Introduction to Airbreathing Propulsion

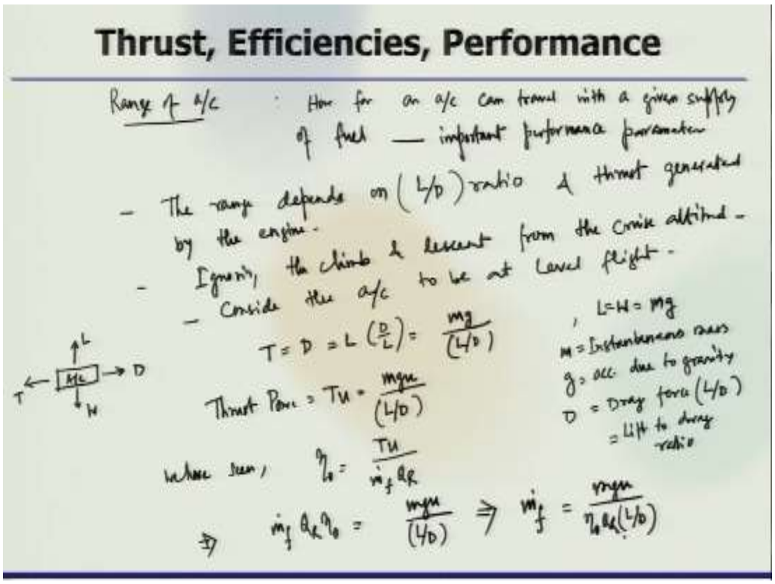
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology – Kanpur

Lecture – 15
Introduction to Gas Turbine Engines (Contd.,)

Okay, so let us looking at this different performance parameter and we are talking about overall efficiency, thermal efficiency, propulsion efficiency and we have looked at how they vary and another important factor that we have looked at is the take-off thrust because take-off thrust is one of the important design parameter for any aircraft engine. As I already mentioned this is the maximum thrust that one engine can produce.

And this required to be produced at the engine when it is on the static condition or rather adjust on the run at the ground level while it is going to take-off with the full payload including the fuel load complete fuel load and everything. So that is the maximum thrust that one engine can produce and how that can be varied that we have already seen. Now we are moving ahead with the other factors like range and endurance and all this.

(Refer Slide Time: 01:19)



So, what we will discuss is the range of aircraft. So, that is we are going to look at it. So the range when we talk about range it means it says how far an aircraft can travel with a given supply of fuel. Given supply of fuel so this is also an important performance parameter and range actually the range depends on L / D ratio which lift to drag ratio and thrust generated by the engine.

Now, if we ignore let us say if we ignore the climb too and descent from cruise altitude that means we are ignoring the climb and descent from the cruise altitude and we can only consider the then consider the aircraft to be at level flight. So, when you talk about the level flight the engines thrust and the vehicle drag must be equal and lift on the aircraft should be equal to its weight.

That means if we look at the free body let us say this is the aircraft then this is thrust, this is drag, this is weight and this is lift so they should be equal. So

$$T = D = L * \frac{D}{L} = \frac{mg}{(\frac{L}{D})}$$

because we have L is W is mg. Now here m is the instantaneous mass, g is the acceleration due to gravity and D is drag force which is L / D or rather lift to drag ratio.

Then we have thrust power that would be

$$Thrust\ power = Tu = \frac{mgu}{(\frac{L}{D})}$$

and what we have seen that overall efficiency is

$$\eta_0 = \frac{Tu}{\dot{m}_f Q_R}$$

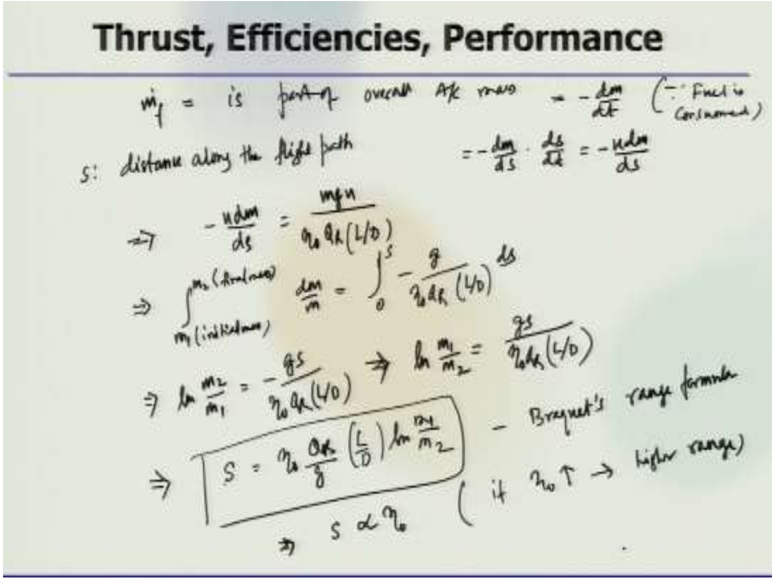
So if we equate this term what we can write

