Introduction to Airbreathing Propulsion
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Lecture – 26
Performance/Cycle Analysis: Pulsejet (Contd.,), Ramjet

So, let us continue the discussion on the Ramjet and we are looking at the individual modules first like we have looked at intake and the burner. Now we are looking at we are going to look at nozzle and then finally the complete engine analysis.

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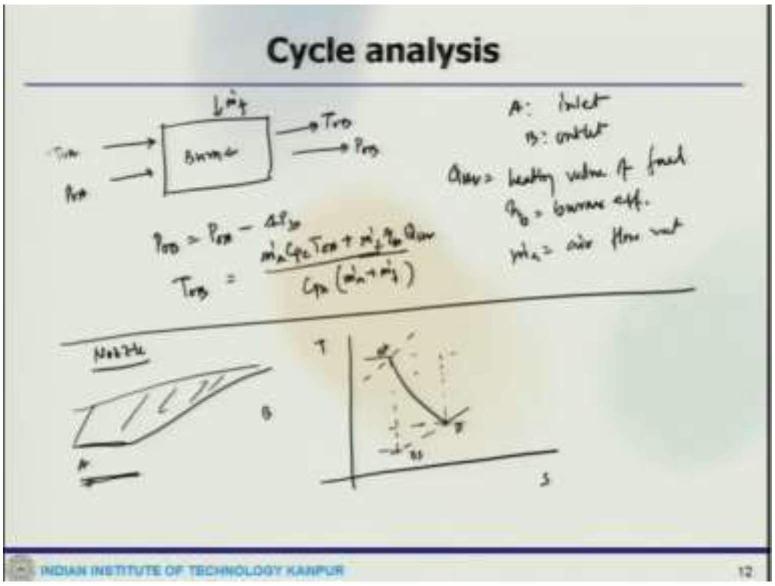

So how the nozzle again the nozzle would look like let us say draw a schematic of the nozzle it may look let us say like this here is some portion here. So again, this is A and B which is used as inlet and exit so the TS diagram for the nozzle module this should be A to B. So, this is B this so this is S this is OA. So, this is let us say we get OB and this portion is $\frac{V_B^2}{2C_P}$

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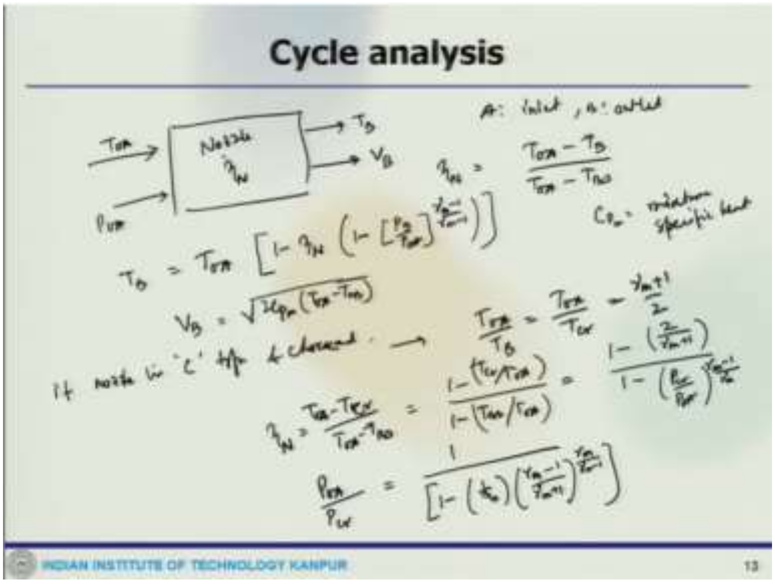

And if I draw the again the block diagram let us say the nozzle here is the nozzle it has nozzle efficiency of η_N exit velocity V_B exit temperature inlet T_{0A}, P_{0A} again A is inlet B is outlet. Now what happens then

$$\eta_N = \frac{T_{0A} - T_{0B}}{T_{0A} - T_{BS}}$$

and what we can do

$$T_B = T_{0A}\left[1 - \eta_N\left(1 - \left[\frac{p_B}{p_{0A}}\right]^{\frac{\gamma_m}{\gamma_m - 1}}\right)\right]$$

So, and we get the exit velocity is

$$V_B = \sqrt{2C_{Pm}(T_{0A} - T_{0B})}$$

here C_{Pm} is the mixture specific heat.

