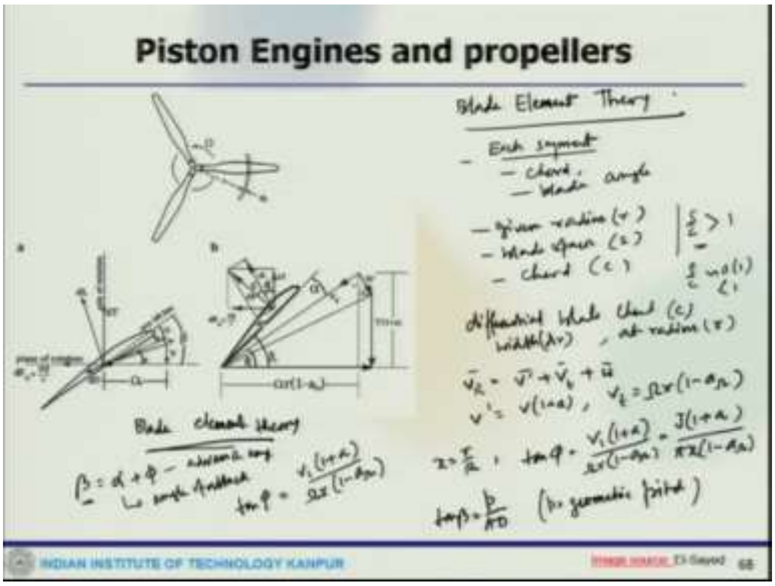
Introduction to Airbreathing Propulsion
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology - Kanpur

Lecture – 23
Piston Engines and Propellers (Contd.,)

Okay, so we have looked at this actuator disk theory and all these for the propeller, now there is a one more topic that I would like to discuss on these things is the blade configurations or the blade element consideration.

(Refer Slide Time: 00:35)



So, let us finish that discussion, so that that will pretty much conclude the discussion on propeller and all these, so just an this is a schematic which give you an idea about the any blade element theory, so just look at the different cross sections and all these things because this is quite involved and there are multiple parameters which are also involved there, so this is some now, this is called blade element consideration or theory.

So, one can think about blade element theory, so this is just like wing theory, so here the blade is divided into small sections which are handled independently from each other, each segment has a chord, a blade angle associated, so each segment they will have a chord, will have a blade angle and associated airfoil characteristics, the isolated airfoil assumptions is found to be substantially correct for 2 or 3 bladed propeller except near hub.

Now, we can have a given radius as, given radius r and circumferential distance between 2 successive blades spacing which is s, while the length of the blade section between leading and trailing edges, so that is chord c, so s by c for 2 or 3 blade propellers are much greater than; so s by c would be much greater than 1 for 2 or 3 bladed propeller, so that justifies the use of isolated airfoil characteristics in this theory.

So, where the multiple blade propeller where this s by s ratio to be; s by c ratio to be order of 1, then or less than 1, so there it can be treated as cascade, so since here is s by c greater than 1, we can treat them an individual aerofoil. Now, one can see the small aerofoil sections and then they are the forces like lift and drag can be applied and we can determine the rest of the calculation.

So, there are, let us say a differential blade chord which is c with dr and at the at radius r, okay from the propeller axis as shown here, so from here this is at r, this is dr. Now, the element is shown acting under the influence of rotational speed V_t and the forward speed V' and the induced velocity w'. So, the resultant velocity vector

$$V_R = V' + V_t + w$$

the axial velocity and the rotational velocities are given as

$$V' = V(1 + a)$$

$$V_t = \Omega r(1 - a_\Omega)$$

The relative fluid velocity varies from hub to tip, primarily under the influence of the rotational velocity which is the component of omega r and the factor a, so here you can see all the velocity vectors and the forces which are shown at the section radius r and $r + \Delta r$, whose chord length is c, so the geometric pitch angle $\beta = \alpha + \phi$, where α is the angle of attack and ϕ is the advance angle, okay.