$$\dot{m}_f Q_R \eta_0 = \frac{mgu}{(\frac{L}{D})}$$

So that gives us

$$\dot{m}_f = \frac{mgu}{Q_R \eta_0 (\frac{L}{D})}$$

(Refer Slide Time: 06:17)



Now $\dot{m}_f$ is also is part of overall aircraft mass and this is what $\dot{m}_f$ is also getting consumed so dm / dt. Since fuel is consumed then what we can write that

$$-\frac{dm}{ds}\cdot\frac{ds}{dt} = -\frac{udm}{ds}$$

where S one can assume that the this is the distance along the flight path. So now we put this back in this $\dot{m}_f$ expression so what we get

$$-\frac{udm}{ds} = \frac{mgu}{Q_R\eta_0(\frac{L}{D})}$$

Now, if we integrate

$$\int_{m_1(initial\ mass)}^{m_2(final\ mass)}\frac{dm}{m} = \int_0^S -\frac{g}{Q_R\eta_0(\frac{L}{D})}ds$$

So this becomes

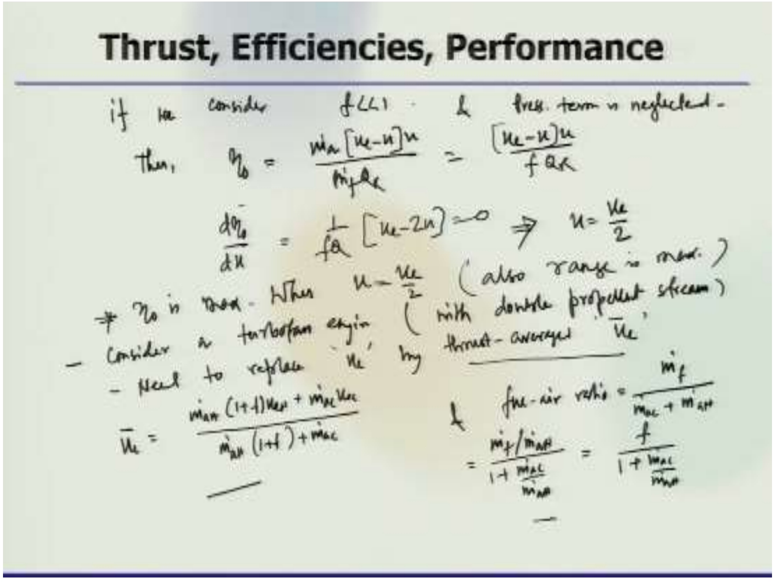
$$ln\frac{m_2}{m_1} = -\frac{gS}{Q_R\eta_0(\frac{L}{D})}$$

So we will get

$$S = \frac{Q_R\eta_0}{g}(\frac{L}{D})ln\frac{m_1}{m_2}$$

So this is known as Breguet's range formula. So what we see here that S is proportional to eta 0 that means if eta 0 is higher so we have higher range.

(Refer Slide Time: 09:51)



Now, if we consider f is small and pressure term is neglected then what we can write

$$\eta_0 = \frac{\dot{m}_a[u_e - u]}{\dot{m}_f Q_R}$$

which is

$$\eta_0 = \frac{[u_e - u]}{f Q_R}$$

So now we take derivative to the eta 0 with respect to u for a given ue. So what we can write this is

