**Introduction to Airbreathing Propulsion**
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**Lecture - 12**
**Introduction to Gas Turbine Engines (Contd.,)**

So we are trying to find out the equation for the thrust or the expression for the thrust and then we will calculate other performance parameter and this is what we started with the free body diagram.

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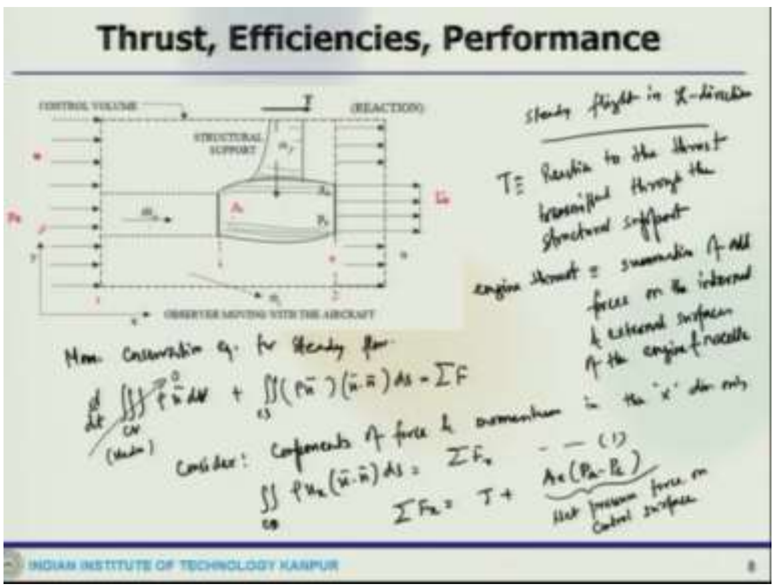


And then finally what we said that we get the mass.

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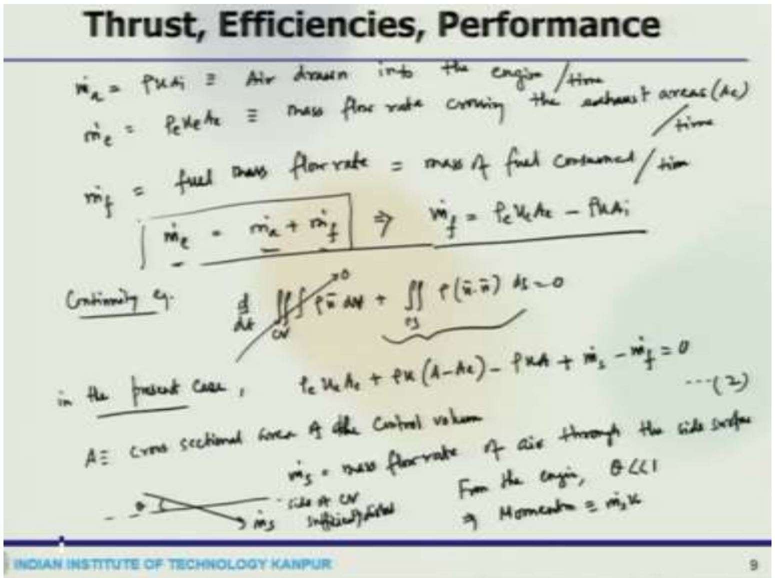


Which is the total exhaust or exit mass flow rate would be sum of the air comes in and the fuel flow rate. So this is what we stopped. Now we write the continuity equation, so what we write that

$$\iiint_{CV} \rho u dV + \iint_{CS} \rho(u \cdot n) ds = 0$$

So now it is a steady. So this component would not contribute and then we will get from this component only. Now in the present situation or in the present case what we will write

$$\rho_e u_e A_e + \rho u(A - A_e) - \rho u A + \dot{m}_s - \dot{m}_f = 0$$

So across all the controlled surfaces, this is what we get from the continuity equation and A here is the cross sectional area of the control volume Now what we have is that this is the structural support, where we have m dot s, this is with angle theta, m dot s is the mass flow rate of air through the side surface. Now this is the side of CV control volume at sufficient distance or sufficiently distant. Now from in the engine, this theta is very, very small, which leads that momentum is