So, this gives us

$$tan\phi = \frac{V_1(1 + a)}{\Omega r(1 - a_\Omega)}$$

now we can define a dimensionless radius that is

$$x = \frac{r}{R}$$

then this

$$tan\phi = \frac{V_1(1+a)}{\Omega r(1-a_\Omega)} = \frac{J(1+a)}{\pi x(1-a_\Omega)}$$

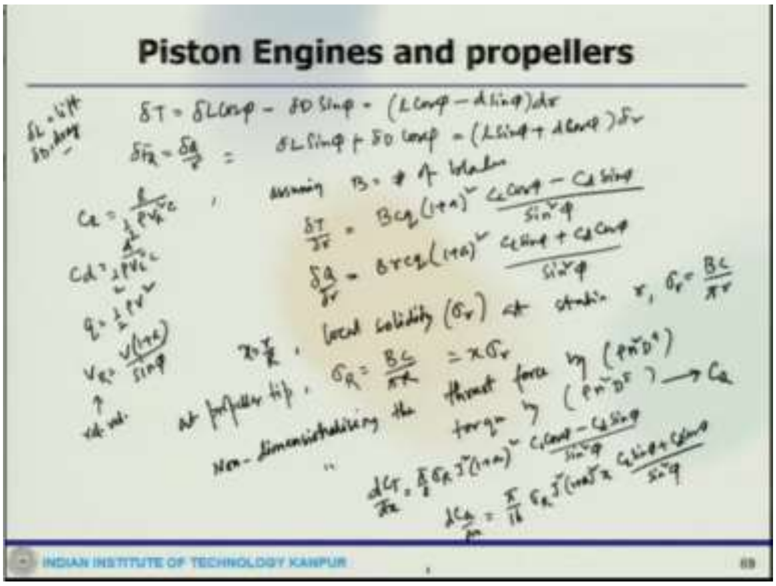
where beta is calculated based advanced propeller during the revolution with geometric pitch is P, so

$$tan\beta = \frac{P}{\pi D}$$

here P is geometric pitch.

Now, the radial pitch distribution will be specified for a given blade but a representative pitch is required as the reference radius. Now, overall the solid body pitch may be varied by rotation of the blade about its radial axis.

(Refer Slide Time: 08:18)



Now, we can calculate the thrust and the torque, now the axial and tangential forces on the blade element generate the thrust, let us say

$$\delta T = \delta L\, cos\phi - \delta D\, sin\phi = (l\, cos\phi - d\, sin\phi)dr$$

torque which is

$$\delta F_Q = \frac{\delta Q}{r} = \delta L\, sin\phi + \delta D\, cos\phi = (l\, sin\phi + d\, cos\phi)dr$$

δL is the lift, δD is the drag force on the blade element.

Now, introducing other definitions of the lift and drag coefficient C_l and C_d with the dynamic pressure q, this C_l could be defined as

$$C_l = \frac{l}{\frac{1}{2}\rho V_R^2 c}$$

$$C_d = \frac{d}{\frac{1}{2}\rho V_R^2 c}$$

$$q = \frac{1}{2}\rho V_R^2$$

so these are the; so also we can write

$$V_R = \frac{V(1+a)}{sin\phi}$$

the relative velocity, this is relative velocity. Now, we use all these things there and assuming B is number of blades, what we can write;

$$\frac{\delta T}{\delta r} = Bcq(1+a)^2 \frac{C_l\, cos\phi - C_d\, sin\phi}{Sin^2\phi}$$

And

$$\frac{\delta Q}{\delta r} = Brcq(1+a)^2 \frac{C_l\, sin\phi + C_d\, cos\phi}{Sin^2\phi}$$

now with $x = \frac{r}{R}$ and the local solidity is

$$\sigma_r = \frac{Bc}{\pi r}$$

then the solidity at the propeller tip, at propeller tip the

$$\sigma_R = \frac{Bc}{\pi R} = x\sigma_r$$

Now, so the now we can non dimensionalizing the thrust force, so non dimensionalizing the thrust force by $\rho n^2 D^4$

