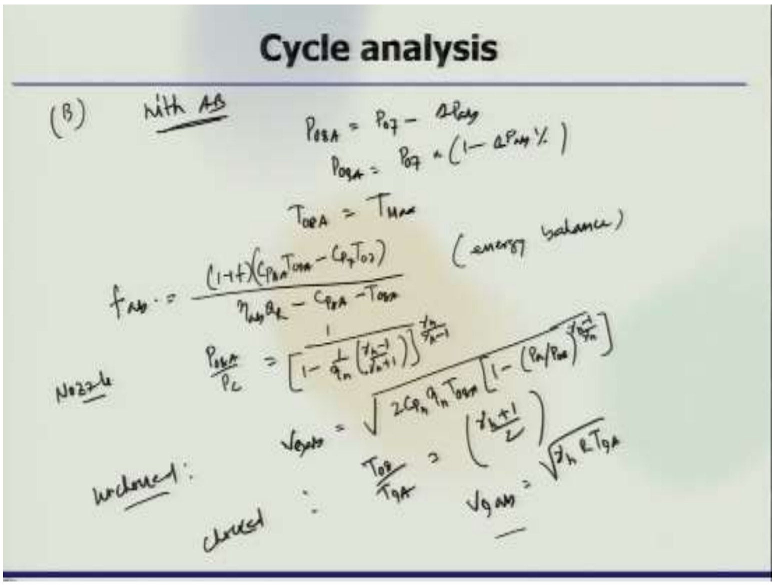
Introduction to Airbreathing Propulsion
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology – Kanpur

Lecture – 30
Performance/ Cycle Analysis: Turbojet (Contd.,) Turbofan

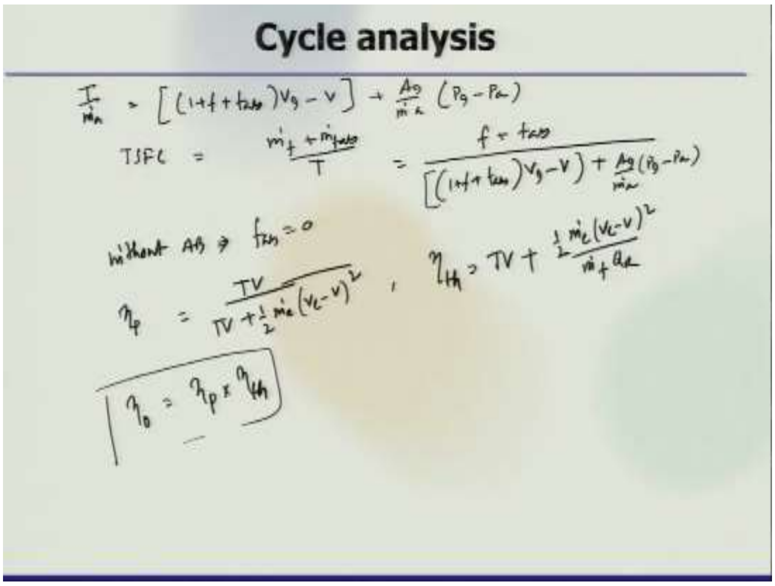
So we are looking at this dual spool turbojet engine and we have looked at with afterburner calculations and now look at the performance parameter.

(Refer Slide Time: 00:25)



So this is where we have just stopped with the state both the different scenario with or without afterburner and we have stopped here. So now we will look at some of the performance parameter.

(Refer Slide Time: 00:41)



Like for the dual spool engine like one of the important parameter is the specific thrust which would be again

$$\frac{T}{\dot{m}_a} = [(1 + f + f_{ab})V_9 - V] + \frac{A_9}{\dot{m}_a}(p_9 - p_a)$$

Now once we get that we can also find out the TSFC like TSFC would be

$$TSFC = \frac{(\dot{m}_f + \dot{m}_{fab})}{T} = \frac{f + f_{ab}}{[(1 + f + f_{ab})V_9 - V] + \frac{A_9}{\dot{m}_a}(p_9 - p_a)}$$

So, obviously when the afterburner is not on.

