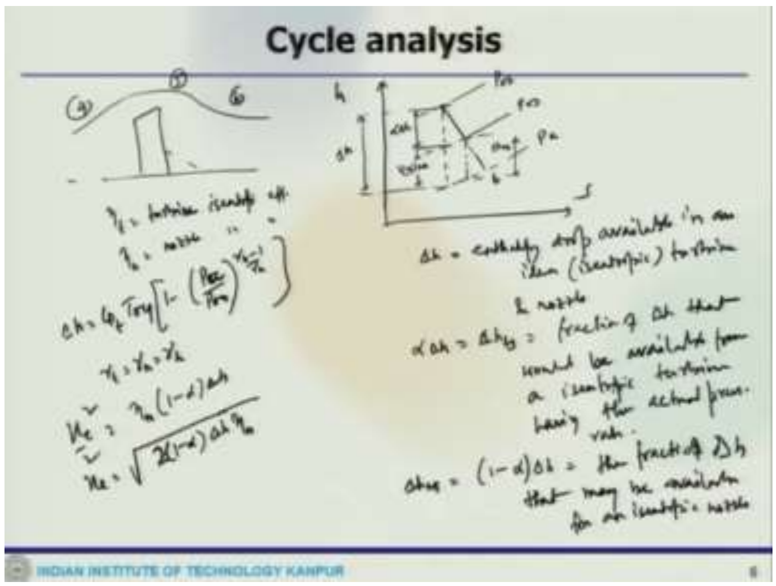

So we can look at a simple diagram to explain this like we have the turbine here. So this is 4, this is 5 and this is 6. So if we draw an diagram of enthalpy entropy diagram, so this is P_{04} where it was there coming to P_{05} and then coming down to P_a. So this is 6. So this portion is Δh, then this portion is $\alpha\Delta h$, then obviously this portion would be $(1-\alpha)\Delta h$ and this is called the Δh_{ns}.

So here η_t is turbine isentropic efficiency, η_N is nozzle isentropic efficiency. So what we can write here that also Δh is the enthalpy drop available in an ideal which is isentropic turbine and nozzle exhaust nozzle $\alpha\Delta h = \Delta h_{ts}$, which is the fraction of delta h that would be available from an isentropic turbine, which is having the actual pressure ratio and Δh_{ns}, which is $(1-\alpha)\Delta h$ is the fraction of Δh that may be available from an isentropic nozzle.

So what we can write that

$$\Delta h = C_{Pt}T_{04}\left[1 - \left(\frac{p_a}{p_{04}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]$$

Now

$$\gamma_t = \gamma_h = \gamma_n$$

then exhaust gas speed would be calculated

$$\frac{u_e^2}{2} = \eta_N(1-\alpha)\Delta h$$

$$u_e = \sqrt{2\eta_N(1-\alpha)\Delta h}$$

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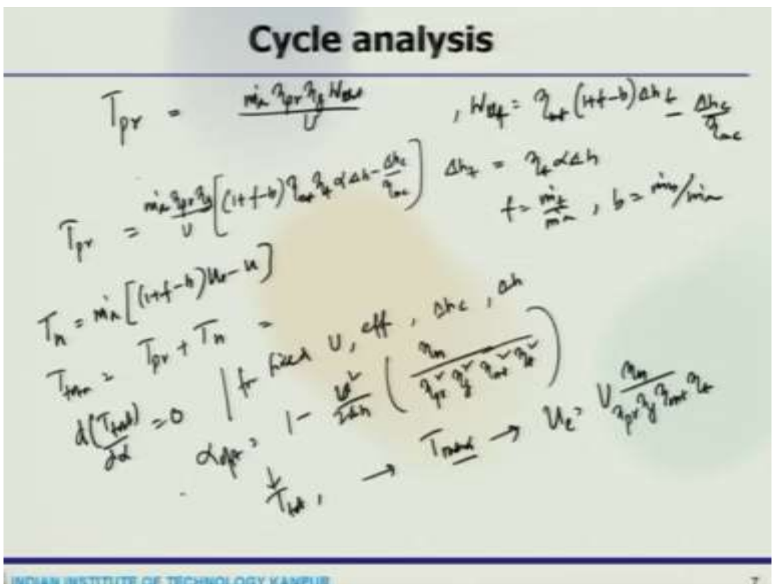