$$\frac{d\eta_0}{du} = \frac{[u_e - 2u]}{f Q_R} = 0$$

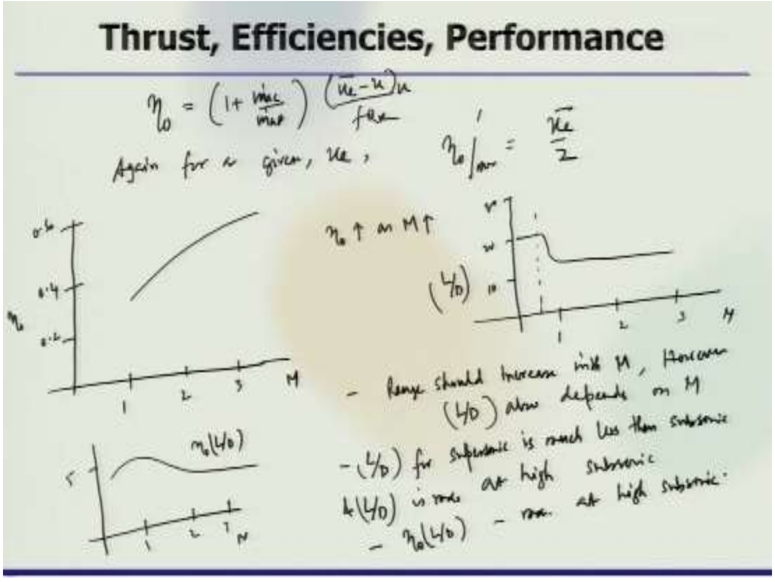
that gives u is ue / 2 so we have already seen that eta 0 is max when u = ue / 2 and that is the same time also range is max because range is also proportional to the eta 0.

Now let us consider a turbofan engine which is essentially with double propellant stream. So, here we need to replace because turbofan has one cold stream and hot stream so we need to replace u_e by the thrust averaged quantity like $\overline{u_e}$. So, where $\overline{u_e}$ is defined as

$$\overline{u_e} = \frac{\dot{m}_{ah}(1+f)u_{eh} + \dot{m}_{ac}u_{ac}}{\dot{m}_{ah}(1+f) + \dot{m}_{ac}}$$

so using this average ue and this fuel air ratio.

(Refer Slide Time: 13:51)



One write that

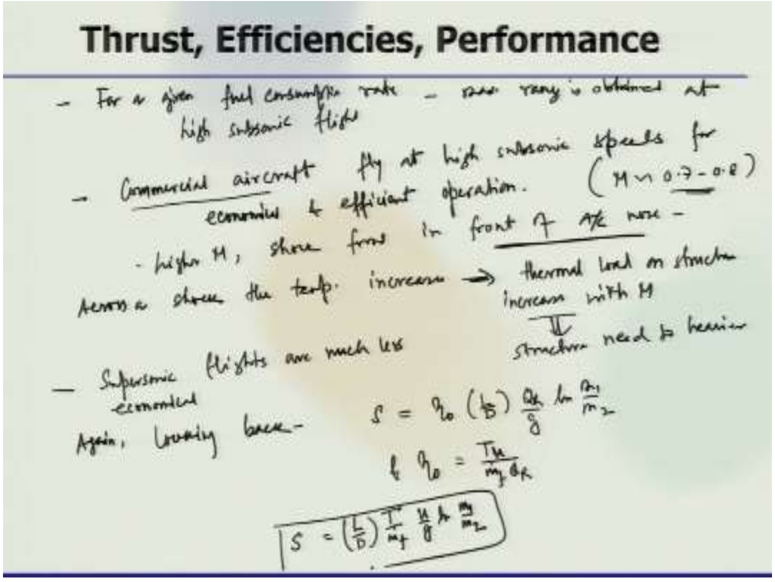
$$\eta_0 = (1 + \frac{\dot{m}_{ac}}{\dot{m}_{ah}})\frac{[u_e - u]}{f Q_R}u$$

Again, for a given ue η_0 max would be ue bar / 2. So we can take the simple derivative of that and find out that this would be ue / 2. So eta 0 also depends on flight Mach number. So one can plot η_0 like let us say 1, 2, 3 like this, this is flight Mach number 0.2, 0.4, 0.6. So eta 0 this varies like this.

So η_0 increases as flight Mach number increases and at the same time what we can look at is the L / D variation how that goes with flight Mach number 1, 2, 3 this is flight Mach number 10, 20, 30 so this is how L / D with the flight Mach number and other things Mach number this is 5 and this is η_0 into L / D. So the range should increase with Mach number how L / D also depends on Mach number that means range should increase with flight Mach number.

However, L / D also depends on M. Now L / D for supersonic is much less than subsonic and L / D is maximum at high subsonic. For the product of η_0 L / D this is also maximum at high subsonic.

(Refer Slide Time: 17:27)



Now, for a given fuel consumption rate the maximum range is obtained at high subsonic flight. For a given fuel consumption rate maximum range is obtained at high subsonic flight. Now the other things like commercial aircraft fly at high subsonic speed for two reason one is economical and second is that efficient operations. So this is where that all our civilian commercial aircraft that fly most of the time at the high subsonic.

So, typically they operate around Mach 0.7 to 0.8 in that range so this is high subsonic range. So what happens that higher Mach number shock forms in front of aircraft nose. Now once that happens if there is a shock formation so we have already seen that across a shock the temperature increases. So when the temperature would increase obviously the thermal load will increase.

