Introduction to Airbreathing Propulsion
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Lecture – 17
Introduction to Gas Turbine Engines (Contd.,)

Okay so let us come to the last portion of this performance and thrust all this. So we have a looked at in details this different thrust coefficient and different efficiency level and then we looked at about the turboprop and the other factors which can really impact the performance parameter. So now just we would like to connect between the aircraft performance and the propulsion system, so what we have looked at is the to some extent the performance in terms of the propulsion system like how much fuel is going to be consumed and due to the fuel consumption how far it can go or with the endurance limit of the; all this. So that is where we go actually.

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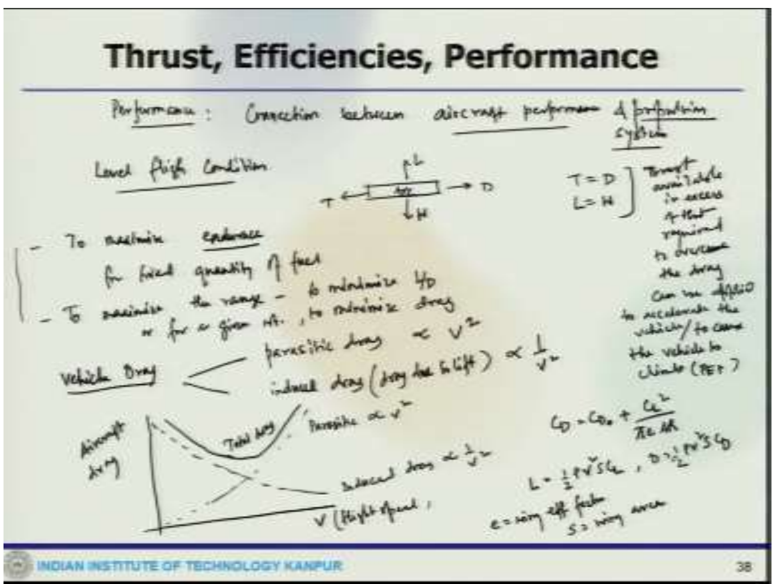

So we look at this last portion of the; again this is something you can think about performance but this is making a connections between; so the connection between aircraft performance and propulsion system. So what we have looked at is this one this quantity but now this is the one which we want to look at now so that these two are kind of connected and one can see how these are kind of affecting.

So again we are assuming the Level Flight Condition, so if you recall what we talked about in the level flight that if this is the aircraft then in level flight this is how thrust is being to be working; this is the direction of the weight, this is lift and this is weight. So obviously when it is in the level flight condition then the thrust force is balanced with the drag force and the lift is balanced with the weight of the body.

So any thrust which is available in excess so that would be work on the drag can be applied accelerate the vehicle so that is going to increase the kinetic energy or so what you can see when talking about this level flight so this; the thrust which is when we talk about that this means the thrust which is available in excess of that required to overcome the drag so that this can be applied to accelerate the vehicle so that means increasing the kinetic energy or to cause the vehicle to climb up, that is increasing the potential energy. So this is increasing the potential energy, okay.

So to maximize the time aloft or the endurance so to maximize endurance or time aloft for a fixed quantity of energy, let us say for fixed quantity of fuel it is necessary to minimize the rate of energy uses, so that means the power required which is dragged into flight velocity. And so, and also to maximize the range necessary to minimize L/D for a given weight or for a given weight to minimize drag, so these are the two important criteria that one hand we would like to maximize the endurance so that the; and that is possible for a given amount of fuel and the other hand we would like to maximize the range, so that to maximize the range we have to minimize the L/D or for a given weight to minimize drag.

Now we have vehicle drag so look at those component, so there are two component of that one is the parasitic drag, the other one is the induced drag, okay. So this parasitic drag is proportional to the flight velocity square, okay or V square. And the part which is in this induced drag or this is drag due to lift that decreases in proportion to the inverse of the flight velocity, so that means this also the flight; so this is 1/V.

