

Introduction to Airbreathing Propulsion
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Lecture – 29
Performance/ Cycle Analysis Turbojet (Contd.,)

So let us continue the discussion on turbojet analysis. So we have looked at the this is what we are doing right now single spool turbojet analysis and we started looking at the nozzle right now and also nozzle we have to look at whether afterburner on or off.

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Cycle analysis

$$f_{th} = \frac{(1+f)(C_{P1}T_{044} - C_{P5}T_{05})}{\eta_{noz} \dot{m}_a - C_{P1}T_{022}}$$

(b) Nozzle
 (a) without AS

$$\frac{P_{07}}{P_c} = \left[1 - \frac{1}{\gamma} \left(\frac{V_7}{a_7} \right)^2 \right]^{\frac{\gamma}{\gamma-1}}$$

$M_7 = 1 \Rightarrow P_7 = P_c$ (checked)

$\left(\frac{T_{07}}{T_7} \right) = \frac{\gamma+1}{2}, V_7 = \sqrt{\gamma R T_7}$

checked $P_7 = P_c, V_7 = \sqrt{2 C_{P1} \eta_{noz} (T_{044} - T_7) \left[1 - \left(\frac{P_7}{P_{01}} \right)^{\frac{\gamma}{\gamma-1}} \right]}$

$$= \sqrt{\frac{2 \gamma R \eta_{noz} (T_{044} - T_7) \left[1 - \left(\frac{P_c}{P_{01}} \right)^{\frac{\gamma}{\gamma-1}} \right]}{(\gamma-1)}}$$

So this is exactly where we have stopped like started with the nozzle and there could be two difficult situation or scenario.

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Cycle analysis

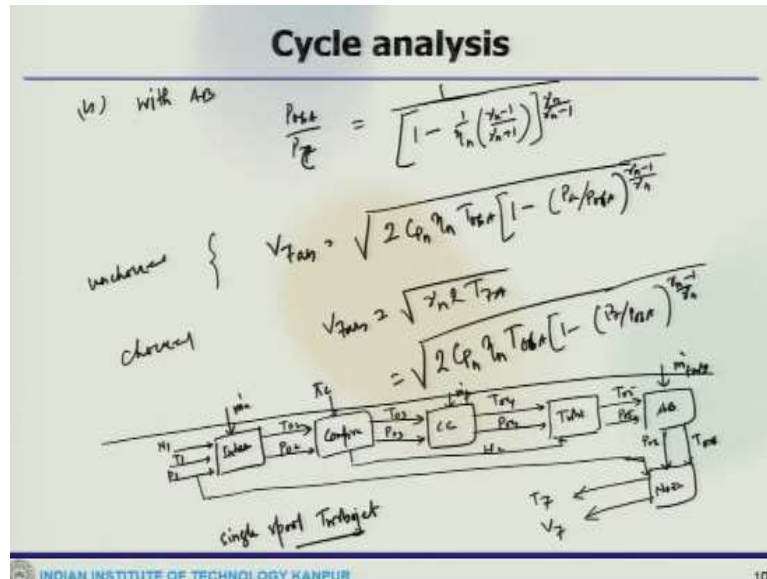
1: free stream
 1-2: Inlet
 2-3: Compression (dynamic compressor)
 3-4: Combustion Chamber
 4-5: Turbine
 5-6: Afterburner
 6-7: Nozzle

$0.7 < \eta_d < 0.9$
 $0.85 < \eta_c < 0.9$
 $0.7 < \eta_b \leq 0.95$
 $0.90 < \eta_f < 0.95$
 $0.7 < \eta_{noz} \leq 0.95$
 $0.75 < \eta_m < 0.98$

Actual cycle: without after burner

So, if you recall the TS diagram of air this is the picture I mean just to give you quick idea whether you have afterburner here or not between 5 and 6 accordingly this things will happen. So now we are looking at the nozzle so the first case we have taken without AB.