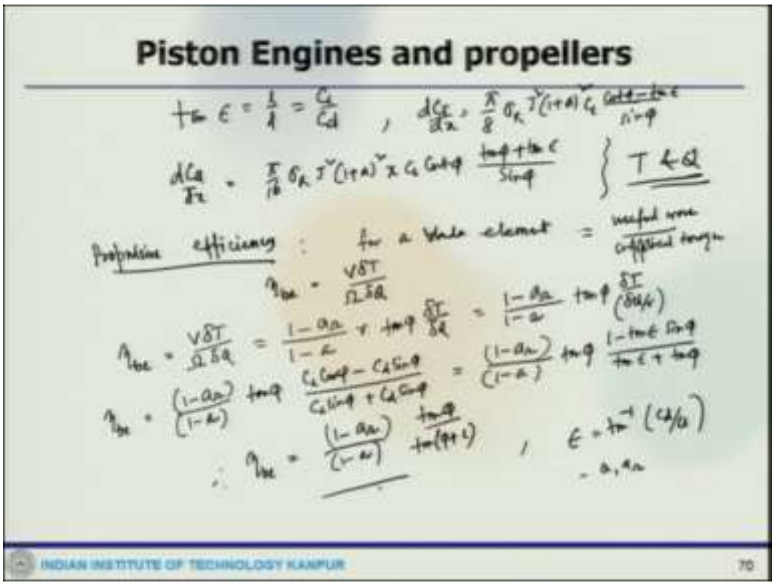
And non dimensionalize the torque by $\rho n^2 D^5$, we can get the torque coefficient C_Q, so let us see how they look like,

$$\frac{dC_T}{dx} = \frac{\pi}{8}\sigma_R \cdot J^2 (1+a)^2 \frac{C_l\, cos\phi - C_d\, sin\phi}{Sin^2\phi}$$

$$\frac{dC_Q}{dx} = \frac{\pi}{16}\sigma_R \cdot J^2 (1+a)^2 x \frac{C_l\, sin\phi + C_d\, cos\phi}{Sin^2\phi}$$

so we get these 2,

(Refer Slide Time: 14:13)



Now, the relation between the drag and lift forces on blade element can be written as

$$tan \in = \frac{l}{d} = \frac{C_l}{C_d}$$

now once we put this back, we write

$$\frac{dC_T}{dx} = \frac{\pi}{8}\sigma_R \cdot J^2(1+a)^2 C_l \frac{cot\phi - tan \in}{sin\phi}$$

okay and similarly, we write

$$\frac{dC_Q}{dx} = \frac{\pi}{16}\sigma_R \cdot J^2(1+a)^2 x C_l cot\phi \frac{tan\phi + tan \in}{sin\phi}$$

So, once we integrate this, by integrating we can find T and Q, the thrust and the torque, so those things can be obtained easily. Now, the other thing which can be calculated is the propulsive efficiency, so now we can look at propulsive efficiency. So, propulsive efficiency for a blade element is defined as the ratio between useful work to the supplied torque, so that means

$$\eta_{be} = \frac{V\delta T}{\Omega\delta Q}$$

Now, already the derived equation for $\frac{\delta T}{\delta Q}$, if we use all these what we can write;

$$\eta_{be} = \frac{1-a_\Omega}{1-a} r tan\phi \frac{\delta T}{\delta Q} = \frac{1-a_\Omega}{1-a} tan\phi \frac{\delta T}{\delta Q/r}$$

okay. So, what we can get is that we write this here as let us say,

$$\eta_{be} = \frac{1-a_\Omega}{1-a} tan\phi \frac{C_l\, cos\phi - C_d\, sin\phi}{C_l\, sin\phi + C_d\, sin\phi} = \frac{1-a_\Omega}{1-a} tan\phi \frac{1 - tan \in sin\phi}{tan \in + tan\phi}$$

So, hence we can get

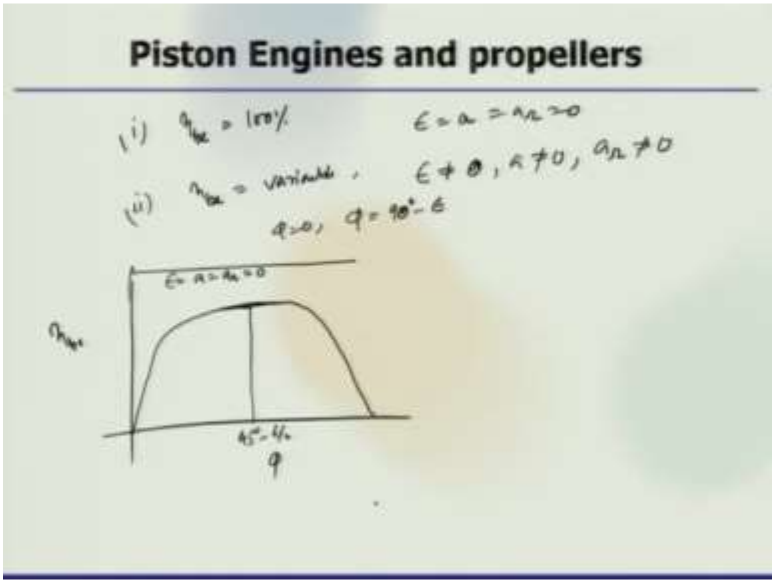
$$\eta_{be} = \frac{1 - a_{\Omega}}{1 - a} \frac{tan\phi}{\tan(\phi + \epsilon)}$$