So that means what happens if you look at this let us say V the flight speed and this is the actual drag or rather aircraft drag so with flight velocity parasitic drag will increase so it is goes like that so this is parasitic one which is proportional to V square and induced drag would be 1/V square, so this will come down like this, so this is induced drag which is 1/V square. So in between to compensate that the total drag will go like this, so that is the total drag. So these are the different component of the drag.

Now what we have

$$C_D = C_{D_0} + \frac{C_L^2}{\pi\ e\ AR}$$

where

$$L = \frac{1}{2}\rho V^2 S C_L$$

e is the wing efficiency factor, S is the area and then ρ density C_L and C_D is the lift and drag coefficient.

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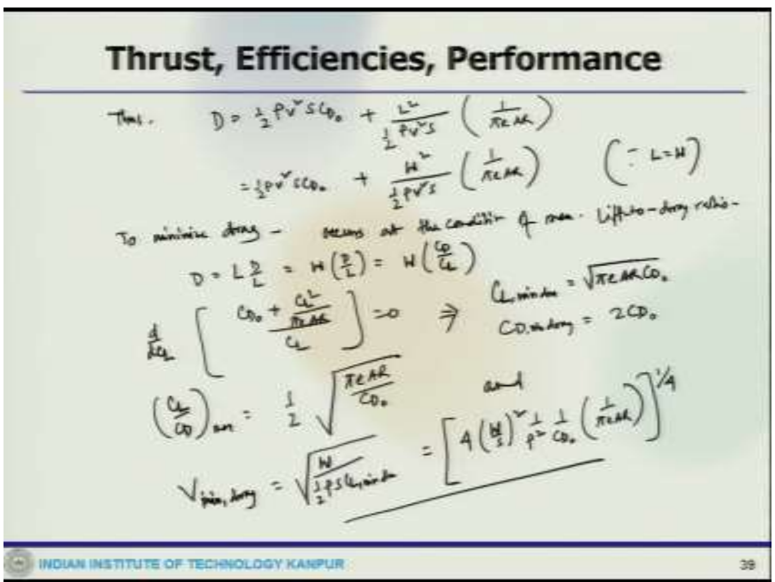

So what we can write then that

$$D = \frac{1}{2}\rho V^2 S C_{D_0} + \frac{L^2}{\frac{1}{2}\rho V^2 S}\left(\frac{1}{\pi\ e\ AR}\right)$$

so one thing to just to mention here is the aspect ratio, okay. So now what we get is that; now we can write

$$\frac{1}{2}\rho V^2 S C_{D_0} + \frac{W^2}{\frac{1}{2}\rho V^2 S}\left(\frac{1}{\pi\ e\ AR}\right)$$

so since at level flight L and W they are same. Now to the minimum drag condition is our interest so we have to minimize drag.

So for a given weight it can occurs at the condition of the maximum so this occurs at the condition of maximum lift to drag ratio so

$$D = L\frac{D}{L} = W\frac{C_D}{C_L}$$

okay. Now we can find out maximum lift to drag ratio by setting

$$\frac{d}{dC_L}\left(\frac{C_{D_0} + \frac{C_L^2}{\pi\ e\ AR}}{C_L}\right) = 0$$

So this is to for maximum. And what we can find that from here

$$C_{L,\min drag} = \sqrt{\pi\ e\ AR C_{D_0}}$$

and

$$C_{D,\min drag} = 2C_{D_0}$$

So what it gives us

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{2}\sqrt{\frac{\pi\ e\ AR}{C_{D_0}}}$$

and we get the flight speed for minimum drag condition is

$$V_{\min drag} = \sqrt{\frac{W}{\frac{1}{2}\rho S C_{L,\min drag}}}$$

which is essentially

$$V_{\min drag} = \left[4\left(\frac{W}{S}\right)^2 \frac{1}{\rho^2}\frac{1}{C_{D_0}}\left(\frac{1}{\pi\ e\ AR}\right)\right]^{\frac{1}{4}}$$

so that is what you get, that is the; for the minimum drag condition what could be the flight velocity and the things like that.

