UAV Design - Part II
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Lecture - 07
Stability and Criteria for Longitudinal Static Stability

Dear all, welcome back. During our previous discussion, we considered aircraft as a rigid body which has six degrees of freedom in space. And we classified the motion of the aircraft into two phases, the first one is longitudinal case and the second is lateral directional case. And the longitudinal case is a single vertical plane motion, where it can translate along the body x and body z axis.

And all associated with the rotation about y axis call pitching moment, right. And we also looked at what is lateral directional motion which is which involves two rotations and one translation along y axis and the rotations here or rolling motion as well as yawing motion which is about x axis and z axis respectively, right.

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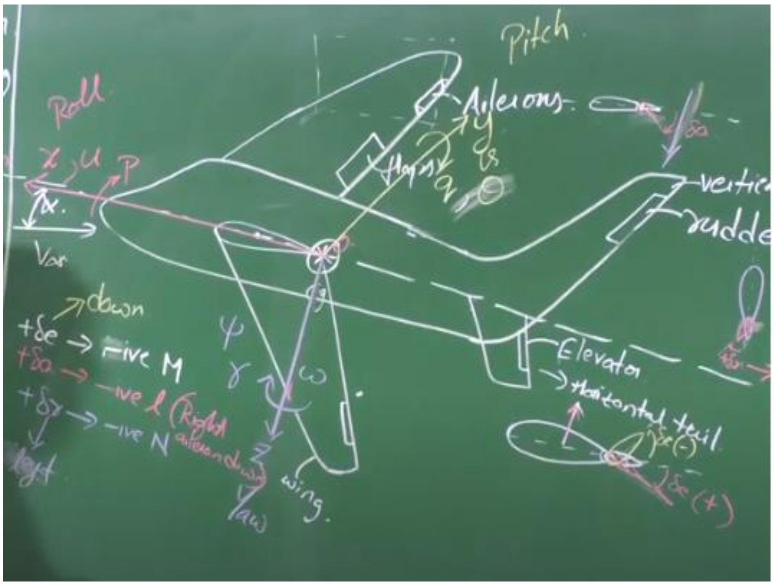

So we have an aircraft. So let us define an axis with respect to which all the components of this aircrafts are mounted. Let us say this as fuselage reference line, right. So we will use this fuselage reference line while talking about stability of an aircraft here. So now let us consider a body access system. So we have wing yeah, let us quickly go through this major components of this aircraft.

So we have wing, right and a horizontal tail and a vertical tail, right. So these are the three major components which are indeed fixed components of aircraft along with the propulsion system, right. So and we also have some small moving surfaces that are attached to this aircraft, fixed components, right.

So these moving surfaces, when deflected at certain angle with respect to the flow right, say this aircraft is moving at certain velocity V infinity making an angle of attack alpha with respect to this free stream velocity, right. So when we deflect this control surface with respect to this flow at a certain angle, so this will alter the local aerodynamic forces which generates momentum, right about C g.

Let us say this is my C g. This is my C g. So the change in aerodynamic force aft this C g or at an offset location with respect to C g will produce a moment because of it, right. So this moment is, because of this moment the aircraft orientation changes, right orientation in space changes. So the surfaces with which we control the aircraft orientation is known as control surface right.

So the control surface on the horizontal plane is elevator horizontal tail, which is on the vertical tail is rudder and on wings we have ailerons and we also have some high lifting devices called flaps attached to the wings. Flaps are majorly to enhance the lifting characteristics while takeoff and landing whereas ailerons, rudder and elevator are to control the orientation of the aircraft throughout the flight.

So we call them as control surfaces. So these are the moving components here, right. And then yeah of course we have propulsion system either a propeller driven or a jet driven aircraft or a UAV, right. And we said that we have six degrees of freedom of motion here. So let us consider the origin of this coordinate system at the C g itself, so which we discussed during the previous course.

And let the x axis be pointed towards the nose of this aircraft from the C g location. So this is our x axis of this body coordinate system. And then we have z axis which is towards the undercarriage right. So let O be the origin of this coordinate system, which is coinciding with the C g of the aircraft. And then so let z be towards the undercarriage and then we have so we are considering a right handed coordinate system, so z is in the downward.

So x cross y is z. So what we have is the second axis which is y axis here right, which is into this board here, right. So we have defined the aircraft motion in two modes.