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Now other case where we can take that with AB that means afterburner is operative. So, in that case the expansion process in nozzle starts from 06 A to state 7. So the pressure ratio between this two guy would be

$$\frac{p_{06A}}{p_7} = \frac{1}{\left[1 - \frac{1}{\eta_N} \left(\frac{\gamma_n - 1}{\gamma_n + 1}\right)\right]^{\frac{\gamma_n}{\gamma_n - 1}}}$$

Now, again if the nozzle is unchoked the exhaust pressure is equal to the ambient one and the jet speed would be

$$V_{7ab} = \sqrt{2C_{Pn}\eta_N T_{06A} \left[1 - \left(\frac{p_a}{p_{06A}}\right)\right]^{\frac{\gamma_n - 1}{\gamma_n}}}$$

So this is when the things is unchoked or if it is choked that means

$$V_{7ab} = \sqrt{\gamma_N R T_{7A}}$$

and then we can use the relationship so this would be

$$V_{7ab} = \sqrt{2C_{Pn}\eta_N T_{06A} \left[1 - \left(\frac{p_7}{p_{06A}}\right)\right]^{\frac{\gamma_n - 1}{\gamma_n}}}$$

So that is what we get and just to calculate or rather show the total block diagram this is where your intake. Now intake gets M_1 T_1 P_1 \dot{m}_a from intake it goes to compressor, compressor it goes T_{02} P_{02} where compressor ratio is π_c from where it goes to combustion chamber.

So it goes to T_3 this is P_3 and then from there it goes to turbine. So that means T_4 P_4 and also the compressor this goes W_c . Now from turbine we get afterburner. So here we get T_5 P_5 and afterburner we have \dot{m}_{fab} in the combustion chamber we have \dot{m}_f and then finally from here we go to nozzle so this goes T_6 P_6 and also from here this comes here. So, the nozzle will get out T_7 V_7 so that is block diagram of this, this is for single spool turbojet.

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Cycle analysis

$$\frac{T}{\dot{m}_a} = [(1 + f + f_{ab})V_7 - V] + \frac{A_7}{\dot{m}_a} (P_7 - P_a)$$

$$TSFC = \frac{\dot{m}_f + \dot{m}_{fab}}{T}, \quad TSFC = \frac{f + f_{ab}}{(1 + f + f_{ab})V_7 - V + \frac{A_7}{\dot{m}_a} (P_7 - P_a)}$$

(i) Exhaust Gas Temperature gauge (EGT)
(ii) Engine Pressure Ratio (EPR)
(iii) MACH

Now we can see the performance parameter for this single spool engine. One is important is that specific thrust which is

$$\frac{T}{\dot{m}_a} = [(1 + f + f_{ab})V_7 - V] + \frac{A_7}{\dot{m}_a} (P_7 - P_a)$$

$$TSFC = \frac{(\dot{m}_f + \dot{m}_a)}{T} = \frac{f + f_{ab}}{[(1 + f + f_{ab})V_7 - V] + \frac{A_7}{\dot{m}_a} (P_7 - P_a)}$$

So these are the performance parameter if the afterburner is not operative then f_{ab} goes to 0 otherwise that f_{ab} comes into the picture.

Now some of the important definitions which are going to be important in this analysis. So this definitions why they are important because these are eventually used pretty much every day what I would say for designer practice. One exhaust gas temperature gauge so EGT. So this is exhaust gas temperature which is measured with a thermocouple type parameter by monitoring EGT the pilot can get an idea of the engine air fuel ratio.