Now there is a propeller power or force which is there, then the propeller thrust can be calculated, which is

$$T_{Pr} = \frac{\dot{m}_a \eta_{Pr} \eta_g W_{shaft}}{U}$$

Now

$$W_{shaft} = \eta_{mt}(1+f-b)\Delta h_t - \frac{\Delta h_t}{\eta_{mc}}$$

So where the turbine specific power like

$$\Delta h_t = \eta_t \alpha \Delta h$$

now $\dot{m}_a$ is the air which is inducted,

$$f = \frac{\dot{m}_f}{\dot{m}_a}$$

$$b = \frac{\dot{m}_b}{\dot{m}_a}$$

So this guy T_{pr} can be replaced as

$$T_{Pr} = \frac{\dot{m}_a \eta_{Pr} \eta_g}{U} \left[(1 + f - b) \eta_{mt} \eta_t \alpha \Delta h - \frac{\Delta h_t}{\eta_{mc}} \right]$$

Now the thrust force which is obtained from this exhaust gas nozzle is denoted as T_n, which is

$$T_n = [(1 + f - b) u_e - u]$$

So the total thrust would be

$$T_{total} = T_{pr} + T_n$$

So now that is what we can get. Now if differentiate this total thrust with respect to α and set it to 0, then we get the α optimum, but that maximizes the thrust for fixed efficiencies flight speed, obviously those parameters.

So for fixed u, efficiency Δh_c, delta h for these things, then the α optimum which one can get

$$\alpha_{Opt} = 1 - \frac{u^2}{2\Delta h} \left(\frac{\eta_N}{\eta_{Pr}^2 \eta_g^2 \eta_{mt}^2 \eta_t^2} \right)$$

So that is what one can get. Now for a particular value of α depends this optimum power speed between the propeller and the jet. So once we use this value in T_{total} and then we can get the T_{max} T total max or the maximum value of the thrust force and for that the corresponding

$$u_e = U \frac{\eta_N}{\eta_{Pr} \eta_g \eta_{mt} \eta_t}$$

So that would be the corresponding value of that.

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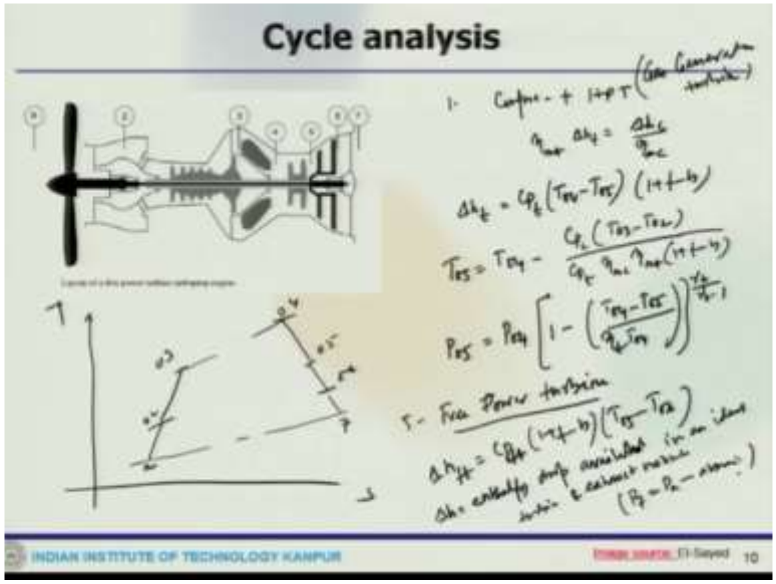