Because now this is after the bond product, so we use that now let us say if nozzle is a convergent type and choked then the Mach number at the exit is sonic. So exit condition would be critical one then the ratio between the total temperature to the static one would be

$$\frac{T_{0A}}{T_{cr}} = \frac{\gamma_m + 1}{2}$$

and the nozzle efficiency that time could be calculated

$$\eta_N = \frac{T_{0A} - T_{cr}}{T_{0A} - T_{BS}} = \frac{1 - \frac{T_{cr}}{T_{0A}}}{1 - \frac{T_{BS}}{T_{0A}}} = \frac{1 - \left(\frac{2}{\gamma_m + 1}\right)}{1 - \left(\frac{p_{cr}}{p_{0A}}\right)^{\frac{\gamma_m - 1}{\gamma_m}}}$$

Now and critical pressure ratio we know that this would be

$$\frac{p_{0A}}{p_{cr}} = \frac{1}{\left[1 - \left(\frac{1}{\eta_N}\right)\left(\frac{\gamma_m - 1}{\gamma_m + 1}\right)^{\frac{\gamma_m - 1}{\gamma_m}}\right]}$$

So, this is what the critical pressure ratio is so now you see why the discussion of compressible flow and all these relationships are important. Because this is what going to be handy.

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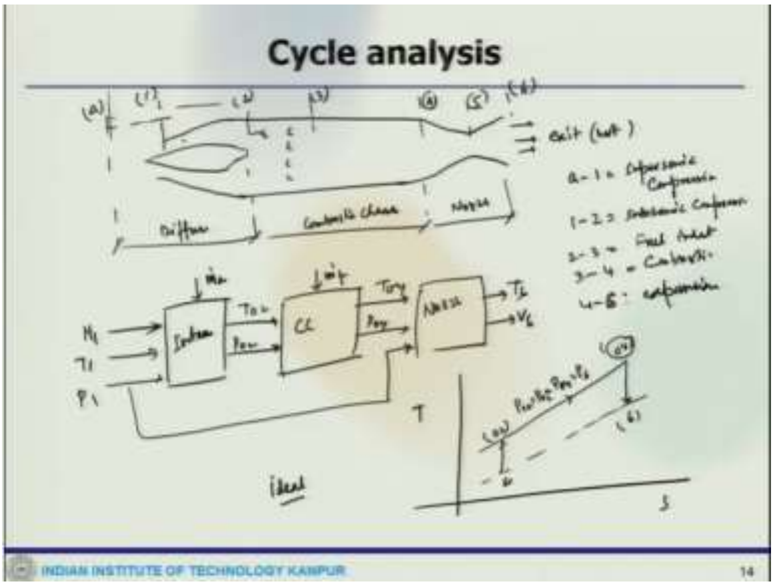

Now with that we move at with the complete engine let us say we draw an engine like that something like that then so then there would be nozzle. So here things are at the exit these are the hot then we have this portion for the inlet this is called the diffuser then we have here let us say somewhere fuel is injected this is the burner so we call this one as the combustion chamber.

And then this portion we call it nozzle now we have different zone let us say a1 then we got it 2, 3, 4, 5, 6. Now a to 1 is supersonic compression 1 to 2 subsonic compression 2 to 3 fuel inlet 3 to 4 combustion 4 to 5 or rather 4 to complete 6 is the expansion process in nozzle. So and if you put a sort of an block diagram to that then it will look like and let us say you have intake where you m1 comes in T_1 P_1 this is where your $\dot{m}_a$ goes T_{02}, P_{02} this is combustion chamber where you inject fuel and then it comes out T_{04}, P_{04} goes to nozzle it gets T_6 gets V_6 and in between there could be so can go there.

So this is the complete block diagram now once you have the block diagram then you can always put the cycle let us say we can put TS diagram let us say TS diagram what happens we will go here this is a and then this is 4 then we come to 6 so this is 02 and then this is 6 this is 04. So, this is an ideal TS diagram where P_{0A} is P_{02} is $P_{04} = P_6$ that is a ideal situation. Now in realistically that would not happen because you will have this oblique shock or shock boundary layer interactions which will take place in the intake there will be losses and that loss will also continue.