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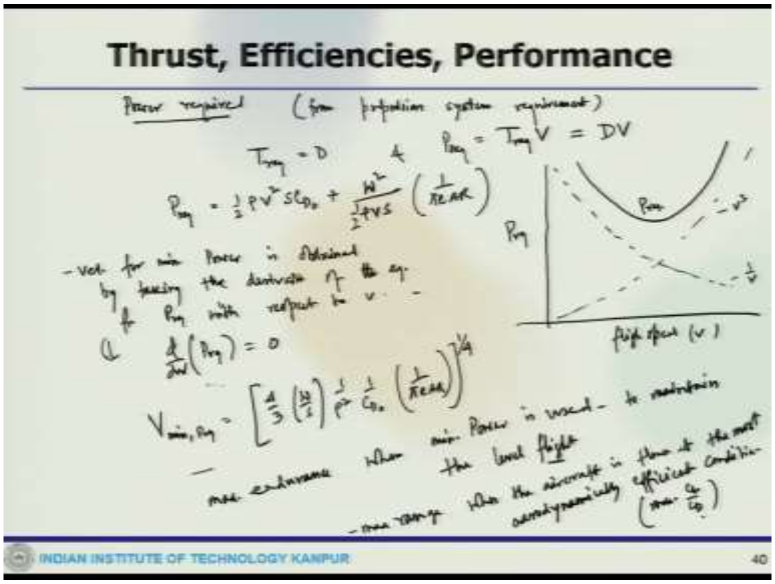

Now we can find out how much power would be required. So now we can look at the propulsion system requirement, so to maintain the steady level flight so this is from propulsion system requirement what we get T required would be drag and power required would be

$$P_{req} = T_{req}V = D * V$$

So what we have power required would be

$$P_{req} = \frac{1}{2}\rho V^2 S C_{D_0} + \frac{W^2}{\frac{1}{2}\rho V^2 S}\left(\frac{1}{\pi\ e\ AR}\right)$$

So again if you look at the sort of an; this is flight speed of v and this is required P required so this changes with and this goes like so this curve is the sort of cube this is 1/V so my power required would lie like this. So this is my P required the actual. So this is a typical power required curve of an aircraft and that's how; now the velocity for minimum power is obtained so the velocity for minimum power is obtained by taking the derivative of the equation for P required with respect to V and setting it to equal to 0.

So when you take that, that means this means so we will take

$$\frac{d}{dV}\left(P_{req}\right) = 0$$

and to find out that velocity which will; so one can do the maths here, we are not going into that calculation because it would be quite straightforward so this is P required condition would be

$$V_{\min Power\, req} = \left[4\left(\frac{W}{S}\right)\frac{1}{\rho^2}\frac{1}{C_{D_0}}\left(\frac{1}{\pi\, e\, AR}\right)\right]^{\frac{1}{4}}$$

So as we will see in the quickly that maximum endurance occurs when the minimum power is used to maintain the steady level flight.

So that means the maximum endurance that will be obtained when minimum power is used to maintain the level flight, okay. And the maximum range that is the distance traveled so maximum range will be obtained when the aircraft is flown at the most aerodynamically efficient condition, okay. So that is maximum C_L/C_D.