so the local propeller efficiency what we can see from here is dependent on the angle epsilon

$$\epsilon = tan^{-1}\left({C_d}/{C_l}\right)$$

and the interference factors which are a, a_{Ω} these factors, so you can, one can plot and see the other things.

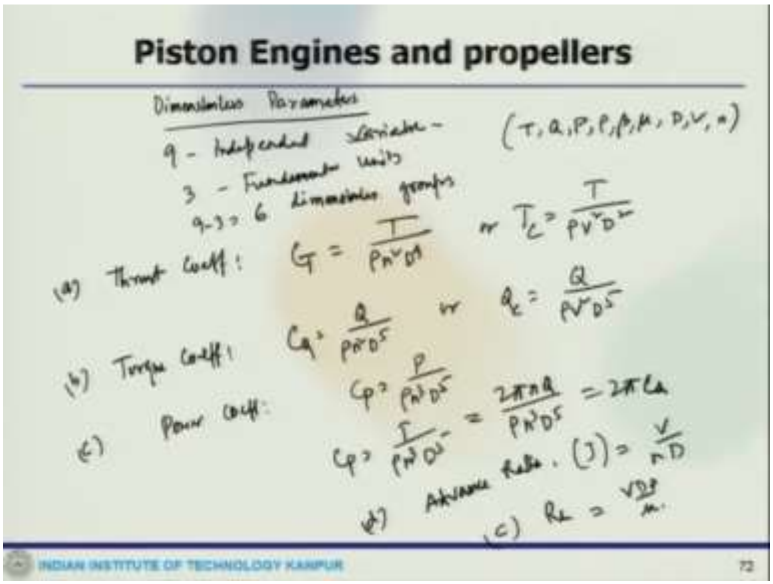
(Refer Slide Time: 19:46)



So, what happens; there could be 2 possibilities, let us say eta be is 100 percent, this is a straight case where $\epsilon = a = a_{\Omega} = 0$ or second case η_{be}, it could be variable, where ϵ is not equals to 0, a is not equal to 0, a_{Ω} not equals to 0. In that case, the blade efficiency has zero value when ϕ is 0 and $\phi = 90 - \epsilon$ So, in between value the blade element efficiency always you can say, so this is how one can look at that η_{be} with this is phi, so this goes like somewhere and comes down.

So, somewhere middle it is the maximum which is around $45 - \frac{\epsilon}{2}$ and so and the other would be a flat line when it is 100 percent, when a, a_{Ω} all 0, so that is how it actually varies.

(Refer Slide Time: 21:40)



And now we can have some dimensionless parameters like there are different dimensionless parameters which could be like, so this define and required to define the performance of the fixed which propeller, so dimension less parameters where we can have; so actually it requires 9 independent variables to define this like in subsonic; 9 independent variables are required.

So, like thrust, force, torque, power, air density, compressibility, air viscosity, propeller diameter, appropriate speed, rotational speed and these are the basket of torque, $Q, P, \rho, \beta, \mu, D, V, n$. Now, we have only 3 fundamental units, so we require 6 dimensionless group, so 9 – 3, 6 dimension less groups. So, number 1 or

a)is thrust coefficient which is C_T is

$$C_T = \frac{T}{\rho n^2 D^4}$$

$$T_C = \frac{T}{\rho V^2 D^2}$$

b) torque coefficient C_Q which is

$$C_Q = \frac{Q}{\rho n^2 D^5}$$

$$Q_C = \frac{Q}{\rho V^2 D^5}$$

Then, we have power coefficient which is C_P that is

$$C_P = \frac{P}{\rho n^3 D^5}$$

then there could be a relation between power and torque coefficients which could be