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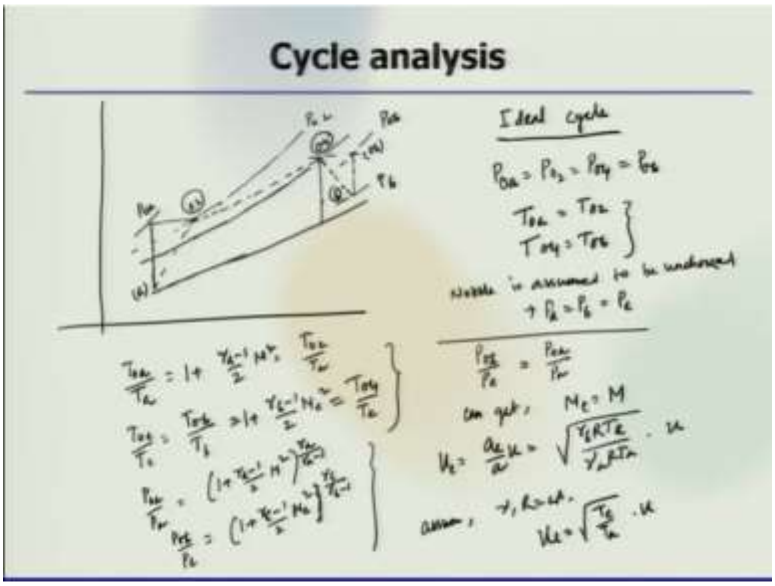

And that is why the actual cycle would be slightly different and if you plot that actual cycle let us say this is the point of 02 so this is P_{02} and so instead of this this is a this is P_{0A} okay so the actual one comes here then you go also to this is 04 okay so the actual is the dotted. So, it comes here and so this is P_6 this is point 6 and this is P_{06} so this would be so here these lines which are dotted lines.

Now the actual one differs from the real one and that because of some of these losses which takes place. Now first we will start with the ideal cycle now when you look at the ideal cycle this is the previous TS diagram that we have what we will have that the ideal cycle the pressure total pressure is constant in the cycle. So P_{0A} is P_{02} is P_{04} which is P_{06} okay so since no work nor heat addition or rejection takes place in the intake and nozzle so we can apply the first law of thermodynamics.

So what will get T_{0A} is T_{02} and T_{04} is T_{06} so that is what we get now nozzle is assumed to be unchoked so then the full expansion of the hot gases within the nozzle to the ambient pressure is assumed and therefore will have $p_a = p_6 = p_{exit}$. Now we can write down the all the total and stagnation condition between 6 and e what will write

$$\frac{T_{0a}}{T_a} = 1 + \frac{\gamma_a - 1}{2} M^2 = \frac{T_{02}}{T_a}$$

and

$$\frac{T_{0e}}{T_e} = \frac{T_{06}}{T_6} = 1 + \frac{\gamma_6 - 1}{2} M_e^2 = \frac{T_{04}}{T_e}$$

And similarly, we can write for the pressure

$$\frac{p_{0a}}{p_a} = \left(1 + \frac{\gamma_a - 1}{2} M^2\right)^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\frac{p_{06}}{p_6} = \left(1 + \frac{\gamma_6 - 1}{2} M_e^2\right)^{\frac{\gamma_6}{\gamma_6 - 1}}$$

So, these are the stagnation condition that we can write now what we also have

$$\frac{p_{06}}{p_e} = \frac{p_{0a}}{p_a}$$

so we can arrive at the relation at that so can get M_e is M.

Now the inlet and exit Mach numbers are equal but the exit speed is not equal to the flight speed so what is clear that

$$u_e = \frac{a_e}{a} u = \sqrt{\frac{\gamma_6 R T_e}{\gamma_a R T_a}} u$$

So, if you assume γ *and* R to be constant within the engine then u exit would be

$$u_e = \sqrt{\frac{T_e}{T_a}} u$$

So, you so now we can replace that with the temperature so we can get this is T_{04}/T_{0A} into u which is equivalent to T_{04}/T_{02} into u.