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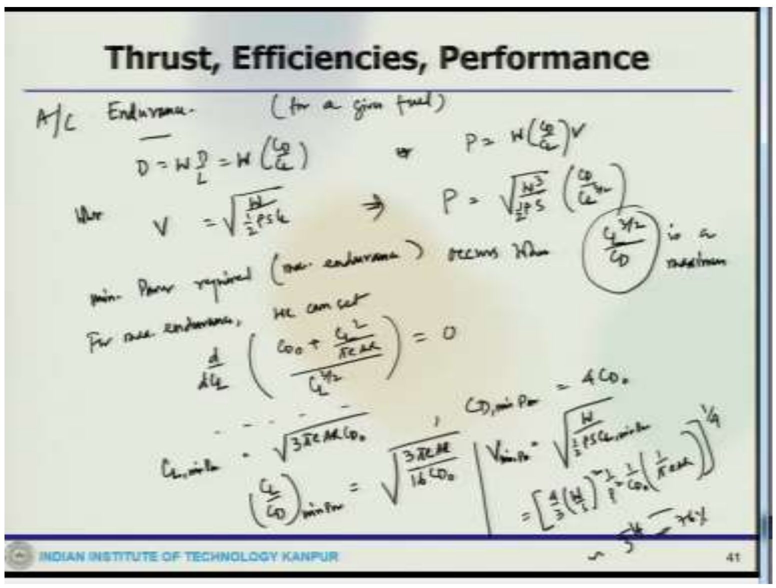

So now we find out that aircraft endurance where; now for a given amount of fuel; for a given fuel what we; the maximum endurance of the time is obtained for the flight condition corresponding to the minimum rate of energy or P, a power required, the minimum power required. So we can find out the aerodynamic configuration which provides the minimum energy expenditure.

So to do that we will again come back to this; and this is what we can write due to level flight and power is

$$P = W\frac{C_D}{C_L}V$$

So where V is

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}}$$

So that gives us power would be

$$P = \sqrt{\frac{W^3}{\frac{1}{2}\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right)$$

So the minimum power required; so minimum power required or maximum endurance occurs when $\left(\frac{C_L^{3/2}}{C_D}\right)$ is a maximum.

So that means this quantity the inverse quantity is maximum. So with them we can do some little bit of algebra here and arrived an expression for the maximum endurance. So for max endurance we can set like

$$\frac{d}{dC_L}\left(\frac{C_{D_0} + \frac{C_L^2}{\pi\ e\ AR}}{C_L^{3/2}}\right) = 0$$

and what we will find from here so you can do this much. So

$$C_{L,\min power} = \sqrt{3\pi\ e\ AR C_{D_0}}$$

$$C_{D,\min power} = 4C_{D_0}$$

$$\left(\frac{C_L}{C_D}\right)_{\min power} = \sqrt{\frac{3\pi\ e\ AR}{16C_{D_0}}}$$

And at the same time V for minimum power would be

$$V_{\min power} = \sqrt{\frac{W}{\frac{1}{2}\rho S C_{L,\min power}}}$$

and if we put back the $C_{L,\min power}$ this will get us

$$V_{\min power} = \left[\frac{4}{3}\left(\frac{W}{S}\right)^2 \frac{1}{\rho^2}\frac{1}{C_{D_0}}\left(\frac{1}{\pi\ e\ AR}\right)\right]^{\frac{1}{4}}$$

So we get the minimum velocity and all these. So the minimum power or the maximum endurance condition occurs at a speed which is roughly this roughly 3 to the power 1/4 76% of the minimum drag or maximum range condition. And the corresponding lift to drag ratio would be roughly; so the 86% or something like that.