So without AB

$$f_{ab} = 0$$

Now also it resembles now the propulsive efficiency η_p which will be

$$\eta_p = \frac{TV}{TV + \frac{1}{2}\dot{m}_e(V_e - V)^2}$$

So, where the $\dot{m}_e$ is the exit mass flow rate and V_e is the exit velocity and similarly we can write thermal efficiency which is

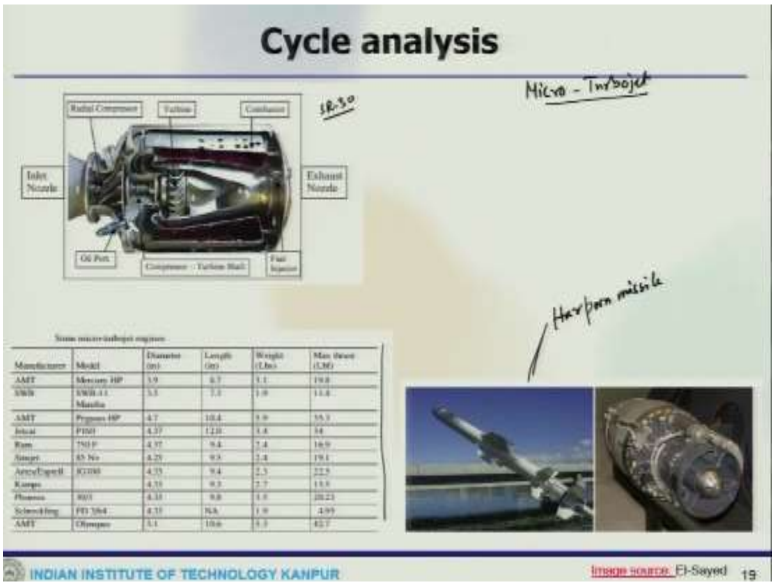
$$\eta_p = TV + \frac{\frac{1}{2}\dot{m}_e(V_e - V)^2}{\dot{m}_f Q_R}$$

So, overall efficiency would be

$$\eta_o = \eta_p * \eta_{th}$$

So that is how you can find out. So, finally when you get this thermo dynamical analysis final m is to find out like getting this performance parameter like specific thrust TSFC and all this.

(Refer Slide Time: 03:10)

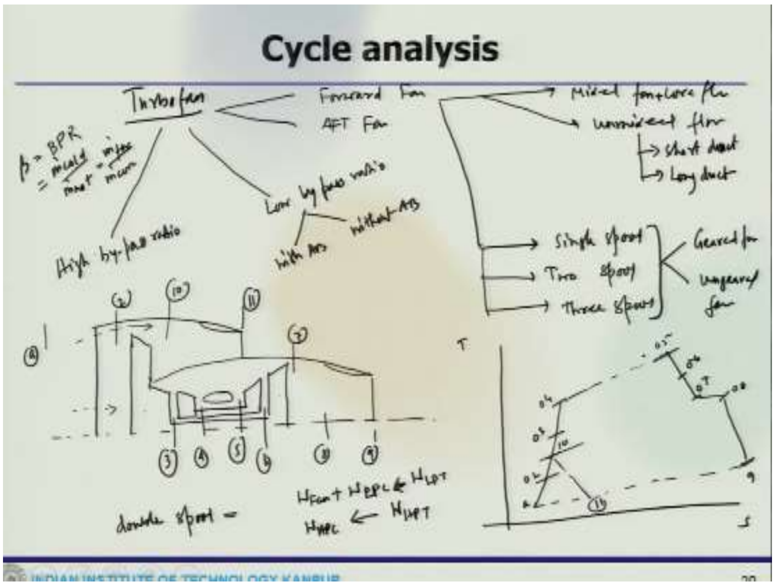


Now moving ahead there would be another thing which I would like to discuss about something like micro turbojet. So, these are basically small turbojet engine there is a cross section of this picture which is a engine of SR-30 this is SR-30 this is a cross section of micro turbojet and you can see the different components of that engine. So these are the small turbojet engines which are developed for using fueled missile, target drone and other small unmanned air vehicle.

Like there are multiple examples of this, this is one of the micro turbojet. The first commercial micro turbine made available by Jet in the early 1990 and right after that shortly after that Germany and Netherland sort out to develop powerful light weight fuel micro turbine. Now also these are some of these some micro turbine engine which you can see which are there.