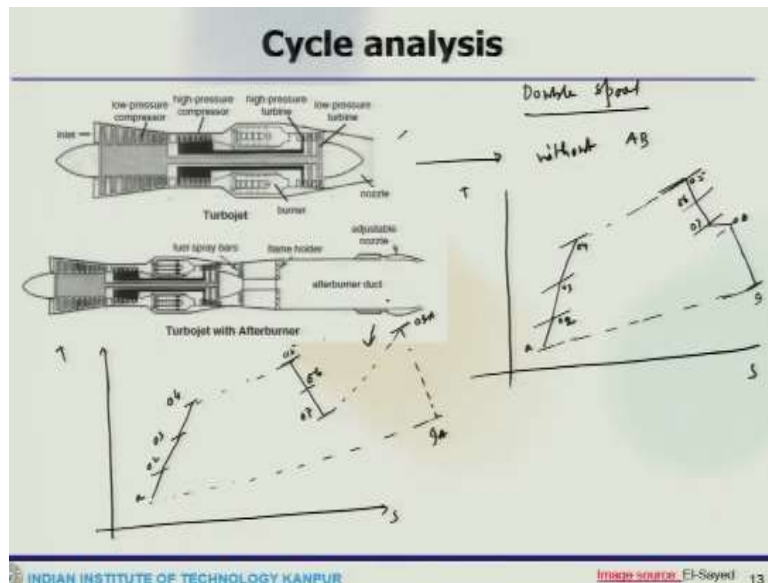
At an stoichiometric fuel air ratio the exhaust gas temperature would be different than at a lean or rich fuel air ratio. High temperature can be an indicator of dangerous condition that can be took as a static failure so this is one. Second one is the engine pressure ratio which is EPR. Engine pressure ratio is also ratio of turbine discharge to compressor inlet pressure. So, EPR is used as an indication of the amount of thrust being developed by an turbine engine.

If EPR gauge is used to indicate the power output of the turbojet and turbofan engine pressure measurements are recorded by probes installing the engine inlet and exhaust. So this data the EPR system design automatically compensate for the effect of air speed and altitude and all this. So change in ambient temperature require a corrections and that can be taken care of. Now the bleed so bleed air is another in aircraft engine is a compressed air that can be taken from within the engine most of an after the compressor stays, but before the fuel is injected in the burner.

Bleed air has high temperature and high pressure typical value would be 200 to 250 degree centigrade, 275 kilopascal. So this compressed air after the compression process is used in aircraft in many different ways. So this is deicing of the wing leading edge, pressurizing the cabin, pneumatic actuators, starting the remaining engine and pressurizing lavatory. Also it is used in deicing of engine intake and fueling of turbine blades and such things.

So, these are the important parameters which are often used in regular design practices and one is supposed to know all these. Now then we will move to the other one is the double spool turbojet. Now the double spool turbojet that means what we will have is.

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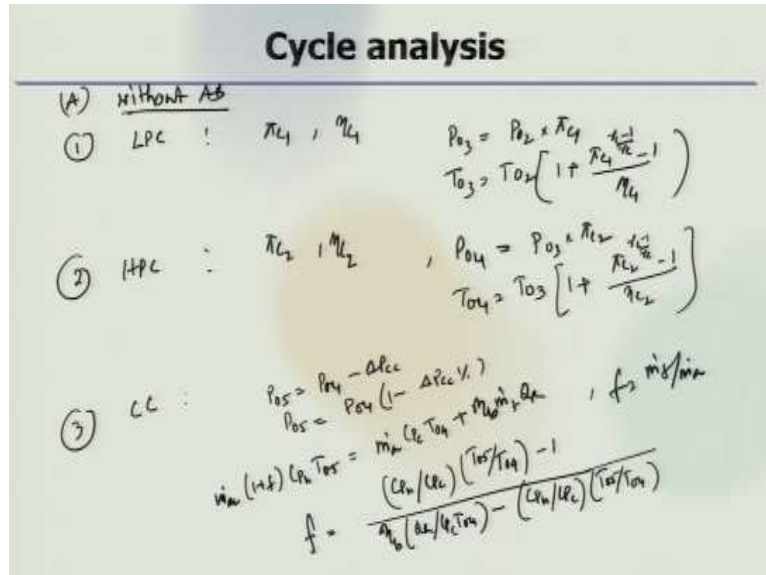


So let us see schematic of the double spool turbojet here. So this is a double spool turbojet there is no afterburner here it is afterburner and as I have already talked about that when you talk about this double spool that means that you have one low pressure compressor LPC, high pressure compressor similarly at the high pressure turbine and low pressure turbine which actually used to so that means that they are connected in a single shaft to operate each other.

Now, we can have a TS diagram for let us say we draw the TS diagram for this one which is without AB one that means if I look at the TS diagram so this is a goes here a then P_{02} so this is 02 then this is P_{03} it is 03 then this is P_{04} that means 04 then from here it will go to 05 that means P_{05} then the expansion takes place let us say so this is 06 P_{06} then it again comes 07 P_{07} then if there is no afterburner this is 08 and then it comes back.