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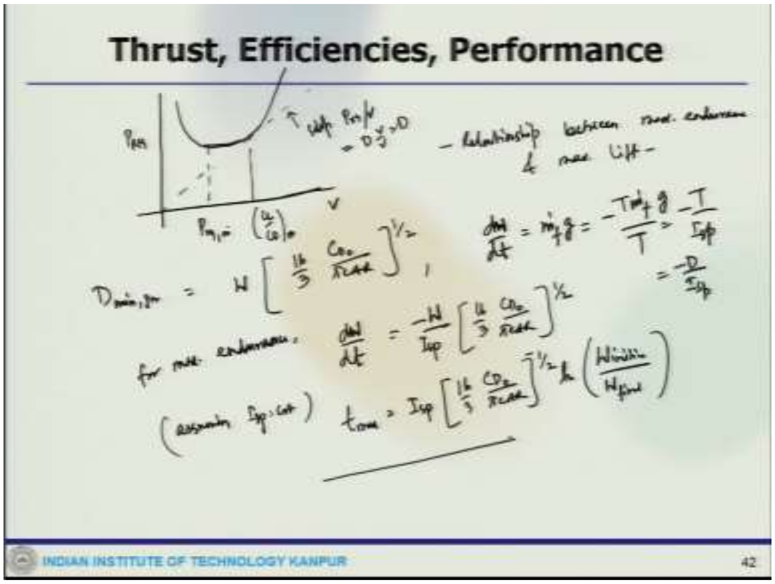

So if we look at that curve like what we have again flight velocity P this is; so this will go like that. So power required and; so this is or required minimum $\left(\frac{C_L}{C_D}\right)_{max}$ and this is the slope of P required by V which is DV/V=D. So this is a relationship between maximum endurance and maximum range. So this is relationship between maximum endurance and max lift. So continuing what we get that

$$D_{\min power} = \left[\frac{16}{3}\left(\frac{C_{D_0}}{\pi\ e\ AR}\right)\right]^{\frac{1}{2}}$$

So this can be substituted into

$$\frac{dW}{dt} = \dot{m}_f g = \frac{-T\dot{m}_f g}{T} = \frac{-T}{I_{sp}} = \frac{-D}{I_{sp}}$$

So it says that for maximum endurance what we get

$$\frac{dW}{dt} = \frac{-W}{I_{sp}}\left[\frac{16}{3}\left(\frac{C_{D_0}}{\pi\ e\ AR}\right)\right]^{\frac{1}{2}}$$

So now one can integrate this one to get the assuming the; so as I_{sp} to be constant so what we get

$$t_{max} = I_{sp}\left[\frac{16}{3}\left(\frac{C_{D_0}}{\pi\ e\ AR}\right)\right]^{-\frac{1}{2}} \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

So that is what we get.

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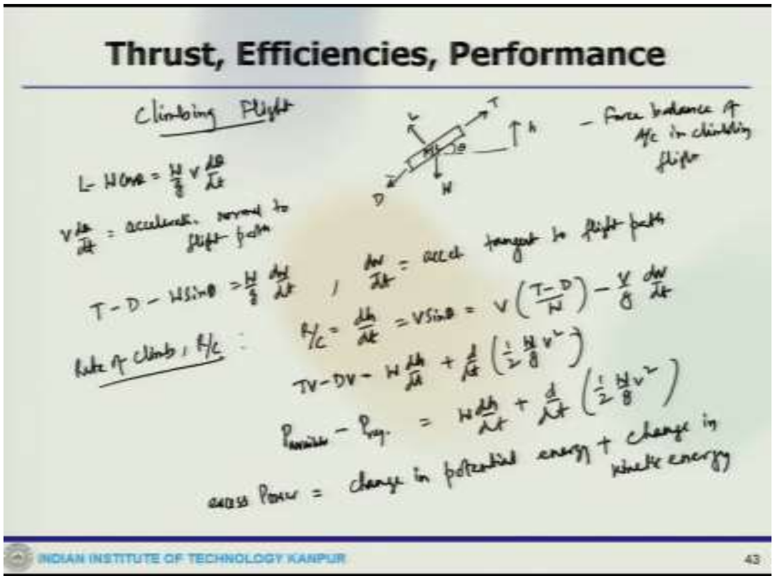

Now there is one more condition that will take the climbing flight, so climbing flight condition. So here we can say that the; if aircraft is like this the free body diagram. Let us say this is theta or let us put like that, let us say this is theta, this is thrust and this goes the height. So this is L, this is drag and this is weight. So any excess in power beyond that is required to work on the drag will cause the vehicle increase it is kinetic or potential energy.