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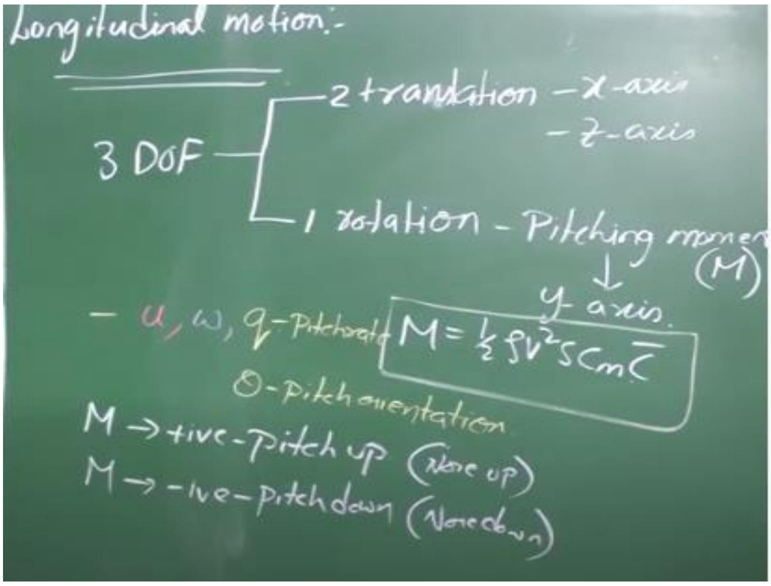

So the first one corresponds to longitudinal motion. You can also say this as a single plane motion right. So for an aircraft or a UAV we assume there exists a plane of symmetry. What exactly it is? We have a vertical plane about which the geometry though it is it does not look symmetric here, but in reality, we have the geometric symmetry about that vertical plane as well as mass distribution, the symmetry in mass distribution, okay.

So if the aircraft is if the aircraft is moving in this in that particular vertical plane or the motion is constrained to that particular vertical plane or the plane of symmetry, we call such a motion corresponds to longitudinal motion right. So the aircraft can translate along x axis and z axis which corresponds to this vertical plane of symmetry right. So there are two translation motions.

And it can also have a rotational motion in this vertical plane. So the aircraft is not crossing this vertical plane. So the motion is still constrained to this vertical plane even during that rotation, right. So we have translation along x. So in longitudinal plane we have 3 degrees of freedom, 2 translation so which is along x axis and z axis, right and 1 rotation.

So rotation is due to moment as we discussed. So we have a pitching moment, which is the rotation about y axis, right. So what we have is pitching moment, which is given by q, right. So the pitching moment is so for a longitudinal motion, what we have is, so let u be the translational velocity along x and w be the translation velocity along z axis and say v be the translation velocity along y axis.

Now we what we have is so you have u, w and u, w and q, right. So the corresponding what is q is the corresponding pitch rate, the rate at which it is the aircraft is changing its pitch orientation. So because of this pitching moment say M, let this pitching moment be M in the longitudinal plane denoted by capital M, which is given by half rho V square, S C m.

Where C m is non-dimensional pitching moment coefficient multiplied with a characteristic length along this longitudinal axis which is the mean aerodynamic chord of this wing, right. So you can right so this pitching moment creates a change in orientation and the corresponding changes are the orientation change in theta, right. So we have because we have a rotation about y axis, which changes the orientation of this aircraft given by theta.

And the rate at which this theta is changing is q, right when there is no other rotation involved. So q is the pitch rate. We call it as pitch rate. Theta is the corresponding pitch orientation, okay. So u and w are the translation velocities here. So how can we generate this pitching motion? How this moment is generated? We know if we can generate a force at an offset distance from the C g in the longitudinal plane that can create a moment here, right.

So about y axis if I have to produce a moment, then I have to produce a force in this vertical plane at an offset to the C g, right. So we have a control surface for this longitudinal control called elevator right. When you deflect an elevator, it produces, it changes the aerodynamic force there, which produce a moment about the C g, right.

So the corresponding deflection at this control surface, let us say if I take the cross-section of this tail. So this particular attachment is a moving surface for this horizontal tail, right which is a shorter version a smaller version of this wing, right. And in general this is a symmetric wing. We will discuss why it is. So when we deflect this particular control surface, say if you deflect this control surface down, right.

So this deflection obstructs the flow here. You can say changes the camber of the airfoil or you can say you obstruct the flow right. So the flow gives a reaction in the upward direction. So this produces a force net force upward direction. So about C g this force produce a moment right, pitch down here, right, pitching down right. So let us also define the convention here.