$\dot{m}_s * u$

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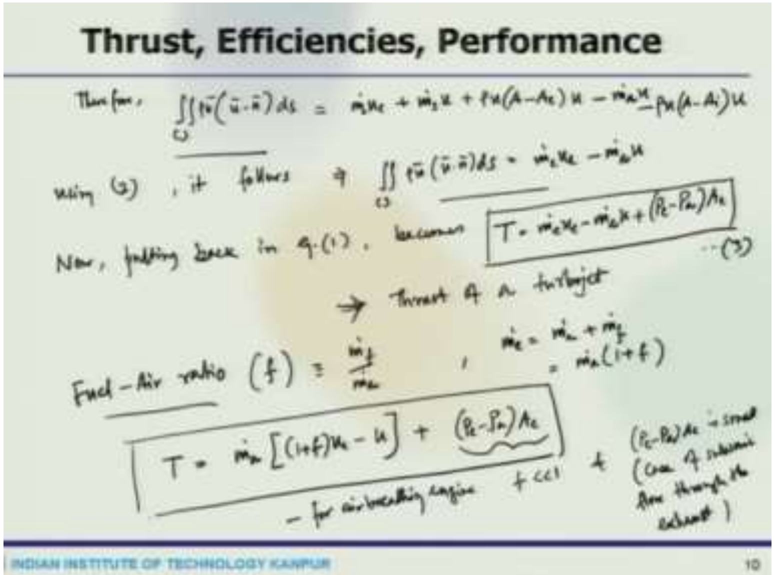


So what we write that therefore,

$$\iint_{CS} \rho(u \cdot n)ds = \dot{m}u_e + \dot{m}u + \rho u(A - A_e)u - \dot{m}_a u - \rho u(A - A_i)u$$

So this is a component which we get in the steady momentum equation. Now we are using 2, it follows that, so we put equation 2 that is what we get from the continuity, this guy into this one and once you do the math, what we will get that the

$$\iint_{CS} \rho(u \cdot n)ds = \dot{m}_e u_e - \dot{m}_a u$$

So once we get this, now putting back in equation 1, it becomes

$$T = \dot{m}_e u_e - \dot{m}_a u + (P_e - P_a)A_e$$

So that is what we get. This is equation 3. So this is what also one can say. This is the thrust of a turbojet. So you can get finally the expression for the thrust of a turbojet engine. Now we consider the fuel-air ratio, which is usually represented with f, which is

$$f = \frac{\dot{m}_f}{\dot{m}_a}$$

Now when we use this, then what we get? We have also obtained that

$$\dot{m}_e = \dot{m}_a + \dot{m}_f = \dot{m}_a(1 + f)$$

Now using this one in the thrust equation, the thrust equation becomes little bit

$$T = \dot{m}_a[(1 + f)u_e - u] + (P_e - P_a)A_e$$

So given fuel-air ratio, so these are the parameters. They are important because when engine specification, if you look at the engine specification from any book or these things I mean like the pressure ratio, speed, efficiencies, range of operation, fuel-air ratio, these are given.

So that is what you can get. Now typically for air breathing engine, for air breathing engine in general f is very, very small and that this particular term which is Pe - Pa into Ae that is also small, very, very small or almost this is the situation the in the case of subsonic flow through the exhaust.

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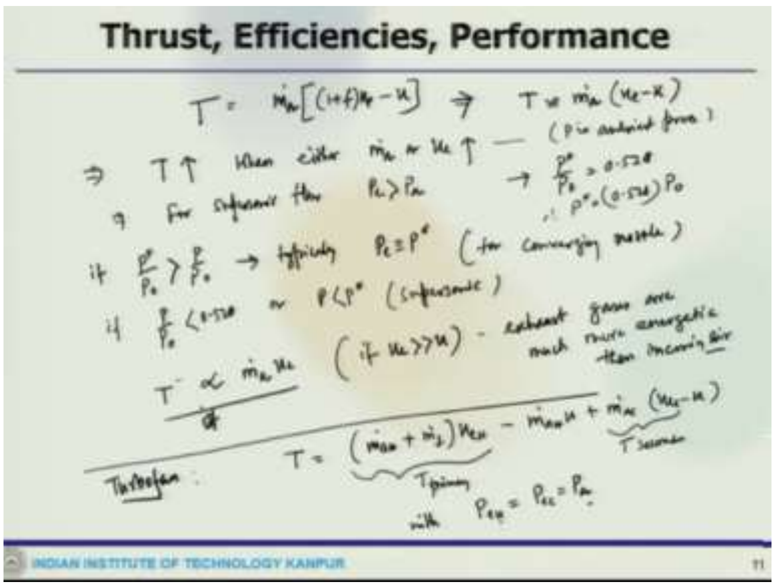


So then what we can neglect this term and that term and then finally we can write

$$T = \dot{m}_a[(1 + f)u_e - u]$$

which immediately says that thrust is proportional somewhat $\dot{m}_a(u_e - u)$. So what happens, thrust always increases when either $\dot{m}_a$ or $u_e$ increases. Now for supersonic flow Pe exit is greater than P atmosphere. So what happens that $\frac{p*}{p_0}$ is 0.528. So the $p*$ would be 0.528 into $p_0$, if P is the ambient pressure and this is the case P is ambient pressure.