And this is one of the example of that micro turbine rather let us say this is the Harpoon missile which is an air to surface or submarine launch anti surface or anti-ship missile. It is powered by a small J402 small turbojet engine which has a solid propellant booster so which is now we can see this there are also range of micro turbine which are available in the market and they are sort of used to for this kind of applications.

(Refer Slide Time: 05:13)



Now with that we will move to turbofan so this is now the engine which will have we have seen already multiple example of this kind of engines where there would be a fan. So, the turbojet where we have added fan and this fan is going to have some bypass and then like one can divide the turbofan engine different ways, one can have like it is divided into two ways like whether forward fan or AFT fan that is one.

Then forward fan can be taken again into different ways like mixed fan and core flow mixed fan plus core flow or it could be unmixed flow and then if it is unmixed again it could have short duct or it could be having a long duct then again forward fan it could have some another branching could be single spool then it will have two spool then you could have three spool also.

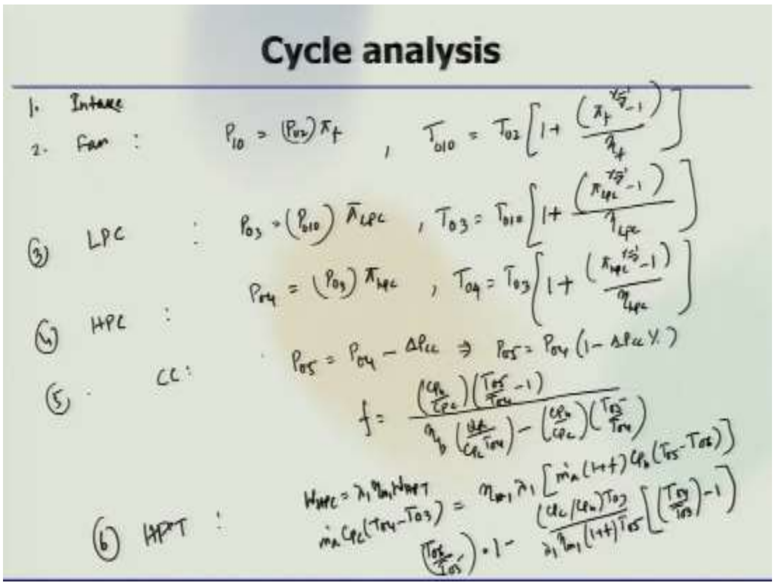
And then all these things can be clubbed to have two more segment one could be geared fan or ungeared fan then the AFT fan can we know then again this is another way one can classify high bypass ratio engine or this could have low bypass ratio. Now low bypass ratio it could be with AB or it could be without AB that means with and without afterburning. So, there are different category of engines which are available.

And bypass ratio when we talk about this bypass ratio that means we talk about this factor so the bypass ratio beta we call is the bypass BPR bypass ratio which is m dot cold / m dot hot. So that means the hot so this could be m dot fan / m dot core. So how much pass through the engine and how much being there so that is what it is there. So now in the turbojet so let us have a layout of a engine let us start with this it goes like this.

Then I will have this it comes so this goes up and this then I can have another one here which is connected like this in between I have this. So this one is connected and this is the nozzle. So there are different stations now let us say this is a this is 2 then this is 3 this is 4, this is 5, this is 6 and this is 7, 8, this is 9, this could be 10, this could be 11. So this is a double spool layout where fan double spool system where the fan plus LPC are driven by the LPT.

And HPC is driven by HPT here is the fan then after that some portion of the air goes to the core, some goes here this is bypass then it is low pressure compressor, high pressure compressor then high pressure turbine, low pressure turbine then if AB is there it would be sitting there. So already now we can look at the TS diagram. So here let us start with a so this is a we go to 2 then we go to somewhere 3. Then 3 to 4 then we will go to 5, 5 to 6, 7 if there is no afterburning it is 8, 8 to 9 and this is 10 where 10 to this is 11. So this is how it looks like.