So about y axis so if you stretch your thumb along the positive y axis the curl of your fingers will give you the corresponding positive rotation right. So pitch-up is positive. So nose-up, what is a positive rotation about y if the aircraft rotates its nose up right. So whatever the moment that helps the aircraft to rotate its nose up is called positive moment, right. Corresponding moment is the positive moment.

And if the aircraft is rotates its nose down due to that moment we call it as pitch-down moment right and it is considered negative, right. So and q is considered positive when it is pitching up and the t rate is changing in the like with the nose-up direction and theta is also considered positive when the aircraft nose is up, right. So pitch-up is positive.

So for moment is positive when it is pitch up, right which is nose up. And this moment is negative if it is pitch down or nose down, okay. So similarly, we have lateral directional motion.

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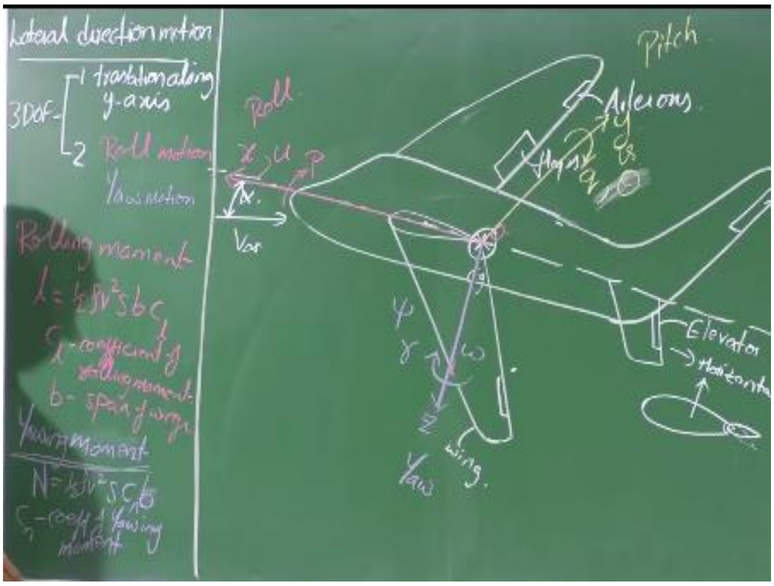

So any motion that cuts this vertical plane of symmetry is known corresponds to a lateral direction corresponds to this lateral directional motion. So what are all the possible cases? Translation along y axis. So this aircraft has to cut this plane of symmetry say let this blackboard be the vertical so this board be the vertical plane of symmetry.

So aircraft craft moving in and out of this aircraft, sorry of this blackboard is translation along y axis and this corresponds to lateral directional motion. So it has again 3 degrees of freedom where the first degree of freedom is due to translation along the y axis. And you have two degrees of freedom by means of rotation here. So one rotation can be the wings rolling right, rolling up and down.

And the rotation about x axis is known as rolling motion, right. So and the positive rolling and the positive roll is given by so what we have is roll and what x axis and we have pitch about y axis and you have about z axis, okay. So the positive roll is given by stretch your thumb along the positive x axis and the curl of your finger will give you the direction of positive roll.

So corresponding rate is the, the roll rate here given by P, right. So what you have is roll and yaw, right. So these two are rolling motion and yawing motion are due to rotations, right. So which are caused by moments. So the corresponding moment about x axis is known as rolling moment. So rolling moment is given by is rolling moment.

So is represented by small l right, which is half rho V square S b times C l, where C l is the coefficient of rolling moment and b is the span of this span of the wings, right. So C l is coefficient of rolling moment. And b is span of wings. And we also have a moment about z axis which makes the aircraft to rotate about this particular axis that will yeah that will help to yeah orient the nose in the horizontal plane, right.

So that change the orientation of this aircraft in the horizontal plane. So that is known as yawing motion. So it is again or it is due to so this that particular motion is known as yawing motion. So this is rolling motion which is rotation about x axis and rotation aboard z axis. So rolling motion yeah z axis is yawing motion and is given by, the corresponding yaw rate is given by r right and it.

So due to this it results in a orientation, change in orientation in the horizontal plane, right. So the aircraft knows initially say along this particular direction. Now due to rotation about yaw axis or say z axis the aircraft nose is now oriented in a different direction. So this particular change in orientation is given by psi right. So under no other rotations, so this psi dot is equals to r, okay.