So now we can move to a two spool configuration like this where again we can look at the TS diagram quickly. This is a go to 2, 3, 4, 5, 6, 7. So this is a. So this is this is a two spool turboprop engine and again the different components can be examined here, again the gas generator where the energy balance between compressor class high-pressure turbine

$$\eta_{mt}\Delta h_t = \frac{\Delta h_c}{\eta_{mc}}$$

So then the specific work generated in the turbine of the gas generator, which is given

$$\Delta h_t = C_{Pt}(T_{04} - T_{05})(1 + f - b)$$

So we know the turbine inlet temperature and the outlet temperature T_{05} can be calculated once, so equals to

$$T_{05} = T_{04} - \frac{C_{Pc}(T_{03} - T_{02})}{C_{Pt}\eta_{mt}\eta_{mc}(1 + f - b)}$$

So I mean as per notation b is the bleed, f is the fuel air ratio, then P_{05} can be calculated like

$$p_{05} = p_{04}\left[1 - \frac{(T_{04} - T_{05})}{\eta_t T_{04}}\right]^{\frac{\gamma_t}{\gamma_t - 1}}$$

So this is what we can talk about gas generator turbine. Now essentially then we can look at the free power turbine. So free power turbine, I think at 5 that is their the work developed by the free power turbine per unit mass would be calculated as

$$\Delta h_t = C_{Pft}(1 + f - b)\,(T_{05} - T_{06})$$

So this is free power turbine. So already these we have discussed for the single spool calculation and a similar procedure can be followed where we can say that delta h is the enthalpy drop. This is available in an ideal turbine and exhaust nozzle, assuming a full expansion to the ambient pressure which is assumed that by assuming P_7 equals to P_a.

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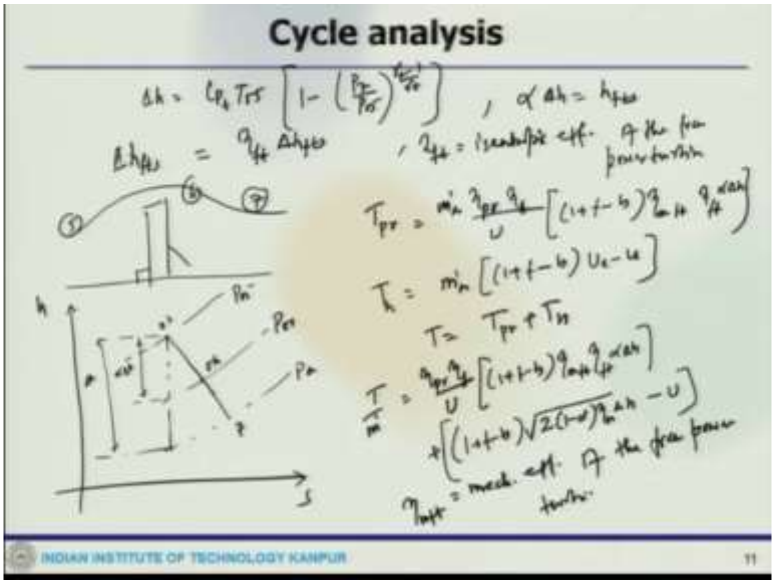

So where then

$$\Delta h = C_{Pt} T_{05} \left[1 - \left(\frac{p_7}{p_{05}} \right)^{\frac{\gamma_t - 1}{\gamma_t}} \right]$$

And

$$\alpha \Delta h = \Delta h_{fts}$$

which is the fraction of the delta s that would be available for an isentropic free turbine having the actual pressure ratio. So what we can write

$$\Delta h_{hfts} = \eta_{ft} \Delta h_{hfts}$$

So η_{ft} is the isentropic efficiency of the free power turbine. So we can follow the similar procedure what we have already done.

