Introduction to Airbreathing Propulsion
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Lecture – 22
Piston Engines and Propellers (Contd.,)

So, let us continue the discussion with the design now, so we have looked at different kind of propeller and how they look like, their nomenclature, now we will go down to slightly different aspect of it like how aerodynamically these are designed.

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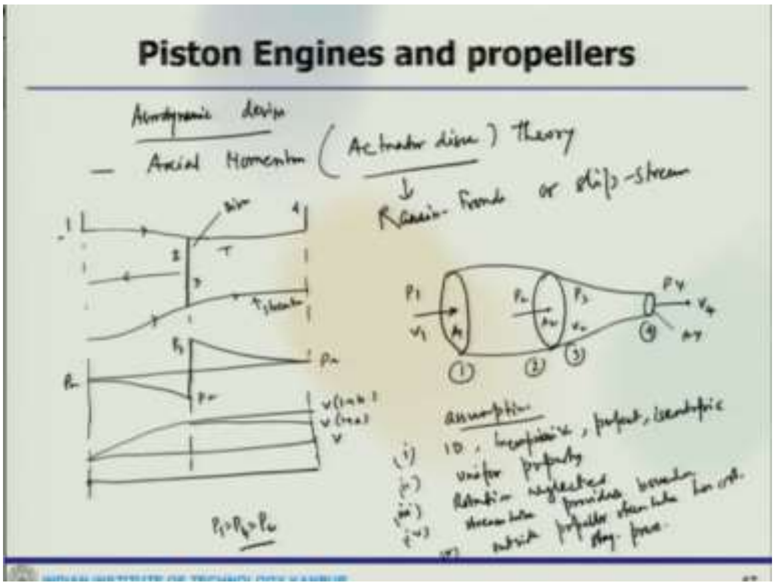

So, let us look at the aerodynamic design and that is done through different theory, so let us start with the axial momentum theory or so we will start with axial momentum or this is called actuator disk theory, okay, so we will start with that. So, actuator disk is also sometime denoted as this actuator disk also called the Rankine Froude or slipstream theory, so these are different naming which are given.

So, on 18; based on 1865, so the actuator disk theory replaces the propeller with the infinitely thin plane or actuator disk, so let us see how it can be done. Let us draw some configuration here, so let us say this is the actuator disk sitting there, then the flow could be like this, could be like this, then this goes like this, so let us say this is the actuator disk sitting there. Now, this is 1, so that is 2, this is the disk, this is T, 3, so this is a stream tube, so that is 4.

And if we plot this, so now this could be v, then this is v(1 + a), v(1 + b), this is velocity, then we can see the similarly pressure, this is P_a, this is P_2, this is P_a, so this is P_3, so and if we plot that in a slightly 3 dimensional way, then it looks like here is the disk sitting there and so this is what v_1, P_1, let us say this is area A_1, this is 2, 3, so P_2, P_3, A_2, v_2, v_4, P_4, area 4, 4, so that is the thin disk kind of now which imparts a certain momentum to the fluid passing through it, when there is this actuator disk.

Now, this theory provides an initial idea regarding the performance of the propeller and also its efficiency but it cannot provide you the detailed design related aspect, so there are certain assumption in this particular approach; one is the fluid is 1 dimensional, incompressible, then perfect, isentropic. Second flow has uniform properties across the that is velocity and pressure across any plane normal to the flow except for the discontinuous jump in pressure across the disk itself.

So, uniform property that is another, third is the rotation neglected that means, the rotation imparted to the flow is also neglected, fourth the streamlines all the edge of the disk define the outer limit of the contracting stream tube which passes through the disk and separates it from the surroundings okay. So, that means the stream tube provides the boundary, so in the stream tubes also a cylindrical section in both for upstream and for downstream.

And then fifth is the flow outside the propeller stream tube has constant stagnation pressure, no work is; so outside propeller stream tube has constant stagnation pressure, so this particular diagram provides you some pressure velocity diagram, so one is far stream here, far stream from the propeller to just in front of the propeller, so let us say, 2 is here, then 3 is here which is just after the propeller, 4 is again further downstream of the propeller.