That is single plane rotation. So psi dot is equals to r. And the positive yaw is given by right wing going back, right. So if you stretch your thumb along the positive z axis, the curl of your fingers will give you the corresponding positive rotation where the right wing going back is corresponds to a positive yaw moment, yawing moment, right. So yeah and right wing going down is positive rolling moment okay.

So stretch your thumb along positive x the corresponding curl of your fingers will give you the rotation, positive rotation of roll. So yawing moment so which is given by capital N, which is half rho V square dynamic pressure times the area will give you the force and the corresponding non-dimension force coefficient times b reference, reference length here, right.

Where C n is the coefficient of yawing moment, right. Now what will be the positive control surface deflections? So the direction in which so the say let us say we deflect a control surface and if that control if that deflection produces a corresponding negative moment right then we can consider that particular deflection as positive.

For example, in case of elevator stretch your thumb along positive y axis again right, y axis and the curl of your fingers will give you the corresponding positive rotation here. So this so elevator moving down, so this particular so this was the reference and now we have deflected it downwards. So say this is the current deflection of this elevator, okay.

This particular angle is known as delta e. When you deflect it down there is some upward force right because of the change in camber here there will be increased lift at the same like for the same angle of attack that we are seeing right. Or you can say you are we are obstructing the flow right. So that flow will give you a reaction as an upward force. And this change in force at the tail will produce a moment here, right.

So this moment will be rotating the aircraft nose down. So nose down is negative moment. So pitching so the moment that you have is negative pitching moment, is it not? So when you have if the deflection contributes towards negative pitching corresponding negative moment, right. So then we consider that as positive deflection. Delta is positive when it is deflected downwards and negative when it is deflected upwards.

So you can say so if this is delta e then it is negative, right. So elevator is what creates the orientation control along the pitch axis is it not in the longitudinal plane. So controlled by elevator here, right. So the corresponding moment that is generated due to elevated deflection is a pitching moment, is it not?

So if this elevator produces a negative pitching moment, deflection of this elevator helps to helps the aircraft with a negative moment then we say that particular deflection is positive deflection. Now similarly, we have ailerons here. So when we deflect an aileron, right. So the deflection so the rotation, the corresponding rotation will be about x axis.

Why because with respect to C g we have two ailerons on the both the wings right at an offset location along y axis. So when there is a force, upward force at an offset, y offset that creates a rolling moment, right. Let us say this is my aircraft, that is the nose of the aircraft and this is or say this is the nose and that is the tail. Say I have one wing here. Let us say I have a wing here right.

So this is my nose and these are my wings, okay. So I have one control surface here and there is another control surface here. So we have two ailerons located on the wing, right. So one on the right and the one on the left wing or one on the port side the other one is on the starboard side, right. So when we deflect this particular aileron, and again we see it is located towards the tip of the wing, right.

So it and this aileron when deflected will produce a force, right. So when I deflect let us say stretch your thumb along the positive x axis. Now say the x axis is towards the nose here. So the corresponding curl of your fingers will give you the corresponding positive rotation of the control surface as well. Not only the rotation of the aircraft about that axis, but also the control surface rotation.

So now, so my fingers are pointing towards downward motion of this particular control surface. When I deflect this downward what happens is it produces a force upward. Is it not? So these two ailerons are deflected in opposite direction. When I deflect it that right aileron down so the left aileron will be deflected upward, right.

So this is how in general the convention is and the average of these two deflections is the actual control surface deflection of the ailerons right. So when you deflect this up and this down right so when you deflect it down, we have an upward force. When you deflect it up you have a downward force. So this upward force will contribute towards the negative moment.

Even the downward force from here multiplied by this offset distance contributes towards a negative moment right. So the positive deflection is deflecting right aileron downwards, right. So if we deflect this downwards, deflect this aileron down, so let us say this is the new location of the deflection. If you deflect this downwards, right if you deflect it down then we say this is positive.

So here if I take a cross-section, if I take a cross section there, so say this is my wing section and this is the corresponding control surface attached to this wing section, right. So if deflect this downward the right aileron. So if I deflect this aileron downward so I consider this deflection as positive delta a. On the other hand when I deflect it up right, so if I deflect so let us say so this is again this is my nose.

So if I deflect this up this the aileron will change the local aerodynamic force there, right. So the resultant local aerodynamic force will be acting downward, right. And then when I deflect that up, I have to deflect this down. That is how the convention for this ailerons is. If I deflect this down, so I this will produce an upward force, so that will produce a positive rolling which is the downward force and the upward force right.