So if $\frac{p*}{p_0}$ greater than $\frac{p}{p_0}$, then typically P exit would be P star. This is for converging nozzle and if P by P knot less than 0.528 or P less than P star, this is the case when it is supersonic. So another thing, one can see that thrust is proportional to $\dot{m}_a$ and $u_e$. So if I mean let us say, if $u_e$ is quite higher than these things, so thrust becomes incoming mass flow rate and the exit velocity.

So this means that exhaust gases are much more energetic than incoming air. Now that we get all these information when you get that. Now we can look at the same thrust equation for turbofan. So we can write the equation. We will derive all these equations also later on, but right now just to define all the different performance parameters, we just need to have the basic equation for the thrust where

$$T = (\dot{m}_{ah} + \dot{m}_f)u_a - \dot{m}_{ah}u + \dot{m}_{ac}(u_c - u)$$

So this is the primary component of the thrust or hot thrust. This is the secondary component of the thrust or the cold thrust and also here the assumption is that the $P_{eh}$ is $P_{ec}$ is $P_a$. This is atmospheric pressure; this is the pressure at the exit of the hot gas. Now just to quickly get back to this here.

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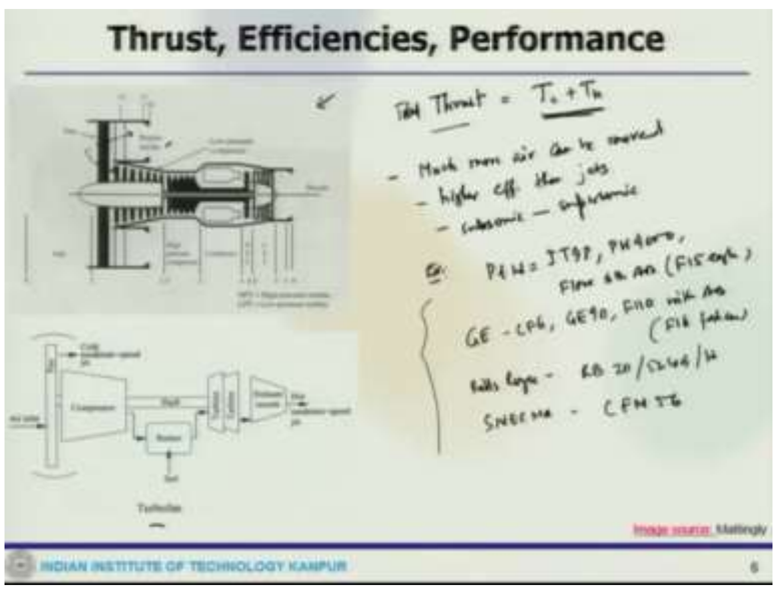


The picture that will give you an idea in turbofan you have a fan and then there is a cold thrust which is going through this bypass and rest go through the engine. So that is what you have the hot component, cold component and from the turbofan engine one can always get back the turbojet engines, where if you do not have this bypass, then this if you remove that out, then essentially it comes down to the same turbojet engine, but since there is a fan and there are two different components of thrust which come into.

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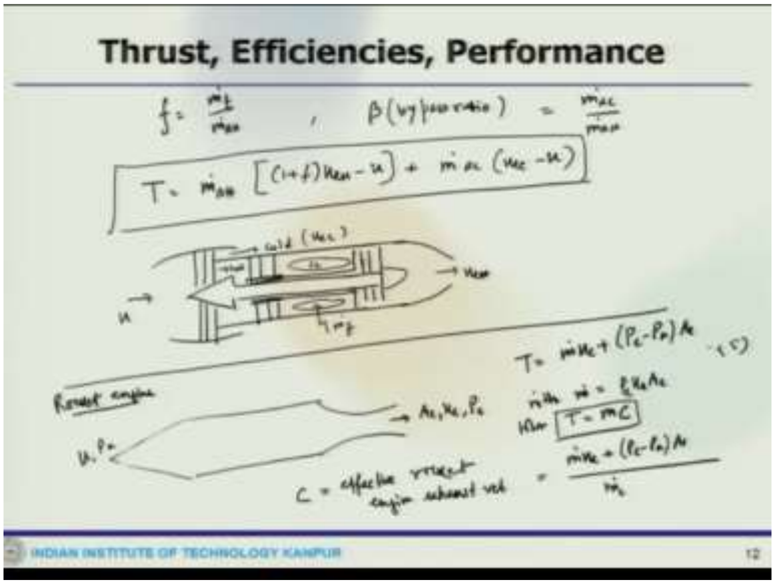