(Refer Slide Time: 12:28)



Now we can do the analysis intake. So we are not going to repeat the same exercise here because the intake would be exactly the similar like what we have done in the turbojet then we have fan. Now a similar analysis to the compressor it can be done. So you can see the fan pressure

$$p_{10} = p_{02} * \pi_f$$

which is the fan pressure ratio then we can find the

$$T_{010} = T_{02}\left[1 + \frac{\pi_f^{\frac{\gamma-1}{\gamma}} - 1}{\eta_f}\right]$$

So that is the fan efficiency that is π_f is the pressure ratio then we have low pressure compressor. So this is

$$p_{03} = p_{010} * \pi_{LPC}$$

So and similarly

$$T_{03} = T_{010}\left[1 + \frac{\pi_{LPC}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{LPC}}\right]$$

then we have HPC which we could find out the HPC and then HPC would be similar like

$$p_{04} = p_{03} * \pi_{HPC}$$

so

$$T_{04} = T_{03}\left[1 + \frac{\pi_{HPC}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{HPC}}\right]$$

So that is what we can do and then 5 we go to combustion chamber where again similarly we find out

$$p_{05} = p_{04} - \Delta p_{cc}$$

$$p_{05} = p_{04}(1 - \Delta p_{cc}\%)$$

Now the temperature of the outlet of the combustion chamber is also the inlet to the turbine and that will dictate the constraint

$$f = \frac{\left(\frac{C_{Ph}}{C_{Pc}}\right)\left(\frac{T_{05}}{T_{04}}\right) - 1}{\left(\frac{\eta_b Q_R}{C_{Pc}T_{04}}\right) - \left(\frac{C_{Ph}}{C_{Pc}}\right)\left(\frac{T_{05}}{T_{04}}\right)}$$

So this again come the energy balance we get it then we go to HPT. So HPT is going to run the HPC so similarly m dot a. So we again assume

$$W_{HPC} = \lambda_1 \eta_{m1} W_{HPT}$$

So which allows us to write

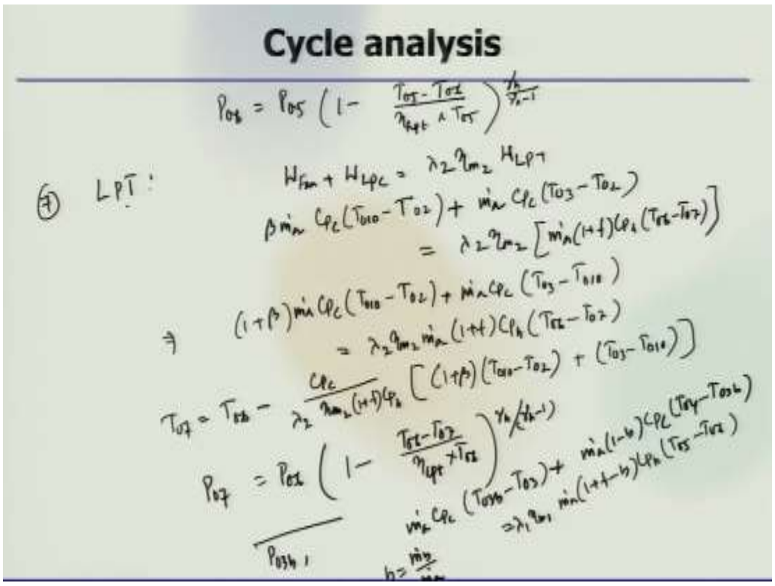
$$\dot{m}_a C_{Pc}(T_{04} - T_{03}) = \lambda_1 \dot{m}_a (1 + f)\eta_{m1} C_{Pc}(T_{05} - T_{06})$$

so the

$$\frac{T_{06}}{T_{05}} = 1 - \frac{(C_{Pc}/C_{Ph})T_{03}}{\lambda_1(1 + f)\eta_{m1}T_{05}}\left[\left(\frac{T_{04}}{T_{03}}\right) - 1\right]$$

So here also η_{m1} is the mechanical efficiency HPC spool mechanical efficiency and the percentage of the HPT which we have developed the λ_1 would go there to run that.