So, the distance between 2 and 3 is infinitesimal because the disk is thin and this is what is assumed to be, now also the stream tube along the stream tube sections 1 and 4, the velocity increases from the free stream value of V_1 at cross sectional area A_1 to the area A_4 and V_4. So, the static pressure at station 1 and 4 could be P_1 and P_4, so which are atmospheric that means, P_1, P_4 it would be P atmospheric.

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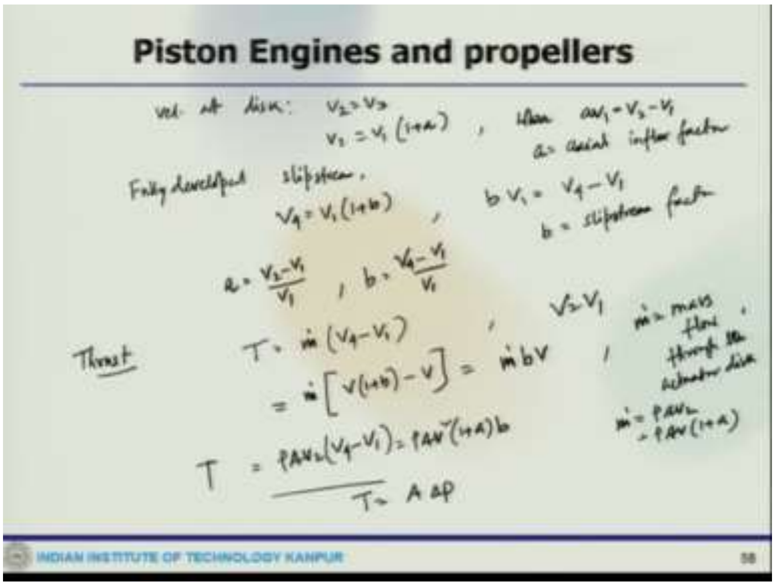

But the pressure difference across the disk is built up and so what we can write, so velocity at disk we can write $V_2 = V_3$, so which is written as $V_2 = V_1(1 + a)$ where $aV_1 = V_2 - V_1$ is the increase in velocity through the disk, where a is called the axial inflow factor, okay. So, now if we have fully developed slipstream, the velocity $V_4 = V_1(1 + b)$, where $bV_1 = V_4 - V_1$

So that is the increase in velocity there and b is called the slipstream factor, so what we can get that

$$a = \frac{V_2 - V_1}{V_1}$$

$$b = \frac{V_4 - V_1}{V_1}$$

so you can estimate the thrust, so we can apply Newton's motion and there is a region, control region between 1 and 4, the thrust could be

$$T = \dot{m}(V_4 - V_1)$$

Now, let us say if we drop the subscript on the free stream velocity V is V_1, then we can write this one is

$$T = \dot{m}[V(1 + b) - V] = \dot{m}bV$$

Now, here m dot is the mass flow rate or mass flow through the actuator disk and also we can write that

$$\dot{m} = \rho A V_2 = \rho A\, V(1 + a)$$

Now, when I plug this back together, so this will give me

$$T = \rho A V_2(V_4 - V_1) = \rho A\, V^2(1 + a)b$$

Now, force balance across the disk require that T would be

$$T = A \cdot \Delta p$$

so this is from that figure that we have drawn.