$$C_P = \frac{T}{\rho n^3 D^5} = \frac{2\pi n Q}{\rho n^3 D^5} = 2\pi C_Q$$

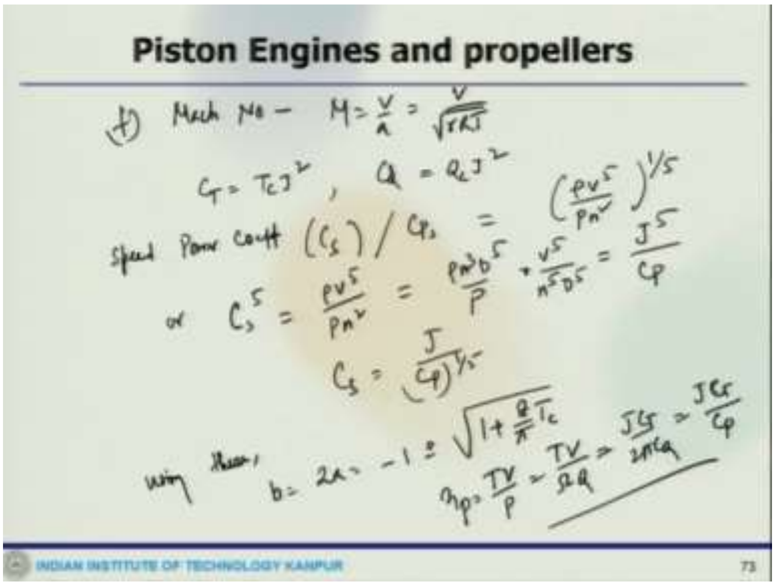
then advance ratio which is

$$J = \frac{V}{nD}$$

then Reynolds number

$$Re = \frac{VD\rho}{\mu}$$

(Refer Slide Time: 24:30)



Then we can have Mach number, which is

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}}$$

then also the first 4 dimensional quantities can be correlated like

$$C_T = T_C J^2$$

$$C_Q = Q_C J^2$$

and another important dimensionless quantity is the speed power coefficient which is denoted as C_s or sometimes C_{Ps} which is

$$C_s = C_{ps} = \left(\frac{\rho V^5}{Pn^2}\right)^{\frac{1}{5}}$$

Or

$${C_s}^5 = \frac{\rho V^5}{Pn^2} = \frac{\rho n^3 D^5}{P} \frac{V^5}{n^5 D^5} = \frac{J^5}{C_P}$$

Or

$$C_s = \frac{J}{(C_P)^{1/5}}$$

so we can also again using this relationship that we obtain again those factors that a and b we can write using these

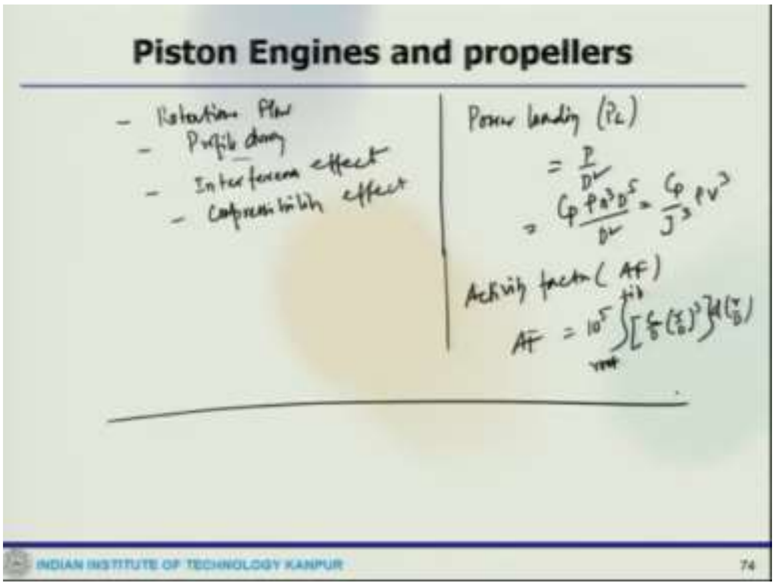
$$b = 2a = -1 \pm \sqrt{1 + \frac{8}{\pi} T_c}$$

Now, the actual propulsive efficiency can be now expressed,

$$\eta_p = \frac{TV}{P} = \frac{TV}{\Omega Q} = \frac{JC_T}{2\pi C_Q} = \frac{JC_T}{C_P}$$

So, typically η_F or the Froude efficiencies could be 1.