Like we can draw a schematic like this where we have this is 5, 6, 7 and we can, let us say, draw the HS diagram. So this is 05 comes here P_{05}, P_{06}, this is 06, so which is kind of 7. So again that is Δh and this portion is $\alpha \Delta h$. So what we get is that the propeller thrust that we can calculate. Now how we do that? We do that like free propeller would be

$$T_{pr} = \frac{\dot{m}_a \eta_{pr} \eta_g}{U}\left[(1+f-b)\eta_{mft}\eta_{ft}\alpha\Delta h\right]$$

and T nozzle would be

$$T_n = \dot{m}_a[(1+f-b)U_e - U]$$

So the total power would be

$$T = T_{pr} + T_n$$

and then again

$$\frac{T}{\dot{m}} = \frac{\eta_{pr}\eta_g}{U}\left[(1+f-b)\eta_{mft}\eta_{ft}\alpha\Delta h\right] + \left[(1+f-b)\sqrt{2(1-\alpha)\eta_n \Delta h}U_e - U\right]$$

and here η_{mft} is the mechanical efficiency of the free power turbine. So when we try to maximize the thrust, so what will happen?

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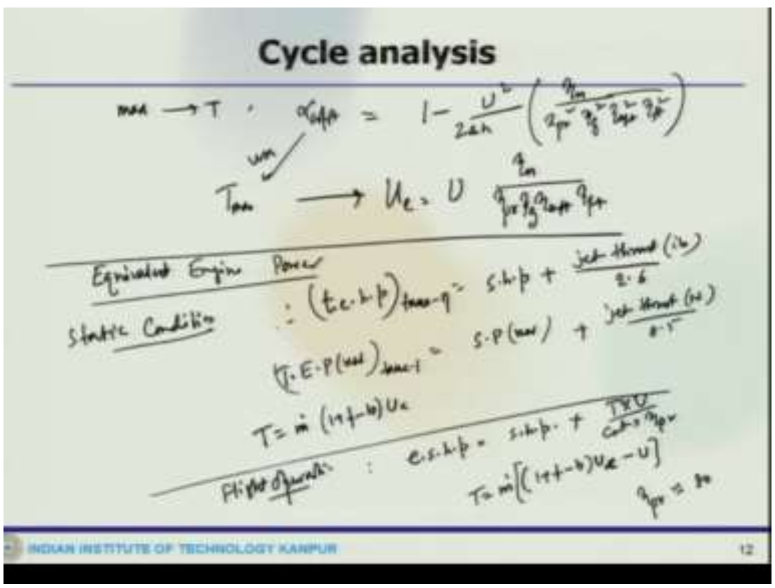

So maximize the trust, so we get alpha optimum which is

$$\alpha_{Opt} = 1 - \frac{U^2}{2\Delta h}\left(\frac{\eta_N}{\eta_{Pr}^2 \eta_g^2 \eta_{mft}^2 \eta_{ft}^2}\right)$$

Again it can be mentioned that this particular value alpha depends the optimum speed between propeller and the jet. So once we use this optimum speed we will get the maximum thrust force. So you can use that to get the T_{max} and corresponding exit velocity would be

$$u_e = U\frac{\eta_N}{\eta_{Pr}\eta_g \eta_{mft}\eta_{ft}}$$

So the outlet conditions are free turbine can be easily calculated by the known value of delta h and alpha optimum. So an alternative to parameter alpha one method can be followed like known as exhaust speed or known the ratio of that. Now then we can look at the equivalent engine power. So when you talk about that, there are two flight phases which can be discussed. One is the static condition.

So this is during testing or take off condition that total equivalent horsepower is denoted by $t.e.h.s$. So one can write this during takeoff

$$(t.e.h.s.)_{take-off} = s.h.p + \frac{jet\ thrust\ (lb)}{2.6}$$

$$(TEP\ (kW))_{take-off} = sp(kW) + \frac{jet\ thrust\ (N)}{28.5}$$

And the thrust on test bench or ground testing, this T would be

$$T = \dot{m}(1 + f - b)U_e$$

Now this is static condition. Other condition could be flight operation, which is for a turboprop engine during a flight the equivalent shaft horsepower which is

$$e.s.h.p = s.h.p + \frac{T * U}{constant * \eta_{Pr}}$$

and

$$T = \dot{m}[(1 + f - b)U_e - U]$$

So now these constants can be depending on the unit. So for example, it can be Knots it could be in mile per hour and where the typical propeller efficiency is around 80% percent.