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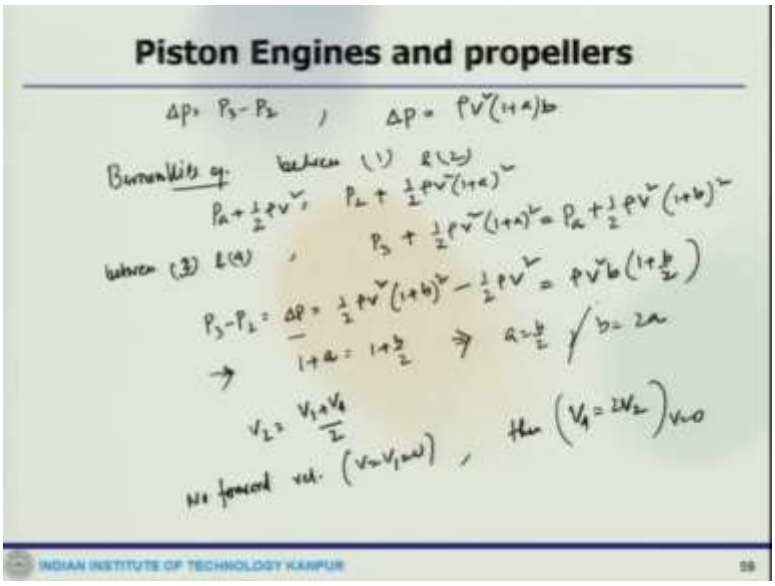

And where

$$\Delta p = p_3 - p_2$$

now if we eliminate T, we get

$$\Delta p = \rho V^2 (1 + a) b$$

now we have assumed the flow is incompressible, so the Bernoulli's equation applied between; now we apply Bernoulli's equation between 1 and 2, we write

$$p_a + \frac{1}{2} \rho V^2$$

$$p_2 + \frac{1}{2} \rho V^2 (1 + a)^2$$

Now, similarly we apply the Bernoulli's equation between 2 and 3 and 4 that gives me

$$p_3 + \frac{1}{2} \rho V^2 (1 + a)^2 = p_a + \frac{1}{2} \rho V^2 (1 + b)^2$$

So, what we get that

$$p_3 - p_2 = \Delta p = \frac{1}{2} \rho V^2 (1 + b)^2 - \frac{1}{2} \rho V^2 = \rho V^2 b (1 + \frac{b}{2})$$

so that is what you get as Δp. Now, if we eliminate Δp from this equation and this equation, we get

$$1 + a = 1 + \frac{b}{2}$$

so that means

$$a = \frac{b}{2}$$

$$b = 2a$$

So, one can prove that

$$V_2 = \frac{V_1 + V_4}{2}$$

Now, if you have no forward velocity or rather zero forward velocity should be

$$V_1 = 0$$

then

$$V_4 = 2V_2$$

when the forward velocity is 0. So, this looks quite simple but important result means that any speed including 0, one of the final increase in velocity in the slipstream has already occurred at the rotor itself.

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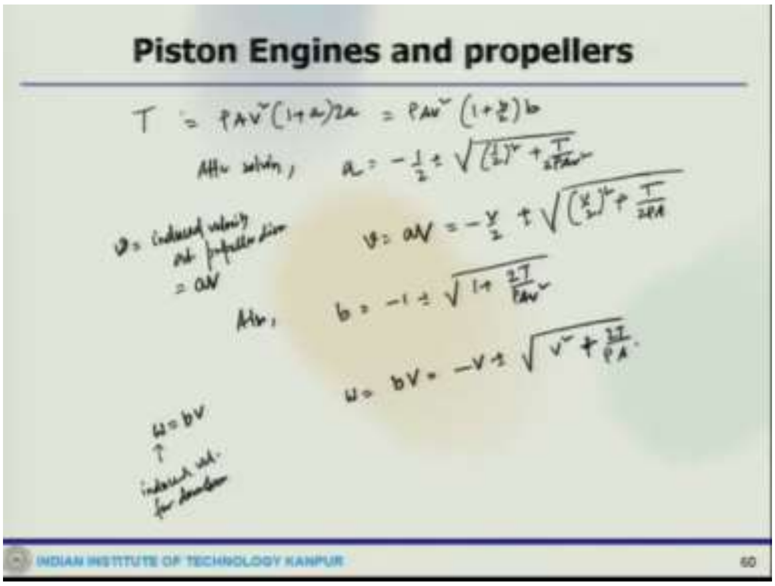

So, now going back to the that thrust expression where we have written

$$T = \rho A\, V^2(1 + a)2a = \rho A V^2(1 + \frac{b}{2})b$$

now this equation is an quadratic equation, so if we solve, after solving what we get

$$a = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + \frac{T}{2\,\rho A V^2}}$$