So this is where this is 9 P_9 or rather instead of this is 03, 02, 05, 06, 07, 08 and 9. So these are the points which are marked and that means you can see where it can be it is upstream then 02 this is 2 to 3, 3 to 4, 4 to 5, 5 to 6, 6 to 7 and 7 to 8 and 8 to 9 and then when it is with afterburner. So this is one with afterburner so 1 a this is 02, 03, 04 this is 05, 06, 07 then it will 08A this is 9A. So with afterburner this is how the things would change and now this points are again A is ahead of the compressor then 2 here, 3, 4, 5, 6, 7, 8 and then 9.

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Now first we will look at a situation where without AB that means the afterburner is not operative. So first is LPC low pressure compressor so the low pressure compressor the pressure ratio is given as π_{c1} and isentropic efficiency η_{c1} then what we will get

$$P_{03} = P_{02} * \pi_{c1}$$

and

$$T_{03} = T_{02} \left[1 + \frac{\pi_{c1}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c1}} \right]$$

Now at the second it goes to HPC.

So here also the pressure ratio is π_{c2} and isentropic efficiency is η_{c2} and isentropic efficiency is η_{c2} . So at the outlet of the HPC it would be $P_{04} = P_{03} * \pi_{c2}$ so π_{c2} and T_{04} which would be

$$T_{04} = T_{03} \left[1 + \frac{\pi_{c2}^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_{c2}} \right]$$

Then we come combustion chamber. So, the temperature at the end of the combustion process is T_{05} which is again generally known because it is the maximum temperature in the engine.

If the afterburner is not on and this should be dictated by turbine inlet temperature so with that we get the pressure loss across the combustion chamber or

$$P_{05} = P_{04} (1 - \Delta p_{cc} \%)$$

Now from the energy balance what we write

$$\dot{m}_a (1 + f) C_{pH} T_{05} = \dot{m}_a C_{pC} T_{04} + \eta_b \dot{m}_f Q_R$$

Now with here

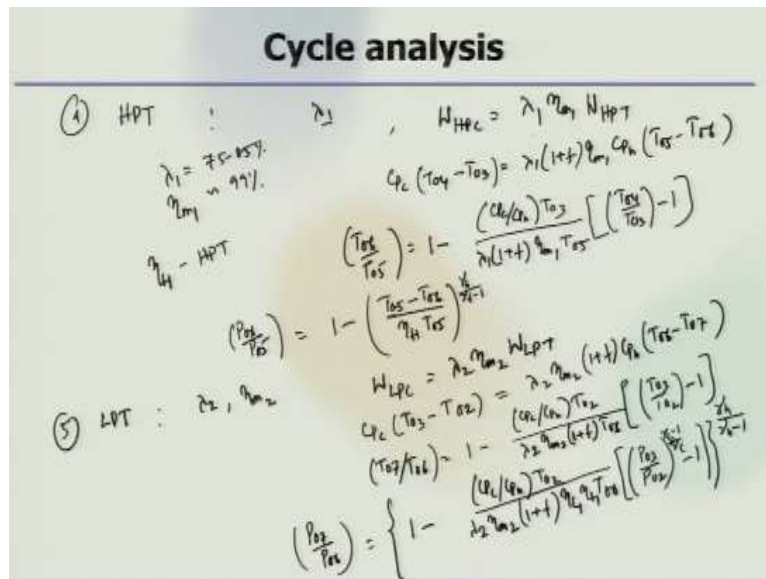
$$f = \frac{\dot{m}_f}{\dot{m}_a}$$

So what we get

$$f = \frac{\left(\frac{C_{Ph}}{C_{Pc}}\right) \left(\frac{T_{05}}{T_{04}}\right) - 1}{\left(\frac{\eta_b Q_R}{C_{Pc} T_{04}}\right) - \left(\frac{C_{Ph}}{C_{Pc}}\right) \left(\frac{T_{05}}{T_{04}}\right)}$$

So, then we have the combustion chamber.