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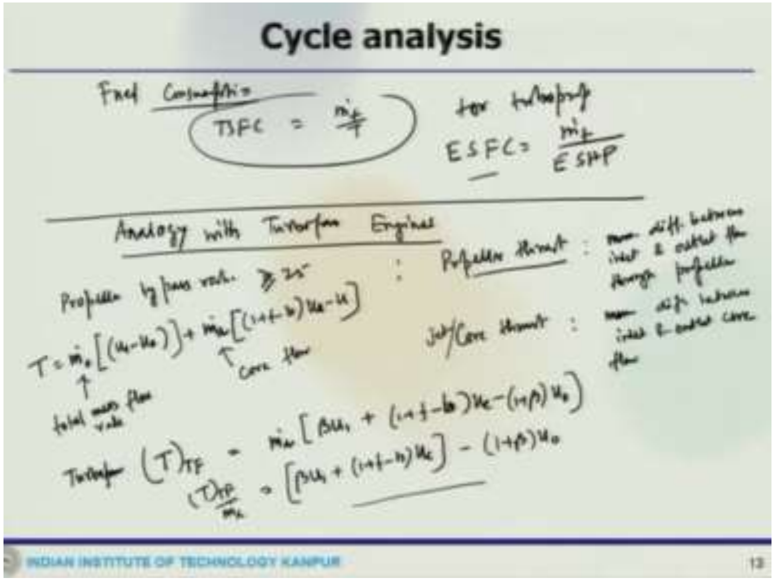

Then, we have fuel consumption. So already we have to define TSFC, which is

$$TSFC = \frac{\dot{m}_f}{T}$$

and for turboprop engine, the fuel consumption which is defined is equivalent. So this is in general for turboprop equivalent specific fuel consumption, which is m dot a to equivalent shaft horsepower. So this is the value typically remains 0.27236 kg fuel per kilowatt horsepower. So that is what for propeller engine. We can define things in that fashion.

So now we can draw a quick correlation like some sort of an one can say like analogy with turbofan engines. So this can be sort of drawn. Now the turboprop engines are kind of analogous to the high bypass ratio turbofan engine. So the propeller itself is an unducted fan with a bypass ratio equal to or greater than so the propeller bypass ratio could be equivalent or greater than equal to 25 or higher.

So this can be looked at that unducted bypass and the airflow through the propeller is slightly accelerated. Thus, it requires speed slightly higher than the aircraft flight speed. Now the another thing is that momentum difference between the inlet and the outlet flows. So the propeller produces the propeller thrust. So there is a propeller thrust, which is getting produced. Then the accelerated air passed through the core of the engine and then where the gas generator energy is added due to fuel burning and then it accelerate to high speed.

So the momentum difference, so this is sort of a momentum difference between inlet and outlet flow through propeller. So this is propeller thrust and then the core thrust or the sometimes one can say the jet thrust, which is a momentum difference between inlet and outlet core flow, so the first force can be written as

$$T = \dot{m}_o[u_1 - u_0] + \dot{m}_a[(1 + f - b)u_e - u]$$

Here $\dot{m}_a$ is the mass flow rate of the air which process through the core flow.

And this is $\dot{m}$ is the total mass flow rate. So now if you look at for the turbofan thrust equation, this is for turbo fan with bypass ratio; this is written at

$$(T)_{TF} = \dot{m}_a[\beta u_1 - (1 + f - b)u_e - (1 + \beta)u_0]$$

So the specific thrust to engine core mass flow-rate to get that is that here that

$$\frac{(T)_{TF}}{\dot{m}_a} = [\beta u_1 - (1 + f - b)u_e - (1 + \beta)u_0]$$

So you can see this nice correlation between the turbo prop engine and bypass ratio turbofan.

They kind of look similar which is sort of an equivalency one can think about, because here the propeller actually behaves like a fan which is sitting in the turbofan engines and it also contributes to the thrust and we have already seen that there are two component of the thrust; one is the propeller thrust, then the other one is the core thrust. So these are the different components, which could be produced from that. Now we stop the discussion part here and pretty much that is the one which I would like to discuss on turboprop engine like turbo shaft and other I will continue in the next class.