Now, let us say v; if we say this is induced velocity at the propeller disk and can be written as

$$v = aV = -\frac{V}{2} \pm \sqrt{\left(\frac{V}{2}\right)^2 + \frac{T}{2\,\rho A}}$$

Also we get

$$b = -1 \pm \sqrt{1 + \frac{2T}{\rho A V^2}}$$

now again we denote

$$w = bV = -V \pm \sqrt{V + \frac{2T}{\rho A}}$$

and this one is the induced velocity for downstream the propeller disk, so we get all these details that we wanted to calculate.

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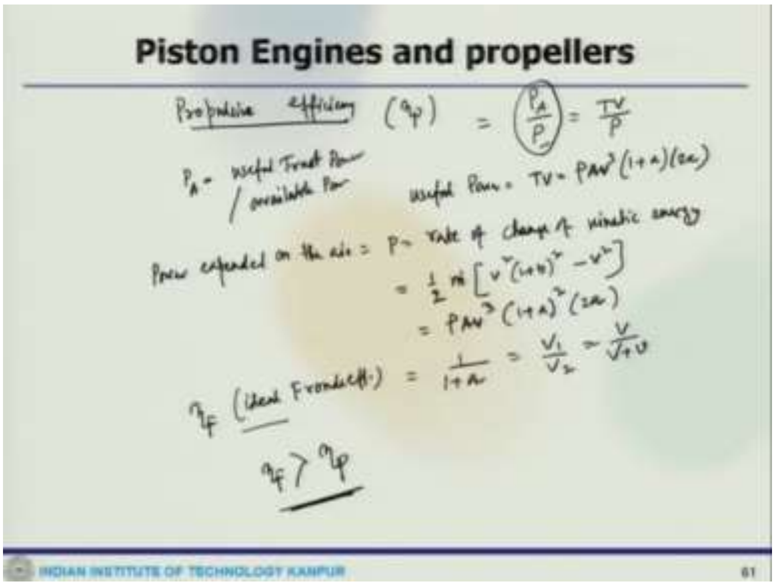

So, now we can look at some other factors like propulsion efficiency or propulsive efficiency, so that is η_p and that is defined as

$$\eta_p = \frac{p_A}{p} = \frac{TV}{p}$$

and where p_A is the thrust power or useful thrust power which is TV, this is the or sometimes called the available power and this is the P is the power delivered, so this is a ratio between available power to the power delivered is the propulsive efficiency.

Now, for the actuator disk model, this efficiency is an ideal propulsive efficiency because it ignores all losses except the associated stream wise kinetic energy, so what we have the useful power is

$$Useful\,power = TV = \rho A V^3 (1+a) 2a$$

Now, the power expended on the air which is also P which is also rate of change of kinetic energy okay, so that becomes

$$P = \frac{1}{2}\dot{m}[V^2(1+b)^2 - V^2] = \rho A V^3 (1+a)^2 2a$$

So, this ratio of these 2 powers for the ideal actuator disk model is called the ideal Froude efficiency that is

$$\eta_F = \frac{1}{1+a} = \frac{V_1}{V_2} = \frac{V}{V+v}$$

so this proves that higher efficiency of propulsion can be achieved by large rotor with very small increase in the fluid velocity so, achieving thrust by large surfaces rather than velocity.

So, ideal Froude efficiency η_F is always greater than actual propulsive efficiency, so that is what it is always like that now, we will go to another theory which is called a simple vortex model.