So we consider this by the; by this case by resolving the forces into different direction of the flight and equating with the acceleration. So this essentially the force balance of aircraft in climbing flight. So what we get

$$L - Wcos\theta = \frac{W}{g}V\frac{d\theta}{dt}$$

where

$V\frac{d\theta}{dt}$ is the acceleration normal to flight paths and

$$T - D - Wsin\theta = \frac{W}{g}\frac{dV}{dt}$$

where dv/dt is the acceleration tangent to flight path.

So the change in height of the vehicle or the rate of climb which is R/C so that is we can write that this is the change in height so this would be $Vsin\theta$,

$$\frac{R}{C} = Vsin\theta = V\left(\frac{T-D}{W}\right) - \frac{V}{g}\frac{dV}{dt}$$

So we can write this in a form

$$TV - DV = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

So which is means power available minus power required is

$$P_{avail} - P_{req} = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

See in other words one can say that, that excess power should be change in potential energy plus change in kinetic energy.

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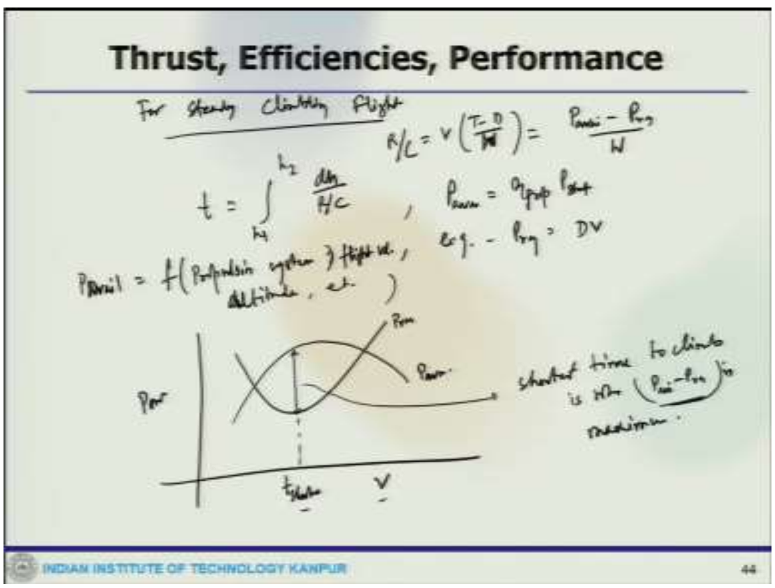

So now for steady climbing flight what we see that this R/C would be

$$\frac{R}{C} = V\left(\frac{T-D}{W}\right) = \frac{P_{avail} - P_{req}}{W}$$

And the time to climb would be

$$t = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

where

$$P_{avail} = \eta_{prop} P_{shaft}$$

For example, P required is D into V. So the power available is a function of propulsion system so it means the P avail is a function of propulsion system flight velocity altitude, so altitude etcetera.

Typically, it takes a form such that like this could be like such as power; this is flight speed V so this goes like this, this comes like this. So this is probably the; this is the power required curve; this is P available curve and this is the shortest time to climb is where P avail minus P required is maximum. So this is a typical behavior of power available as a function of flight speed. So that means the shortest time to climb it will occur at flight velocity where; so this would be the corresponding flight velocity for t shortest.

So that means at that particular velocity your P available minus P required these difference is also maximum and that would take the time to climb flight. So that is pretty much gives you an idea about like; so first we looked at the; all these endurances range and all these in terms of the propulsion system where the fuel is consumed and all this. And this portion of the discussion at least the today's discussion would give you a fair amount of idea, how the aircraft endurance and the range they are connected with the propulsion system and that pretty much that is what we would like to discuss on this performance thrust and we will continue the discussion on the other topics in the next lecture.