Introduction to Airbreathing Propulsion
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Lecture - 10
Review of Compressible Flows (Contd.)

So let us conclude or rather continue the discussion on compressible flows. So from the beginning, we have discussed one-dimensional compressible flow, then normal shock and then we finished in the previous session the oblique shock. Now there are few more topics which I would like to touch upon like the Rayleigh flow and all these things and that pretty much is going to talk about I mean all the fundamental stuff, which we need in further cycle analysis all these.

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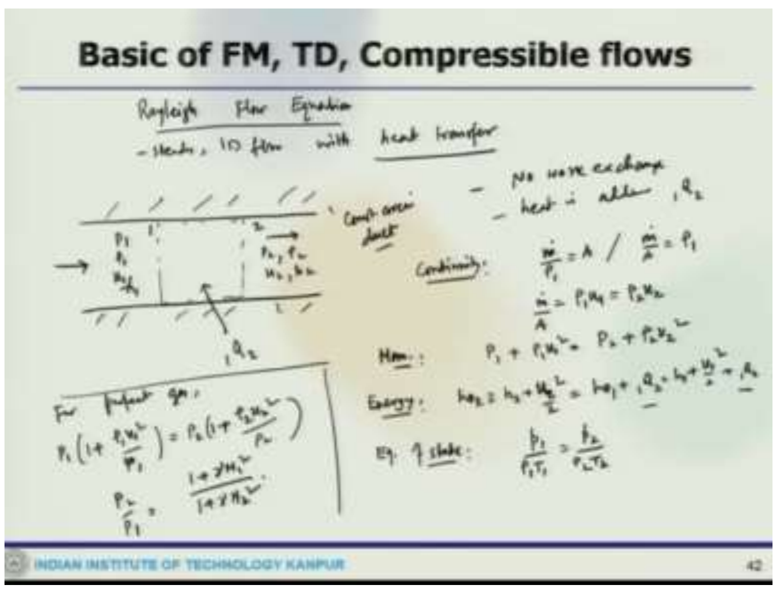

So this just to move ahead, we will start with the Rayleigh flow equation. So this is one of the topic that I would like to discuss Rayleigh flow equation. So this is also a steady 1D flow with heat transfer, so this is important. So thus quite appropriate this kind of flow or appropriate to treat the flow in combustion chamber as a Rayleigh flow case, because where the heat transfer takes place. Now let us put them sort of in schematic to see how we can analyze this.

Let us say, this is a one dimensional flow and we have taken control volume between these two, which is station 1 and station 2. This is upstream. So all your p_1, ρ_1, u_1, h_1 and downstream you have p_2, ρ_2, u_2, h_2 and you can have 1Q2. So this is a constant area duct. So that is another thing that also there is no work exchange while the heat is added, which is 1Q2. So that means heat goes inside; that is the direction. So we write down the governing equations.

First thing that we will write down the continuity equation. So again this is a constant area duct one-dimensional flow. So mass flow rate would be $\rho_1 = A$ or in other way $\dot{m}A = \rho_1$ So we can write

$$\dot{m} = \rho_1 u_1 = \rho_2 u_2$$

right. Now momentum equation where we write

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

So these are already we have derived the equations for steady one-dimensional flow at the starting of the discussion on compressible flow equations.

Now they look pretty similar. The energy equation would be slightly different that we can see

$$h_{02} = h_2 + \frac{u_2^2}{2} = h_{01} + 1Q2 = h_1 + \frac{u_1^2}{2} + 1Q2$$

So this is the portion of the added heat, which shows up here and the last equation of state that we get is

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

Now for a perfect gas, we derived the momentum equation like

$$p_1(1 + \frac{\rho_1 u_1^2}{p_1}) = p_2(1 + \frac{\rho_2 u_2^2}{p_2})$$

So which will give us

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

So that is a relationship that we get now.