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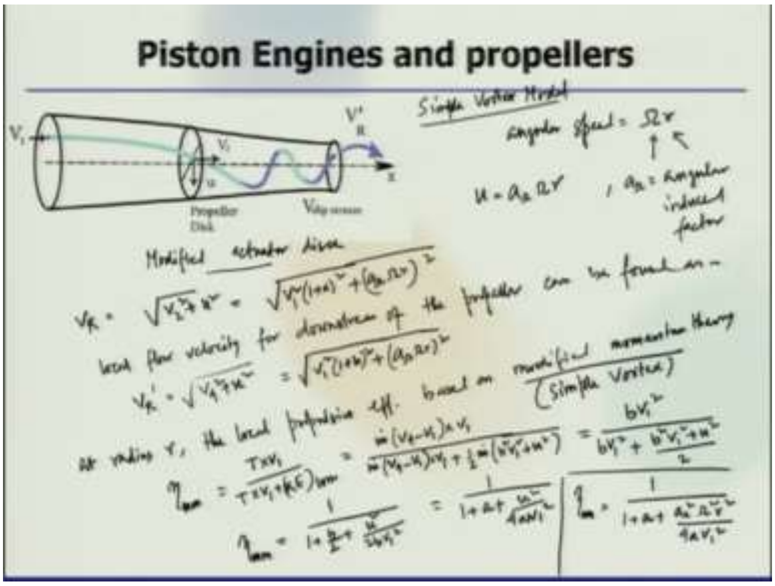

So, this is a sort of an modified actuator disk, now a practical assessment of the propeller performance and the design, so we can consider a stream tube that passes through a radius R and which is shown here, so there is a propeller disk and there is a stream tube the corresponding angular speed at this blade is let us say angular speed that is omega R, so this is the angular speed of the propeller, R is the local radius of the propeller.

When fluid passes through this propeller through the propeller section, it occurs an angular speed due to the swirling because this is called so called simple vortex model, so then the rotational speed at the propeller section would be

$$u = a_{\Omega}\Omega r$$

where a_{Ω} is the angular induced factor, okay. So, the local flow velocity just downstream of the propeller can be estimated which is like let us say

$$V_R = \sqrt{V_2^2 + u^2} = \sqrt{V_1^2(1+a)^2 + (a_{\Omega}\Omega r)^2}$$

So, the local flow velocity for downstream, so the local flow velocity for downstream of the propeller can be found as

$$V_R' = \sqrt{V_4^2 + u^2} = \sqrt{V_1^2(1+b)^2 + (a_{\Omega}\Omega r)^2}$$

Now, that is what you get, so this is where the V_R and u and V_2. Now, the let us say at radius r, the local propulsive efficiency based on modified momentum theory.

This is the, so this is called the modified momentum theory or this is what exactly the simple vortex theory, so sometimes it is called the modified momentum theory or simple vortex theory, so it based on that.

$$\eta_{mm} = \frac{TV_1}{TV_1 + (KE)_{losses}}$$

so which we can write that

$$\eta_{mm} = \frac{\dot{m}(V_4 - V_1)V_1}{\dot{m}(V_4 - V_1)V_1 + \frac{1}{2}\dot{m}(b^2V_1^2 + u^2)}$$

okay.

So, this one can be further simplified and one can write this could be

$$\eta_{mm} = \frac{bV_1^2}{bV_1^2 + \left(\frac{b^2V_1^2 + u^2}{2}\right)}$$

so what we can write is

$$\eta_{mm} = \frac{1}{1 + \frac{b}{2} + \frac{u^2}{2bV_1^2}}$$

okay, which one can write

$$\eta_{mm} = \frac{1}{1 + a + \frac{u^2}{4aV_1^2}}$$

So, finally this one if we replace it back, then we can write that let us write it here that

$$\eta_{mm} = \frac{1}{1 + a + \frac{(a_{\Omega}\Omega r)^2}{4aV_1^2}}$$

So, this is what one can get with the modified momentum theory or simple vortex model, so you can see that either one can do the analysis with like an actuator disk theory, this is one of the simplest way one can do the design simple vortex model or modified momentum theory to get this efficiency and other stuff. So, there are 2 different way you can look at it these are the again the simple theories which are used to calculate this actuator disk analysis.

But these are again assuming that these to be thin and then carrying out this analysis but one can always look at these blades and the detail analysis either put it in the wind tunnel or somewhere by doing testing or sometimes doing the computational approach, where you can calculate all this numerically and look at the flow field around it. So, we will stop this theoretical discussion here then continue some of the part in the next class.