So thermal load on structure increases with M. So to avoid this the payload also so now these two thermal load are to be resistant the structure need to be heavier to withstand all this. This is one of the immediate consequence that one can have as soon as this is obviously having said that it is quite obvious that supersonic flights are much less economical for example when Concorde can take 100 passenger.

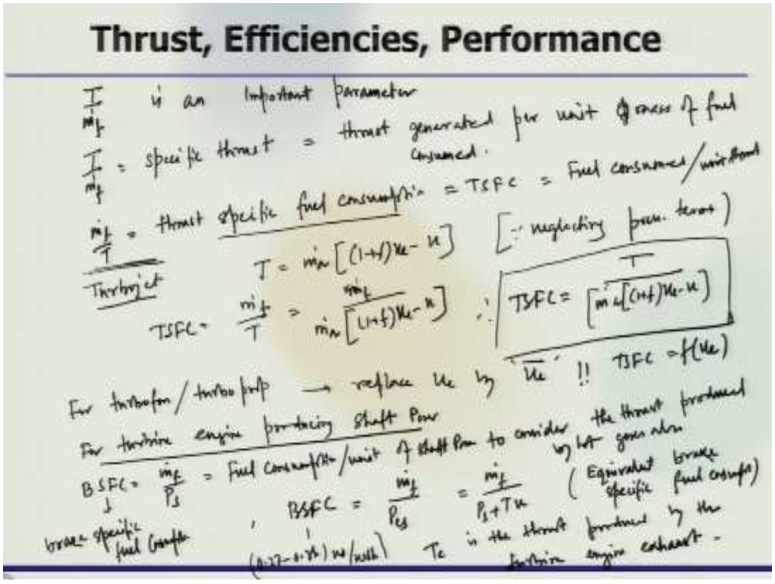
And use more fuel than a Boeing 777 which can carry roughly around 500 passengers so they are not economical either. Again looking back to that so looking back to that range equation what we get or obtain that

$$S = \frac{Q_R \eta_0}{g} (\frac{L}{D}) ln \frac{m_1}{m_2}$$

$$\eta_0 = \frac{Tu}{\dot{m}_f Q_R}$$

$$S = \frac{T}{\dot{m}_f}\frac{u}{g}(\frac{L}{D})ln\frac{m_1}{m_2}$$

(Refer Slide Time: 22:23)



So which tells that $\frac{T}{\dot{m}_f}$ is an important parameter. So $\frac{T}{\dot{m}_f}$ is an important parameter and this is the specific thrust that means the thrust generated per unit mass of fuel consumed. Now the other the reverse of it which is $\frac{\dot{m}_f}{T}$ this is called the thrust specific fuel consumption that is TSFC that means fuel consumed per unit thrust. So just to produce one unit of thrust how much fuel is consumed that is called the TSFC.

And this is one of the parameter which is often provided for providing an engine or configuration or something I mean if you look at or search in internet how this engine specs are provided any engine manufacturer they provide this kind of information the TSFC, PRC, efficiencies and then how much thrust it produces and such like that. Just to look at this things again these are generic terms just to extend these things for turbojet.

Just to show you how things can be written for TSFC

$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f}{[\dot{m}_a(1+f)u_e - u]}$$

So this is for a turbojet how one can estimate the TSFC value. So, similarly for turbofan or turbo propeller let us say for turbofan or turboprop you can just replace this u_e by u_e average.

So one thing is clear here that TSFC depend strongly on u_e that means what is the exit velocity. Now let us say for any turbine engine producing shaft power. So for turbine engine producing shaft power what happens? So we define another quantity called the BSFC which is

$$BSFC = \frac{\dot{m}_f}{P_s}$$

that is called this is fuel consumption per unit of shaft power. To consider the thrust produced by hot gases also.

So this BSFC stand for brake specific fuel consumption this is brake specific fuel consumption. Now equivalent brake specific fuel consumption so this is we write BSFC is

$$BSFC = \frac{\dot{m}_f}{P_{es}}$$

which is equivalent shaft power. So that would be

$$BSFC = \frac{\dot{m}_f}{P_s + Tu}$$

so this is equivalent brake specific fuel consumption. Now what we can see here that T_e is the thrust produced by the so T_e is the thrust produced by the turbine engine exhaust.

So, typically the engine brake specific fuel consumption would be some range like typical range of they should be 0.27 to 0.36 roughly kg per kilowatt hour so that is the typical value of BSFC which is there. So you can see how TSFC and all these things they can be calculated and also the BSFC and range. So, we will stop here and continue this discussion in this next lecture.