Now we also define fuel-air ratio for turbofan. Here it would be

$$f = \frac{\dot{m}_f}{\dot{m}_{ah}}$$

because this is the portion of the air goes through the core and another parameter which we call the beta; this is called bypass ratio and that is defined the percentage of the air which passes through the cold. So using this, we rewrite the equation for the thrust, which is

$$T = \dot{m}_{ah}[(1+f)u_h - u] + \dot{m}_{ac}(u_c - u)$$

So this is what we get for the turbojet. I mean schematically one can see.
This is the nozzle, then you have this now you have the fan sitting there, then it goes through like this. So you have this component, this component then you have the combustion chamber, so you get turbine. So it sort of passes through that. So that is what u comes in. This is the cold and this is where the hot; this is combustion chamber, compressor and then finally this is $u_{eh}$ and this is $u_{ec}$. Here you have $\dot{m}_f$.

Now similarly if you have a rocket engine like let us say you have this. So you get the exit here Ae ue Pe and this so the thrust equation would be

$$T = \dot{m}u_e + (P_e - P_a)A_e$$

that is sort of. Now here with $\dot{m} = \rho_e u_e A_e$, we can write where T = m into c, c is the effective rocket engine exhaust velocity, which is $\frac{\dot{m}u_e+(P_e-P_a)A_e}{\dot{m}_e}$. So this is effective and then you can write the thrust equation like that.

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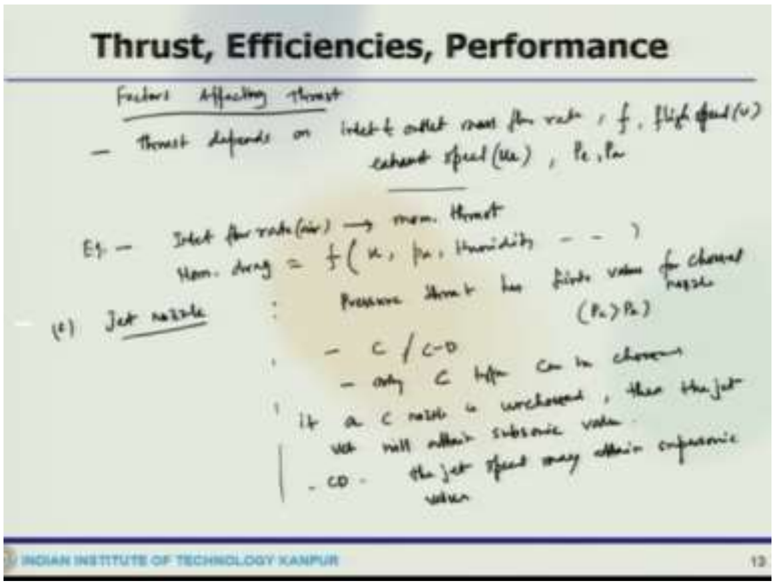


So once you see this, one can identify the things which are important like we can see the factors which are affecting thrust. So obviously any of this turbojet or turbofan or this engine, so thrust depends on inlet and outlet mass flow rate; that is there, fuel-air ratio, flight speed or u, exhaust speed that is ue and then exhaust pressure, ambient pressure. So all these are important factors. So it looks like they are pretty simple to identify these factors, which are listed here.

But each of these parameters what I mean has been listed here, they depend on multiple or several factors. For example, one can take, let us say the inlet air flow rate that inlet flow rate of air or inlet air flow rate influencing both the momentum thrust and the momentum drag. Momentum drag is dependent on several variables including flight speed, ambient temperature, pressure, humidity, altitude, rotational speed of the compressor.

So this affect momentum component of thrust and momentum drag depends on so many things like flight speed u, then pressure, humidity and so on other factors. Similarly, the outlet gas mass flow rate is dependent on fuel added like how much fuel is added there, air bleed, water injections.

Then the pressure thrust term also depends on turbine inlet temperature flight altitude, nozzle outlet area and the momentum thrust also depends on jet velocity.

So these are I mean multiple factors which affects. So some of the important factors like let us say jet nozzle, how? So you can see that pressure thrust has finite values for choked nozzle, where the exit pressure is greater than the ambient pressure that means Pe greater than Pa. So nozzles are either could be convergent or divergent, it could be either convergent type or could be convergent-divergent type. Now only convergent type can be choked.