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We will go to then high pressure turbine now high pressure turbine again it is driving the high pressure compressor so if the ratio of the power which is produced from the high pressure turbine goes to high pressure compressor is λ_1 then what we can write

$$W_{HPC} = \lambda_1 \eta_{m1} W_{HPT}$$

So, typically λ_1 in the range of 75% to 80% and η_{m1} is around 99%.

So, mechanical efficiency is quite high that means it converts the pretty much completely the power which is produced also you can see the percentage around 15% to 25% power goes to run the compressor. So then we can write once we understand this logic of then

$$C_{Pc}(T_{04} - T_{03}) = \lambda_1(1 + f)\eta_{m1}C_{Ph}(T_{05} - T_{06})$$

$$\frac{T_{06}}{T_{05}} = 1 - \frac{(C_{Pc}/C_{Ph})T_{03}}{\lambda_1(1 + f)\eta_{m1}T_{05}} \left[\left(\frac{T_{04}}{T_{03}}\right) - 1 \right]$$

then the pressure ratio of the high pressure turbine.

And high pressure compressor are could be related like

$$\frac{p_{06}}{p_{05}} = 1 - \left(\frac{T_{05} - T_{06}}{\eta_{t1} T_{05}} \right)^{\frac{\gamma_t}{\gamma_t - 1}}$$

So let us say η_{t1} here is the isentropic efficiency of the high pressure turbine. So this is the efficiency of the HPT. Now similarly we would do the similar analysis for LPT there we assume the assume that the λ_2 or the percentage of the low pressure turbine goes to run the LPC because LPC is given by this LPT.

And then the mechanical efficiency would be another η_{m2} so we can write

$$W_{LPC} = \lambda_2 \eta_{m2} W_{LPT}$$

So which again we can write

$$C_{Pc}(T_{03} - T_{02}) = \lambda_2(1 + f)\eta_{m1}C_{Ph}(T_{06} - T_{07})$$

So we write we get the ratio of

$$\frac{T_{07}}{T_{06}} = 1 - \frac{(C_{Pc}/C_{Ph})T_{02}}{\lambda_2(1 + f)\eta_{m2}T_{06}} \left[\left(\frac{T_{03}}{T_{02}} \right) - 1 \right]$$

So then the turbine and compressor pressure ratios are pretty much related. So what we can get that

$$\frac{p_{07}}{p_{06}} = \left\{ 1 - \frac{(C_{Pc}/C_{Ph})T_{02}}{\lambda_2(1 + f)\eta_{m2}\eta_{c1}\eta_{t1}T_{06}} \left[\left(\frac{p_{03}}{p_{02}} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \right\}^{\frac{\gamma_h}{\gamma_h - 1}}$$

So we can use again like the equation from the diffuser part.

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Cycle analysis

$$\left(\frac{p_{02}}{p_0} \right) = \left\{ 1 - \frac{(C_{Pc}/C_{Ph})T_{02}}{\lambda_2 \eta_{m2} (1+f) \eta_{c1} \eta_{t1} T_{06}} \left(1 + \frac{\gamma_c - 1}{2} \left[\left(\frac{p_{03}}{p_{02}} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \right) \right\}^{\frac{\gamma_h}{\gamma_h - 1}}$$

① Jet-Pit $p_{02} = p_{07} - \Delta p_{jet-pit}$
 $T_{02} = T_{07}$

② Nozzle $\frac{p_{02}}{p_c} = \left[1 - \frac{\gamma_h - 1}{\gamma_h} \left(\frac{v_2}{v_1} \right) \right]^{\frac{\gamma_h}{\gamma_h - 1}}$

velocity } $v_2 = \sqrt{2 C_{Ph} \eta_{t2} T_{02} \left[1 - \left(\frac{p_2}{p_{02}} \right)^{\frac{\gamma_h - 1}{\gamma_h}} \right]}$
 chocked : $\frac{T_{02}}{T_0} = \left(\frac{v_2}{v_1} \right)^2, v_2 = \sqrt{\gamma_h R T_0}$

So this can look quite messy, but please keep this in mind one do not need to actually remember this. One can write down the equation and carry out this simple analysis. This is all from the simple thermodynamical analysis I am writing and everywhere we need this isentropic relations and things like that and that is why we have talked about so much about those things at the beginning.