(Refer Slide Time: 17:30)



So what we will get? So we will get the

$$p_{06} = p_{05}\left(1 - \frac{T_{05} - T_{06}}{\eta_{HPT} T_{05}}\right)^{\frac{\gamma_h}{\gamma_h - 1}}$$

Now we will go LPT so again here the low pressure compressor this is both fan and the low pressure compressor would be driven by that. So you can write

$$W_{Fan} + W_{LPC} = \lambda_2 \eta_{m2} W_{LPT}$$

which allow us to write the

$$\beta \dot{m}_a C_{Pc}(T_{010} - T_{02}) + \dot{m}_a C_{Pc}(T_{03} - T_{02}) = \lambda_2 \eta_{m2}[\dot{m}_a (1+f) C_{Ph}(T_{06} - T_{07})]$$

So we can do bit of algebra here like we can write

$$(1+\beta)\dot{m}_a C_{Pc}(T_{010} - T_{02}) + \dot{m}_a C_{Pc}(T_{03} - T_{010}) = \lambda_2 \eta_{m2}[\dot{m}_a (1+f) C_{Ph}(T_{06} - T_{07})]$$

So what we will get

$$T_{07} = T_{06} - \frac{C_{Pc}}{\lambda_2 \eta_{m2}(1+f) C_{Ph}}(1+\beta)(T_{010} - T_{02}) + (T_{03} - T_{010})$$

and the pressure at the outlet we will get

$$p_{07} = p_{06}\left(1 - \frac{T_{06} - T_{07}}{\eta_{LPT} T_{06}}\right)^{\frac{\gamma_h}{\gamma_h - 1}}$$

Now if there is a bleed from the high pressure compressor if there is an air bleed from the HPC at station where the pressure is P_{03b} then the energy balance with high pressure turbine would give. So what we can have there is a bleed then let us say at station where the pressure is P_{03b} then the energy balance would be

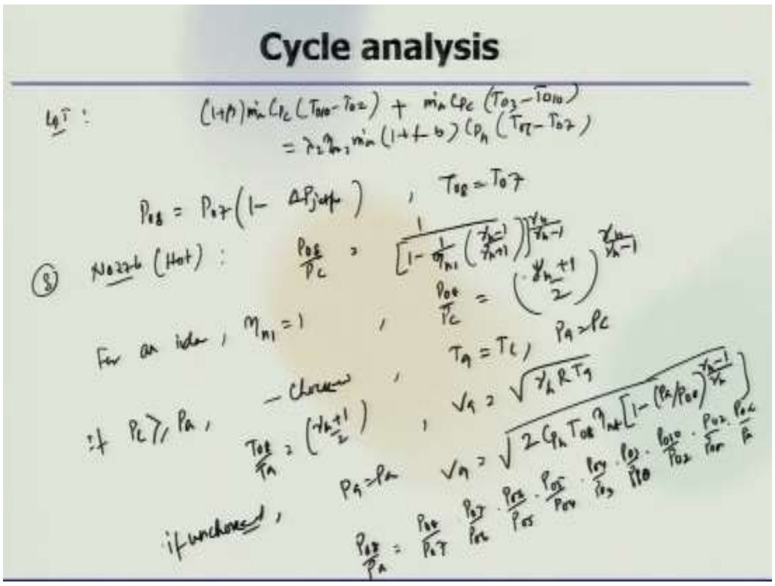
$$\dot{m}_a C_{Pc}(T_{03b} - T_{03}) + \dot{m}_a (1-b) C_{Pc}(T_{04} - T_{03b}) = \lambda_1 \eta_{m1}[\dot{m}_a (1+f-b) C_{Ph}(T_{05} - T_{06})]$$

where b is the bleed ratio which is

$$b = \frac{\dot{m}_b}{\dot{m}_a}$$

Defining the bleed air from the HPC to the core air flow. Moreover, such a bleed has it impact on the energy balance of the low pressure spool and air passing through the low pressure turbine which is also now reduced.

(Refer Slide Time: 22:11)



That means what we can do also in the low pressure turbine LPT in the LPT section

$$(1+\beta)\dot{m}_a C_{Pc}(T_{010} - T_{02}) + \dot{m}_a C_{Pc}(T_{03} - T_{010}) = \lambda_2 \eta_{m2}[\dot{m}_a(1+f-b)C_{Ph}(T_{06} - T_{07})]$$

So if there is a bleed there then the whole analysis between that station has to be changed and the fuel in the jet pipe is frequently accelerate the pressure then we gain to the skin friction. Now the pressure at the upstream of the turbine nozzle is slightly less.