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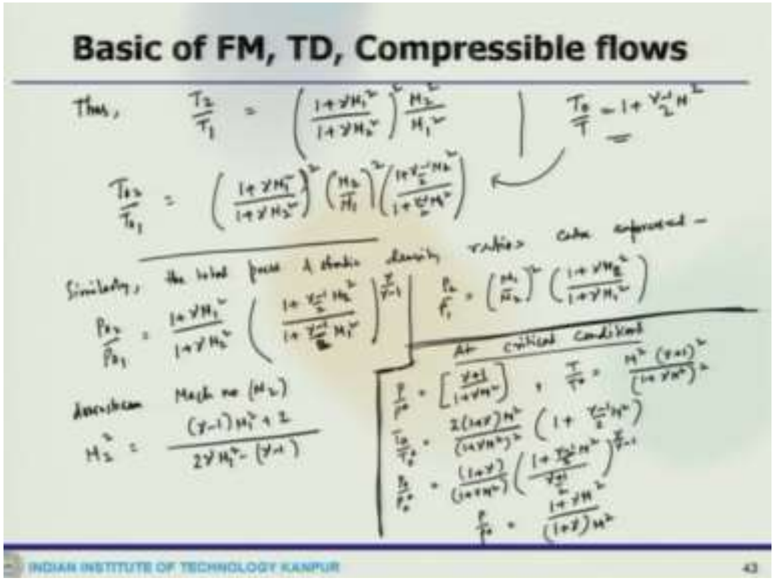

Thus, we can get similarly for temperature relations, which are

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2}$$

Now the relationship between static and stagnation temperature that still holds good. So this is the relationship, which if we use this relationship back in this equation, what we can write that

$$\frac{T_{02}}{T_{01}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{M_2^2}{M_1^2}\right)\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)$$

So that is a sort of a little bit messy relationship, but that follows the standard equations that one can use and derive those things. So now the similar way one can get the total pressure and the static density ratio. Let us say similarly the total pressure and static density ratio that can be expressed. So the

$$\frac{p_{02}}{p_{01}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}}$$

So that is what you get for the stagnation pressure ratio and static density like

$$\frac{\rho_2}{\rho_1} = \left(\frac{M_1}{M_2}\right)^2 \frac{1+\gamma M_2^2}{1+\gamma M_1^2}$$

So you get the static density ratio too. Now once you look at this, here is the Mach number 1.

This side it would be Mach number 2, so the downstream Mach number which is Mach 2. So the downstream Mach number that M_2 one can calculate like

$$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

Now all these relationship, this can be also kind of, for the critical condition it can be reduced to all that thing. So now at critical conditions, what we can write that

$$\frac{p}{p^*} = \frac{\gamma + 1}{1 + \gamma M^2}$$

Then we have

$$\frac{T}{T^*} = \frac{M^2(\gamma + 1)^2}{(1 + \gamma M^2)^2}$$

So we are assuming M_1 is that thing, then similarly

$$\frac{T_0}{T_0^*} = \frac{2(\gamma + 1)M^2}{(1 + \gamma M^2)^2}\left(1 + \frac{\gamma - 1}{2}M^2\right)$$

and similarly

$$\frac{p_0}{p_0^*} = \frac{(1 + \gamma)}{(1 + \gamma M^2)^2}\left(\frac{1 + \frac{\gamma - 1}{2}M^2}{\frac{\gamma + 1}{2}}\right)^{\frac{\gamma}{\gamma - 1}}$$

Finally,

$$\frac{\rho_0}{\rho_0^*} = \frac{1 + \gamma M^2}{(1 + \gamma)M^2}$$

So these are the condition that you can obtain for Rayleigh equations and just I would like to touch upon this thing. The reason is that when we will be dealing with the compressor, then the sorry combustor, the flow field inside the combustor could be treated like that, where there will be flow with some sort of an heat addition.

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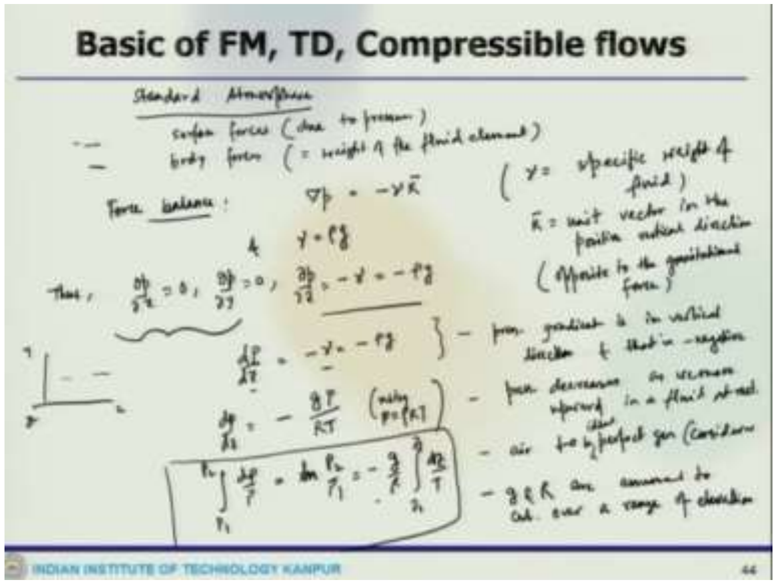