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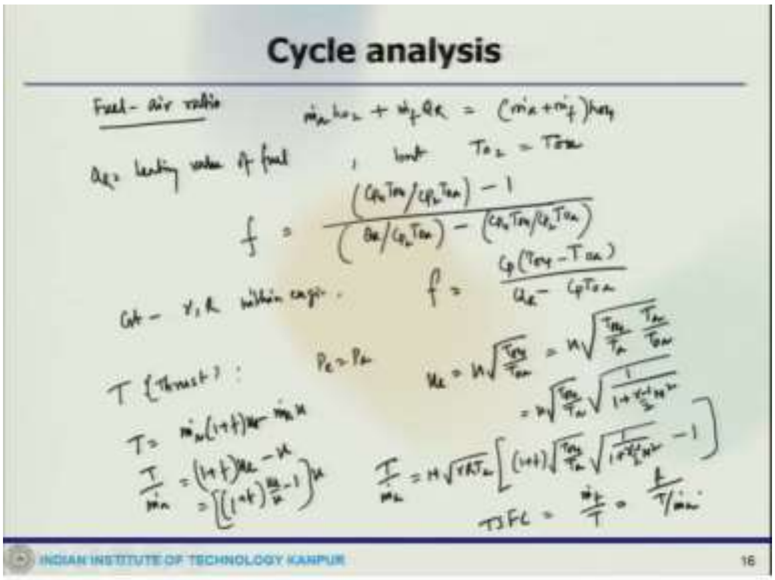

Now we can find out the fuel-air ratio in the burner or the combustion chamber so there will use the energy balance equation which is

$$\dot{m}_a h_{02} + \dot{m}_f Q_R = (\dot{m}_a + \dot{m}_f) h_{04}$$

where Q_R is the heating value of fuel but what we have T_{02} is T_{0a} so we will get

$$f = \frac{(C_{P4}T_{04}/C_{P2}T_{0a}) - 1}{(Q_R/C_{P2}T_{0a}) - (C_{P4}T_{04}/C_{P2}T_{0a})}$$

Now for constant γ *and* R within the engine f was down to

$$f = \frac{C_P(T_{04} - T_{0a})}{(Q_R - C_P T_{0a})}$$

Now we get the thrust force now thrust now since the nozzle is fully expanded so that is p_e is p_a so what we can get is that

$$T = \dot{m}_a(1 + f)u_e - \dot{m}_a u$$

$$\frac{T}{\dot{m}_a} = (1 + f)u_e - u$$

$$\frac{T}{\dot{m}_a} = \left[(1 + f)\frac{u_e}{u} - 1\right] u$$

Now we can use another relationship which is

$$u_e = u\sqrt{\frac{T_{04}}{T_{0a}}} = u\sqrt{\frac{T_{04}}{T_a}\frac{T_a}{T_{0a}}}$$

$$u_e = u\sqrt{\frac{T_{04}}{T_a}}\sqrt{\frac{1}{1 + \frac{\gamma - 1}{2}M^2}}$$

so the thrust force can be expressed which is

$$\frac{T}{\dot{m}_a} = M\sqrt{\gamma R T_a}\left[(1+f)\sqrt{\frac{T_{04}}{T_a}}\sqrt{\frac{1}{1+\frac{\gamma-1}{2}M^2}} - 1\right]$$

And TSFC this would be

$$TSFC = \frac{\dot{m}_f}{T} = \frac{f}{T/\dot{m}_a}$$

so these are the situation what you will get from the ideal cycle. Now in the real cycle the things will deviate and there because there would be nothing like intake there would be losses which we need to take into account but then the combustor there would be a little bit of pressure loss because the combustion efficiency may not be 100% their nozzle there would be losses.