So multiplied by the corresponding momentum here will produce a positive rolling moment. So positive rolling moment is right wing going down. Say this is my right wing and this is my left wing. So positive deflection is right wing going down, right. And if I have to consider rudder here, so if you look at from the top view, so if we look at from this from this top view, right.

So the rudder cross-section will be, again it is a symmetric tail, symmetric vertical tail. So this is how the rudder looks like and then it is attached with a small moving surface called rudder, right. That is what we discussed just now. And when you deflect this rudder it has to produce the corresponding yawing moment right. So this the usage of rudder is to change the orientation in the horizontal plane.

We generally call it as heading, right. We try to change the heading of this aircraft by deflecting this rudder. So which direction if what should be the deflection of this rudder so that it produce a negative moment. Towards the left or towards the right? Yeah, so when you deflect this rudder towards the left, what happens I deflect this rudder towards my yeah towards me, let us say.

If this is the control surface if I deflect this rudder towards me, so this produces a force into the board right. So that force acting at an offset distance with respect to C g.

So it has a longitudinal offset here as well as vertical offset. But let us talk about the longitudinal offset here. So this longitudinal offset and the multiplied with that force that is produced by the rudder creates a yawing moment right.

So that yawing moment is negative, right. So it is rotating left wing out right, left wing back. So the yawing moment that rotates the left wing back is negative yawing moment. That is what we just discussed. So that means when the rudder deflection is producing a negative yawing moment, the corresponding deflection of that rudder is known as positive, right. Is considered to be positive.

So if I have this rudder deflected towards left side right, here. That means what is, so if you place it like this, you just translate it and place it like this. So if you deflect this outside right, so then it is considered as positive delta r. So positive delta r results in negative and negative yawing moment, right. So what we can conclude from here? Positive delta e produces negative pitching moment, right.

And positive okay so positive rudder deflection will produce negative yawing moment. And positive aileron deflection produces negative rolling moment, right. Is it not? So what is positive elevator deflection? Deflecting it down. So the positive, the corresponding positive deflection is deflecting it down, right.

So here it is towards left. So when you are facing the aircraft from the tail right, when you are facing the aircraft from the tail. So then the corresponding deflection towards, rudder deflection towards left side is considered positive, right. Similarly, so aileron deflection right down, so right aileron down. So if I deflect this right aileron down that is considered as positive aileron deflection.

And in general this is taken as average no? The aileron deflection in general is average of this value plus I mean left side and right side aileron deflection value. So if you want to talk about the flying characteristics of this aircraft, first we need to talk about an important property called stability of this aircraft right.

So stability is a property of equilibrium state right and we have defined earlier what is equilibrium and we all know it is a state about which resultant forces and moments are zero, right. So if you want this aircraft to fly right, first of all we need to talk about whether the aircraft can fly or not. So stability is a concept that talks about whether the about the flying qualities of this aircraft, right.

So stability is a property of equilibrium right. And equilibrium is a state about which resultant forces and moments are zero, right.

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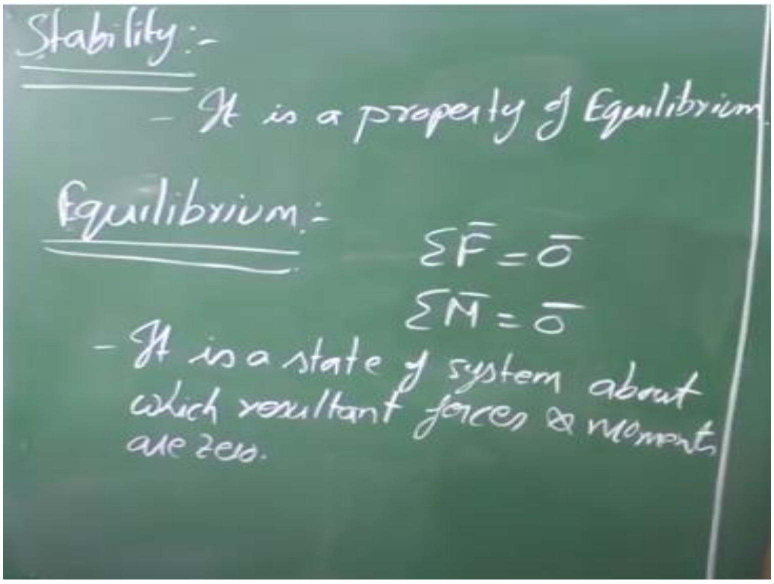

So what we what we are going to talk about is stability. So as I mentioned it is a property of equilibrium, right. So what is equilibrium? When we say a body is in equilibrium? So when the resultant forces and moments acting on that body at that particular state are zero, right. So we say equilibrium corresponds to it is a state of system about which the return forces and moments are zero.