So now some last week comments on some of the other factors, which would be important. One is the we just talked about a little bit of standard atmosphere. So when we talk about the standard atmosphere, now we know when the flow for a fluid in rest without any shearing stresses any elementary fluid element will be subjected to 2 types of forces. One is surface forces which is due to pressure and that is number 1 and number 2 body forces which is essentially equal to the weight of the fluid element.

So that means when we do the force balance, so the force balance can give us this following relation that

$$\nabla p = -\gamma \bar{k}$$

where gamma is written as the specific weight of fluid. So this is not the gamma of air and $\bar{k}$ is the unit vector in positive vertical direction, which means this is opposite to the gravitational force and what we can then write $\gamma = \rho g$ So thus what we will have

$$\frac{\partial p}{\partial x} = 0, \frac{\partial p}{\partial y} = 0, \frac{\partial p}{\partial z} = -\gamma = -\rho g$$

So these first 2 derivatives here, they show that the pressure does not depend on x or y. So as we move from one point to another in a horizontal plane in a xy plane like this, let us say xy plane like this, then the pressure does not change, p only depends on z and so this particular equation we can write in an ordinary differential equation like

$$\frac{\partial p}{\partial z} = -\gamma = -\rho g$$

So this is a fundamental equation when the fluid is at rest and can be used to determine the pressure change with elevation or height.

And this particular equation, this indicates that the pressure gradient is in vertical direction. So and that is negative. So that is a very important information that this is when you go in the vertical directions that is when you feel the pressure gradient and that is also negative, so which means the pressure actually decreases as we move upward. That means, from the sea level as you go up the pressure actually decreases in a fluid, which is at rest.

Now for the atmosphere where the variation in heights are large or something on the order of thousands of feet or kilometer, so we need to pay some attention here that how this variation actually occur. Now in doing that, we can consider the air to be perfect gas. This can be considered ideal perfect gas rather. So ideal perfect gas and also the equation of state $p = \rho RT$ can be used and what we can write this

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT} =$$

this is using $p = \rho RT$

So if we separate the variables and integrate,

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

So here this g and R are assumed to be constant over a range of elevation. Otherwise they cannot be taken out of the integration and do this integration. So this actually provides the variation of pressure in Earth's atmosphere.

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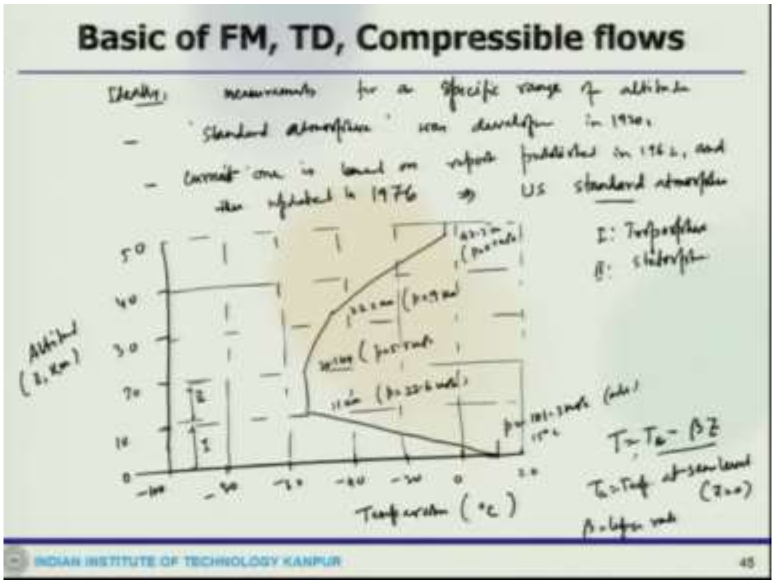

Now ideally one would like to have a measurement of pressure versus altitudes over specific range of altitude. So measurements for a specific range of altitude; however, this kind of information is usually not available. So thus a standard atmosphere has been determined and that can be used for the design of aircraft. Now this was this concept of standard atmosphere. This was developed in 1920s and since then there are many US and international committees and organizations have pursued and developed a standard.