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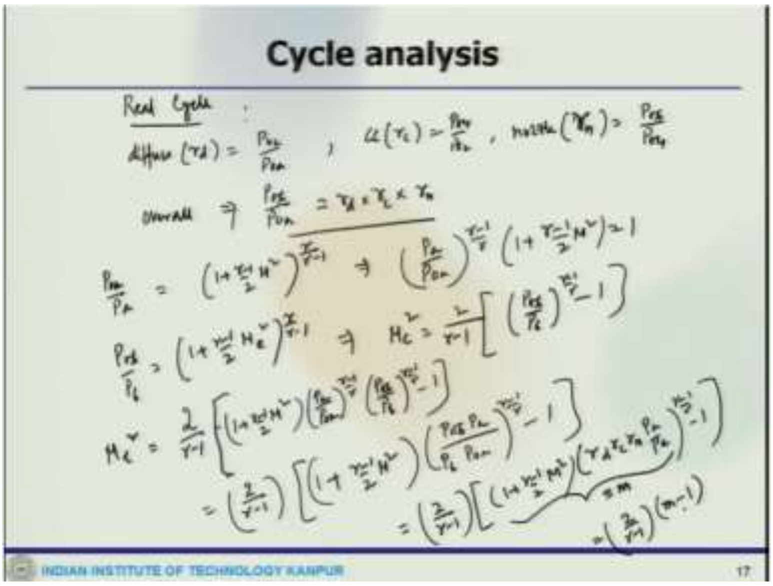

So what would happen that when we look at the real cycle now we have to have different pressure ratios or the stagnation pressure ratio like let us say in the diffuser you have r_d which is a pressure ratio which is called p_{02}/p_{0a} then combustion chamber it is r_c which is p_{04}/p_{02} then we have nozzle which is it let us say $r_n = p_{06}/p_{04}$. So, these are the stagnation pressure ratio that we can define and the overall pressure ratio is p_{06}/p_{0a} which is r_d into r_c into r_n.

So that is the overall pressure ratio but the properties γ, R as it passed through the different portion of the engines, we can still assume constant. So now what we write that stagnation pressure there is

$$\frac{p_{0a}}{p_a} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

so what we get from here

$$\left(\frac{p_a}{p_{0a}}\right)^{\frac{\gamma-1}{\gamma}}\left(1+\frac{\gamma-1}{2}M^2\right)=1$$

similarly

$$\frac{p_{06}}{p_6}=\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

So we get

$$M_e^2=\frac{2}{\gamma-1}\left[\left(\frac{p_{06}}{p_6}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

so we can write

$$M_e^2=\frac{2}{\gamma-1}\left[\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{p_a}{p_{0a}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{p_{06}}{p_6}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

$$M_e^2=\frac{2}{\gamma-1}\left[\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{p_{06}}{p_6}\frac{p_a}{p_{0a}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

$$M_e^2=\frac{2}{\gamma-1}\left[\left(1+\frac{\gamma-1}{2}M^2\right)\left(r_d r_c r_n\frac{p_a}{p_e}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

Let us say we define these quantities completely something equivalent to m then we can write

$$M_e^2=\frac{2}{\gamma-1}(m-1)$$

so that is what you can write okay.

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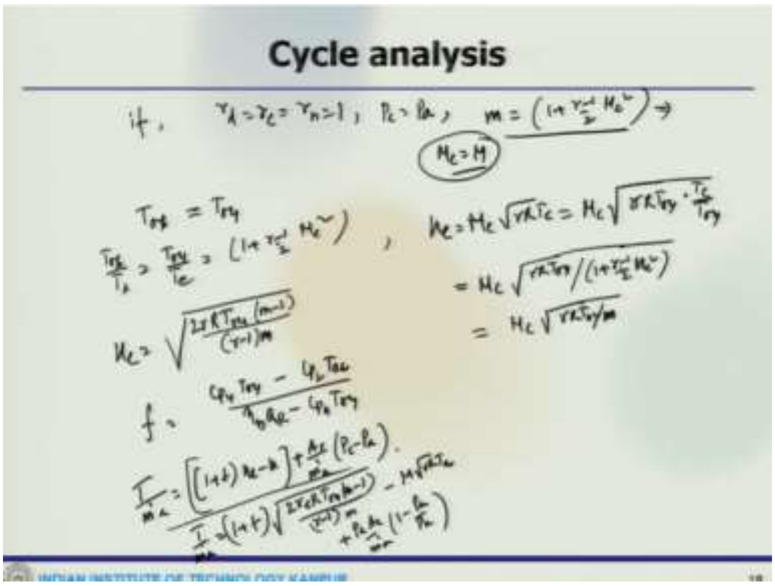