So what we can get

$$p_{08} = p_{07}(1 - \Delta p_{Jet\ pipe})$$

and T_{08} would be T_{07}. Now finally we come to the nozzle this is hot nozzle let us say. So here the exhaust velocities in both the hot gases from the turbine nozzles are obtain again we have to check whether it is choked or not. So, typically see

$$\frac{p_{08}}{p_c} = \frac{1}{\left[1 - \frac{1}{\eta_{n1}}\left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}}$$

So for an ideal case $\eta_{n1} = 1$.

So this would be reducing to

$$\frac{p_{08}}{p_c} = \left[\frac{\gamma_h + 1}{2}\right]^{\frac{\gamma_h}{\gamma_h - 1}}$$

Now if P_c is greater than P_a then the nozzle is choked and the temperature pressure of the gases leaving the nozzle at critical values. So T_9 would be T_c and P_9 would be P_c which is obtained like

$$\frac{T_{08}}{T_9} = \frac{\gamma_h + 1}{2}$$

$$V_9 = \sqrt{\gamma_h R T_9}$$

or if it unchoked if unchoked then $P_9 = P_a$ then

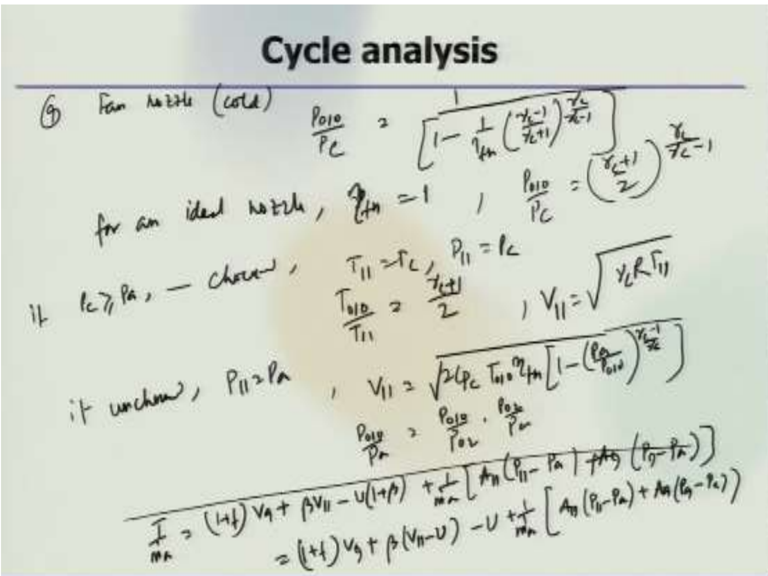
$$V_9 = \sqrt{2C_{Ph}\eta_{nt}T_{08}\left[1 - \left(\frac{p_a}{p_{08}}\right)\right]^{\frac{\gamma_h - 1}{\gamma_h}}}$$

So the pressure ratio in the nozzle which could be obtained

$$\frac{p_{08}}{p_a} = \frac{p_{08}}{p_7} * \frac{p_{07}}{p_{06}} * \frac{p_{06}}{p_{05}} * \frac{p_{05}}{p_{04}} * \frac{p_{04}}{p_{03}} * \frac{p_{03}}{p_{010}} * \frac{p_{010}}{p_{02}} * \frac{p_{02}}{p_{0a}} * \frac{p_{0a}}{p_a}$$

So this is how we get.