But the currently accepted atmospheric standard is the current one, is based on report published in 1962 and then updated in 1976, so which is so-called US standard atmosphere. So that is a sort of idealized representation of the middle latitude with year round mean condition of earth atmosphere. In that particular chart, what one can see? This is let us say -100, 80, 60, 40, 0, 20. This is 0, 10, 20, 30, 40, 50. This is altitude that is Z in kilometer.

This is temperature in degree C. So up to certain range of this and this, so this is known as this is zone 1 and let us say this is zone 2. So zone 1 is troposphere and zone 2 is stratosphere. So somewhere the curve looks like these, then it goes like that and like this. So this is at P = 101.3 kPa which is absolute at 15 degree C. This is at 11 kilometer is roughly 22.6 kPa; this is something at 20.1 kilometer 5 kPa. This is 32.2 km, P is 0.9 kPa, 47.3 kilometer P is 0.1 kPa.

That is the kind of variation with the altitude one can find. So there are troposphere and stratosphere. So what one can use the temperature variation like

$$T = T_a - \beta z$$

So Ta is a temperature at sea level or let us say Z=0 and beta is the sort of lapse rate or rate of change of temperature with height, then you can find out. Now these particular equations if we use along with the previous equation, so what we get?

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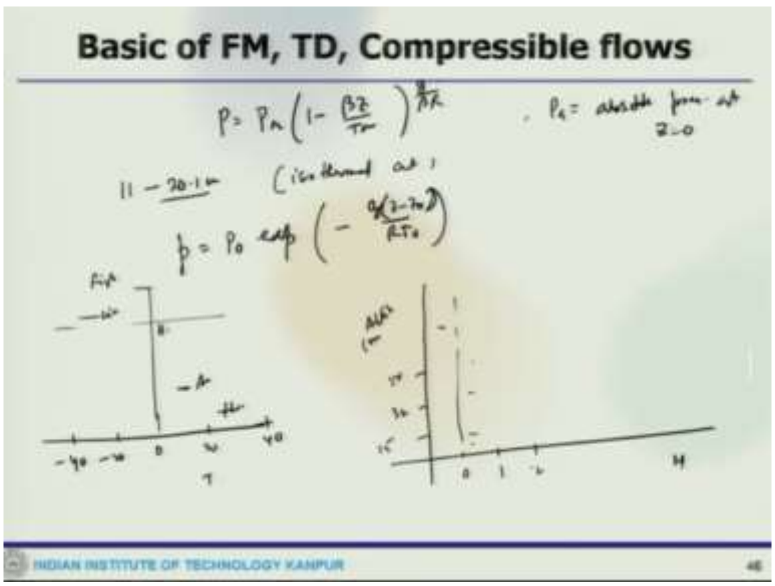

That pressure is

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{\beta R}}$$

So again Pa is the absolute pressure at Z=0 and then one can find out. Now for the atmospheric layer between 11 to 21 one kilometer, so temperature has constant value. This is isothermal conditions and then again the pressure elevation relationship which we can write just

$$p = p_0 \exp\left(-\frac{g(z - z_0)}{RT_0}\right)$$

So that is what one can use.

So p_0, T_0, z_0 are the pressure, temperature and altitude of lower edge of the atmosphere and then and the higher edge one can find out, so you can form a table like that. So this is how one can see the essential parameters like this and this is where let us say, this is 0, 20 degree, 40 degree. This is temperature this is – 20, - 40 and altitude in some kilometers in this side. So typically this is where your helicopter works. This is some low transport aircraft.

This is somewhere above; this is around somewhere 11 kilometer or something. This is where the civilian aircraft and then top of that fighter. So this is the typical picture and if you see the with the Mach number and the altitude, so that is what the another envelope you will get. This is Mach number, which would be close to let us say 0, where there is a curve, then 1, then 2 and this is 15, 32, 50, these are in altitude kilometer.

So low range here again your helicopter, piston engine, turbo propeller, then you get turbofan, then military fighter, ramjet, then commercial jet works between 65 to 80, then you have other fighter jet, so this is another envelope which you will get. So one can see that this atmospheric variation with temperature and pressure, which are also important for aircraft design. This is also just to give you an idea, because this would be sort of required when we will do the analysis for the cycle analysis or the performance analysis. So pretty much completes some of the fundamental discussion on compressible flows and fluid mechanics. So we will continue with the performance in the next session.