So also, if r_d, r_c, r_n is 1 p = p_a then we get

$$m = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)$$

so which essentially means M_e is M okay. Now the heat if the heat transfer from the engine is assumed negligible then the exhaust total temperature T_{06} would be T_{04} and what we can get is that

$$\frac{T_{06}}{T_6} = \frac{T_{04}}{T_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)$$

so we get

$$u_e = M_e\sqrt{\gamma R T_e} = M_e\sqrt{\gamma R T_{04}\frac{T_e}{T_{04}}} = M_e\sqrt{\frac{\gamma R T_{04}}{\left(1 + \frac{\gamma - 1}{2} M_e^2\right)}} = M_e\sqrt{\frac{\gamma R T_{04}}{m}}$$

so we can also see that from here we get that these things is this. So now what we can write this is m now substituting this what we get

$$u_e = \sqrt{\frac{2\gamma R T_{04}(m - 1)}{(\gamma - 1)m}}$$

So irreversibility's they have no effect on total temperature throughout the engine. So we can find out the fuel air ratio for the real

$$f = \frac{C_{P4}T_{04} - C_{P2}T_{0a}}{\eta_b Q_R - C_{P4}T_{04}}$$

and the specific thrust

$$\frac{T}{\dot{m}_a} = \left[(1 + f)\frac{u_e}{u} - 1\right]u + \frac{A_e}{\dot{m}_a}(p_e - p_a)$$

$$\frac{T}{\dot{m}_a} = (1 + f)\sqrt{\frac{2\gamma R T_{04}(m - 1)}{(\gamma - 1)m}} - M\sqrt{\gamma R T_a} + \frac{p_e A_e}{\dot{m}_a}(1 - \frac{p_a}{p_e})$$

So that is the specific thrust expression so this can be derived.

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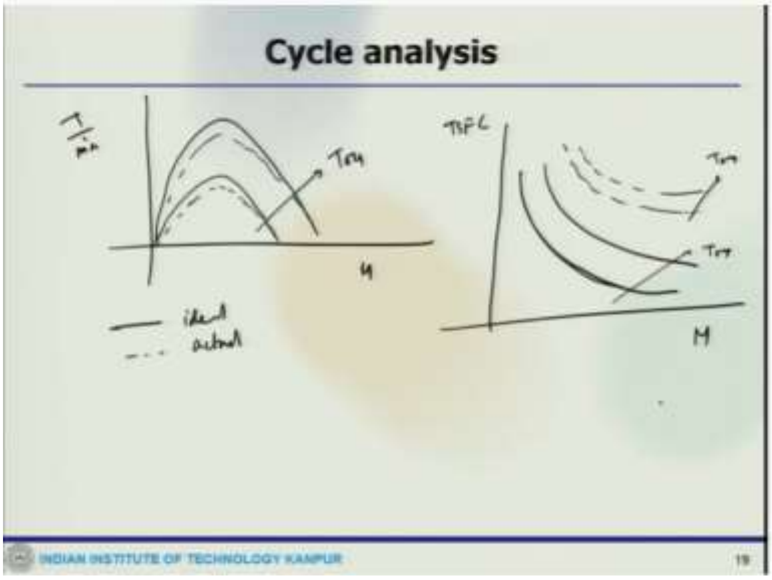

And so, if you look at the picture like if I plot T/m dot a mass as m so this is how thing would so this is increasing T04 for this is the actual ideal cycle and the actual cycle would be like this for both the cases. So, this is ideal 1 and this is actual cycle how it look like similarly if you see that TSFC versus Mach number. So, this is how it look like this is again. So, increasing T04 and this is how actual cycle would work.

So, this is also T04 so you can see how the specific thrust and my TSFC they vary between actual and ideal cycle. So, one thing is clear here that when you consider the actual cycle wanted to calculate or take into account the losses and irreversibility's which take care and then the actual cycle always deviates from the ideal one. But it is good to see the ideal one first because that short of in special case of the actual one when all the efficiencies and everything becomes unit and all this but realistically there are losses so we will continue this discussion in the next class from here.