Now finally we can come to jet pipe so here this is following the low pressure turbine, this is before the nozzle. So total temperature remain constant so they are only things may happen that jet pipe and T_{08} would be T_{07} and finally we come to nozzle again first the thing that needs to be checked whether nozzle choking or not then the critical pressure ratio is important. So

$$\frac{p_{08}}{p_c} = \frac{1}{\left[1 - \frac{1}{\eta_N} \left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}}$$

So if the nozzle is unchoked then the outlet pressure is equal to the inlet pressure and then

$$V_9 = \sqrt{2C_{ph}\eta_N T_{08} \left[1 - \left(\frac{p_a}{p_{08}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]}$$

so this is the case when this is unchoked and if it is choked then the

$$\frac{T_{08}}{T_9} = \frac{\gamma_h + 1}{2}$$

$$V_9 = \sqrt{\gamma_h R T_9}$$

Now that is what happens when we have the engine without the afterburner.

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Cycle analysis

(B) with AS

$$P_{08A} = P_{07} - \Delta P_{07}$$

$$P_{08A} = P_{07} + (1 - \Delta P_{07} \%)$$

$$T_{08A} = T_{07A}$$

$f_{AB} = \frac{(1+f)(C_{pA}T_{08A} - C_{pA}T_{07})}{\eta_{AB}R_e - C_{pA}T_{07A}}$ (energy balance)

Nozzle

$$\frac{P_{08A}}{P_c} = \left[1 - \frac{1}{\eta_N} \left(\frac{\gamma_h - 1}{\gamma_h + 1}\right)\right]^{\frac{\gamma_h}{\gamma_h - 1}}$$

velocity:

unchoked:

$$V_{9AB} = \sqrt{2C_{pA} \eta_N T_{08A} \left[1 - \left(\frac{P_a}{P_{08A}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]}$$

choked:

$$\frac{T_{08A}}{T_{9A}} = \frac{\gamma_h + 1}{2}$$

$$V_{9AB} = \sqrt{\gamma_h R T_{9A}}$$

Now the second case could be with AB. Now with AB so the same treatment would be there till the afterburner is on or the upstream of the afterburner that may still the like. So if you go back up to this point the analysis would be same like here beyond the LPT then only after that afterburner is on so we can take into account that and there the

$$p_{08A} = p_{07} - \Delta p_{ab}$$

$$p_{08A} = p_{07}(1 - \Delta p_{ab} \%)$$

So the maximum temperature in the cycle that would be T_{08A} which is T_{max} and also the fuel air ratio at the afterburner can be calculated which is f_{ab} . Again from the energy balance equation there

$$f_{ab} = \frac{(1 + f)(C_{P8A}T_{0A}) - C_{P7}T_{07}}{\eta_{ab}Q_R - C_{P8A}T_{0A}}$$

Now once we do that we go to the nozzle again similarly the checking has to be done for the nozzle like P_{08A} / P_c which would be again calculation of that critical pressure.

And finding out whether the nozzle is choked or not. So again if it is unchoked nozzle and then the jet speed

$$V_{9ab} = \sqrt{2C_{Pn}\eta_N T_{08A} \left[1 - \left(\frac{p_a}{p_{08}} \right)^{\frac{\gamma_h - 1}{\gamma_h}} \right]}$$

or if the nozzle is choked so then the exhaust gases left the nozzle with the temperate T_{9A} which is

$$\frac{T_8}{T_{9A}} = \frac{\gamma_h + 1}{2}$$

$$V_{9ab} = \sqrt{\gamma_h R T_{9A}}$$

So the sum of the performance parameter would be calculated and now so the important part here is that one has to see every time. So the only difference with ab is that the calculation as I said calculations up to low pressure turbine would remain the same. So whatever we have done here all these calculations would be valid and this would be valid till the point that LPT calculations.

And then after LPT only there would be a pressure loss in the afterburner and then we can find out from the energy balance of the afterburner this is the energy balance in the afterburner we will find out the fuel air ratio then again coming back to the nozzle we find out the critical

pressure ratio and then check the nozzle is choked or unchoked so this is how so we will look at the other just for dual spool engine the performance parameter and all this in the next class.