(Refer Slide Time: 27:10)



Now, also in actual propulsion efficiency is much less than the Froude efficiency because of some losses which are not involved in the actuator disk theory which could arise from let us say rotational flow, so this losses could arise from there, profile drag, the flow over the propeller blades when there would be profile drag or the from there, I got the viscous friction, so these are also ignored in the disk theory, so this is a rotational flow about the rotor axis in the wing.

Then interference effect that is also ignored, then compressibility effect, so these are actually not there and 2 other important parameters relevant to propeller power are power loading and this, so the power loading or

$$PL = \frac{P}{D^2} = \frac{C_P \rho n^3 D^5}{D^2} = \frac{C_P}{J^3} \rho V^3$$

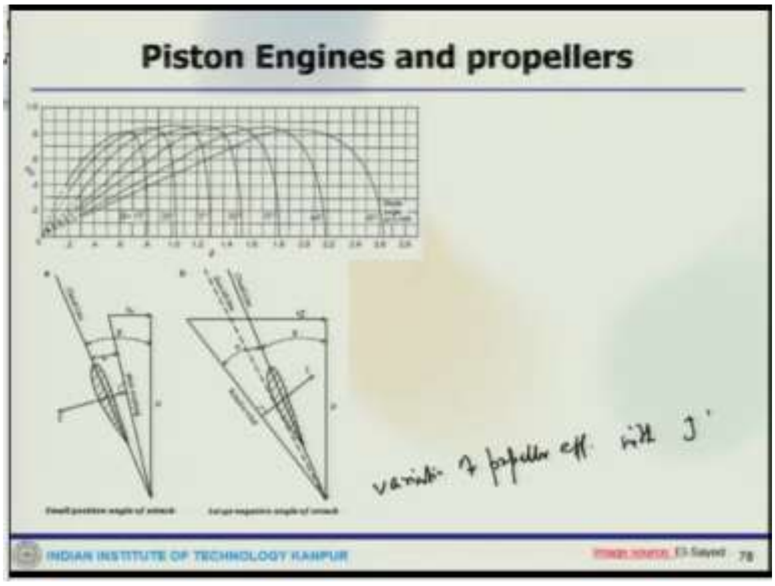
and activity factor which is AF, which is defined as

$$AF = 10^5 \int_{root}^{tip} \left[\frac{C}{D}\frac{r}{D^3}\right] d\left(\frac{r}{d}\right)$$

(Refer Slide Time: 29:10)

So, these are also can be calculated and now there are propeller performance, so the I mean, there are different kind of propellers which are used based on like NACA series, so these are based on NACA series. So, it could be 4424 or something like that, 4418, 2415, and one can do and there are different series which are available.

(Refer Slide Time: 29:36)



And their performance are kind of determined how this, I mean one can see this curve how the propeller efficiency and the advance ratio, now the different blade, so this is eta P versus J for 2 bladed propeller at different pitch, so you can see how and the beta is the pitch angle like beta, so one can see that how these things are varying, this curve give you an idea how things changes with the change in this kind of properties as such.

And the other thing which could be interesting to see is that how the variation of the propeller efficiency that is also important like this, this is also the variation of propeller efficiency with J, advance ratio. So, there is a small positive angle of attack, there could be large positive angle of attack, so one can see the different variations and then that can be sort of estimated like this.

So, essentially these factors are one has to take into kind of consideration when talking about these propeller blades like their material, their design consideration and we have talked about 2 different theories which are simplistic model obviously, one can do detailed testing in kind of wind tunnel or with properly equipped instrument or one can do computational analysis to look at the flow field and all these effect.

And then finally find out all these power coefficient, torque efficiency and everything, so that is pretty much will kind of concludes the discussion on propeller and all this, we will move to the other engines in the next class onward.