For choked convergent nozzle, the pressure thrust depends on both the area of the exhaust nozzle and also on the difference between the exit and ambient pressure. Further the exhaust speed is equal to the sonic speed, which is mainly influenced by the exhaust gas temperature. Now if a convergent nozzle is unchoked, then the jet velocity will attain subsonic values and for CD nozzle the jet speeds may attain supersonic values. So you will see this, how these things happen when we will talk about these nozzles and all these things in the later half of the lecture.

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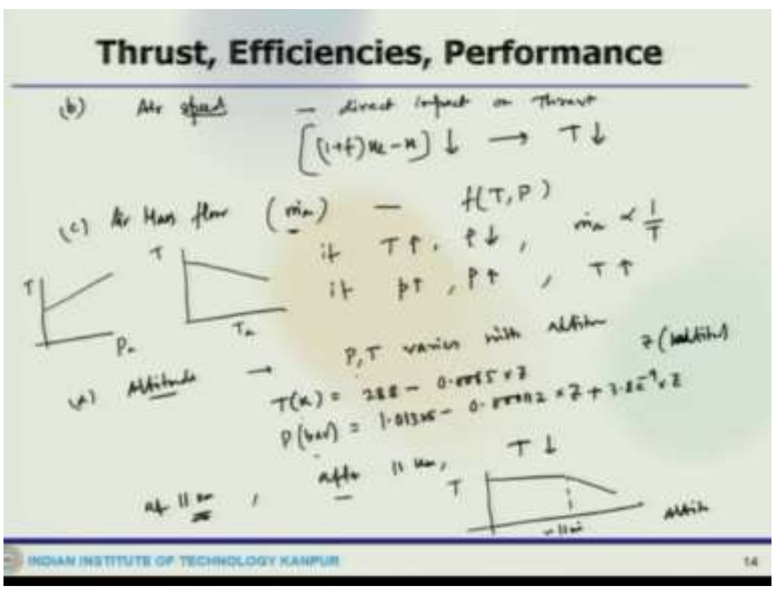


But the interested person may look at the compressible flow book for the time being. Now air speed, this also sometimes depends on the approach speed also is equal to the flight speed in the thrust force such a parameter has a direct impact on the net thrust. So there is a direct impact on thrust. If the exhaust gas velocity is constant and the air velocity is increased, then the difference between let us say 1 + f into ue – u. This also decreases which means the thrust also decreases.

So if the air mass flow rate and the fuel to air ratio are assumed constant, then the linear increased and the decrease in the net thrust is enhanced. So the air mass flow rate, so this is m dot a. This is a significant parameter in thrust. It depends on ambient temperature or air temperature or rather temperature-pressure and sometimes both together with the density. In free air, if there is a rise in temperature, if temperature is increasing, which will decrease the density and the mass flow rate will be also inversely proportional with the air temperature.

So on the other hand, the ambient pressure and increase in pressure; if pressure increases of free air also it increases its density. So that means the thrust also increases. So one can quickly see what happens, let us say this is my thrust and this is Pa, this is how it increases, if this is T ambient and this is my thrust, this is how it decreases. So the air mass flow rate though has important impact on the thrust, but this also depends on pressure and temperature which are at the ambient and how that depends.

Then the altitude, so this is another already during discussion on atmosphere, we have already talked about these that altitudes has direct impact, because in the altitude the pressure temperature that varies with altitude. So once pressure temperature varies, then with the height like one can write that how these things changes for temperature with the altitude, it changes like

$$T(K) = 288 - 0.0065z$$

where z is the height or altitude and pressure changes in bar

$$P(bar) = 1.01325 - 0.000112z + 3.8e^{-9}z$$

So after 11 kilometer, the temperature stops falling, but the pressure continues to drop steadily with increasing in the altitude. So consequently what happens above 11 kilometer, the thrust will drop most rapidly. So after 11 kilometer thrust drops up rapidly. So this mix around this 11 kilometer is an optimum altitude for long-range cruising normal speed just prior to rapidly increase of the effect of the altitude of the thrust.

So one can in one word, one can conclude that this thrust is really a function of density, because when you change the altitude, pressure changes, temperature changes, so density changes, obviously mass flow rate changes and the thrust is going to be affected. So just a schematic of that how it can vary like thrust and this is altitude. So you can see there is a linear variation and then there is a drop of thrust.

So this probably around roughly 11 kilometer. So this is how these factors affect and there are some more other factors also and we will continue this discussion in the next lecture.