(Refer Slide Time: 25:57)



And then there is fan nozzle or one can say the cold nozzle where is there and there also critical pressure has to be checked where

$$\frac{p_{010}}{p_c} = \frac{1}{\left[1 - \frac{1}{\eta_{fn}}\left(\frac{\gamma_c - 1}{\gamma_c + 1}\right)\right]^{\frac{\gamma_c}{\gamma_c - 1}}}$$

So again for an ideal version for an ideal nozzle $\eta_{fn} = 1$ so this guy

$$\frac{p_{010}}{p_c} = \left[\frac{\gamma_c + 1}{2}\right]^{\frac{\gamma_c}{\gamma_c - 1}}$$

If $P_c > P_a$ it is choked nozzle so T_{11} would be T_c and P_{11} would be P_c so

$$\frac{T_{010}}{T_{11}} = \frac{\gamma_c + 1}{2}$$

and exit velocity would be

$$V_{11} = \sqrt{\gamma_c R T_{11}}$$

If unchoked then P_{11} would be P_a then the exit velocity would be calculated as

$$V_{11} = \sqrt{2C_{Pc}\eta_{fn}T_{010}\left[1 - \left(\frac{p_a}{p_{010}}\right)\right]^{\frac{\gamma_c - 1}{\gamma_c}}}$$

and

$$\frac{p_{010}}{p_a} = \frac{p_{010}}{p_{02}} * \frac{p_{02}}{p_a}$$

So the net thrust force which can be now calculated there would be two component of that. So the specific thrust will have

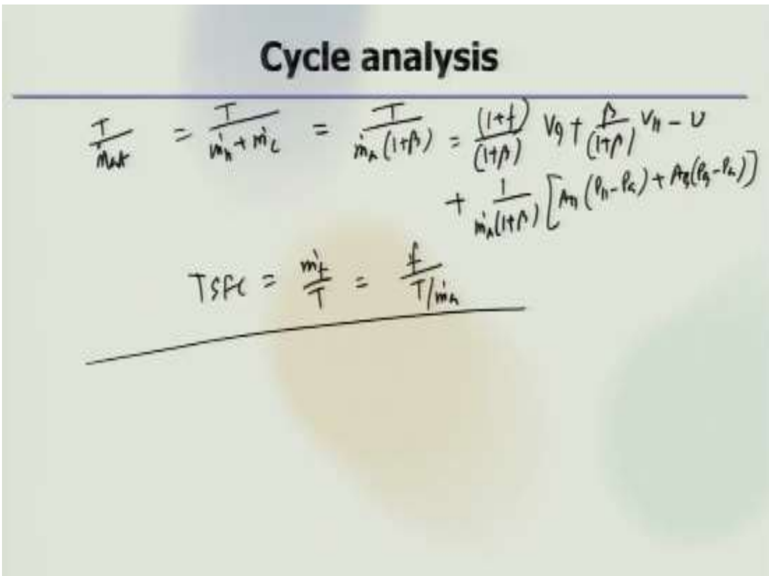
$$\frac{T}{\dot{m}_a} = (1 + f)V_9 + \beta V_{11} - U(1 + \beta) + \frac{1}{\dot{m}_a}[A_{11}(p_{11} - p_a) + A_9(p_9 - p_a)]$$

which one can write

$$\frac{T}{\dot{m}_a} = (1 + f)V_9 + \beta(V_{11} - U) - U + \frac{1}{\dot{m}_a}[A_{11}(p_{11} - p_a) + A_9(p_9 - p_a)]$$

So that is what and specific thrust can be calculated.

(Refer Slide Time: 28:36)



So one can do that which is

$$\frac{T}{\dot{m}_{at}} = \frac{T}{\dot{m}_h + \dot{m}_c} = \frac{T}{\dot{m}_a(1+\beta)}$$

$$= \frac{(1+f)}{(1+\beta)}V_9 + \frac{\beta}{(1+\beta)}V_{11} - U + \frac{1}{\dot{m}_a(1+\beta)}[A_{11}(p_{11}-p_a) + A_9(p_9-p_a)]$$

and TSFC which would be

$$TSFC = \frac{\dot{m}_f}{T} = \frac{f}{T/\dot{m}_a}$$

So this is how all the performance parameter for this double spool engine one can calculate along with all this information. So what we have looked at both for the turbojet and turbofan.

What is important is that the station wise calculation you have to pressure temperature, you have to find out and in the actual cycle there would be losses so we need to consider their efficiencies and by considering their efficiencies one should calculate this pressure temperature finally come down to the nozzle and then first thing you check whether the nozzle is chocked or unchoked accordingly you get the exit velocity and then put it back in the thrust and get all the thermal efficiency, specific thrust, TSFC and all this. So, we will stop here and continue the discussion in the next lecture.