Or no force no net forces and moments acting on the system, right. So let us consider the following three cases. Just to talk about the equilibrium state, what are the various equilibrium states, right? So we should first consider an example.

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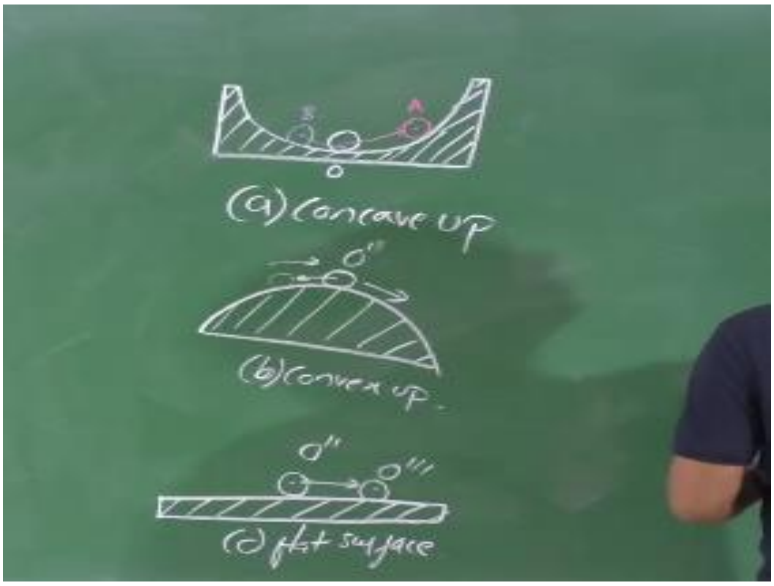

So let us let us talk about that with the help of this example. So let us consider a concave up right surface and let us place a ball on this particular surface. Let this point be O, right. Say this one is concave up, right. At the same time consider another surface on which the ball is again resting, right. Let this position or the location be O prime right, O prime.

So this is convex up. Let us assume this as a flat surface, right. And the same ball is placed on this flat surface here. Let this location or position be O double prime. So this is on flat surface. So let us understand what are various types of equilibrium with the help of this example. So in all these cases, if you observe the ball is at rest. That means the resultant forces and moments acting are zero.

Which implies the ball is in equilibrium. In all the three cases irrespective of their, of the surface on which it is resting in all the three cases the ball is at equilibrium, right. In the first case, so when we displace this ball from this equilibrium, let us see what happens in this different cases, right. Now how can we display this? With a gentle push, is it not? So let us use such a gentle push for this case a where the ball.

So due to this push the ball will try to roll up the surface. Is it not? It will try to go up the surface here and then it may reach a location say A. So this corresponds to the point where the velocity of this wall becomes zero, right. And once it becomes zero then what happens is we know it will come down right and it will come down towards

O and then it will overshoot O and my reach point B. The same ball may reach point B.

So again B corresponds to zero velocity. After that again it may start moving back towards O and this oscillations continues, right. So at some point, let us say once the disturbance is dissipated, right. So this particular ball will try to reach at or attain this point O, which is the initial equilibrium state, right. So that particular equilibrium is known as or is defined as a stable equilibrium, right.

So consider the second case. So we and the ball is at equilibrium which is O prime here right. So when you displace this ball what happens is it will try to roll down the surface. It will try to move up the surface completely. So it may not come back to its O prime right. So it is same case like even if you displace to the other side. So it the ball may not come back to this O prime, right.

So the system here the ball is not able to attain the initial equilibrium, which is O prime right. So in this particular equilibrium is defined as unstable equilibrium for this system, right. So in the third case, we have a ball here. So when you displace this what happens it will translate to a new location and try to attain a new equilibrium let us say O triple prime, right.

So in this case, it is not coming back to the new equilibrium, but it is trying to attain another. So sorry it is not coming back to the previous equilibrium O double prime, instead it is trying to attain new equilibrium O triple prime. So such a case is so such a case is defined as neutrally stable equilibrium, okay.

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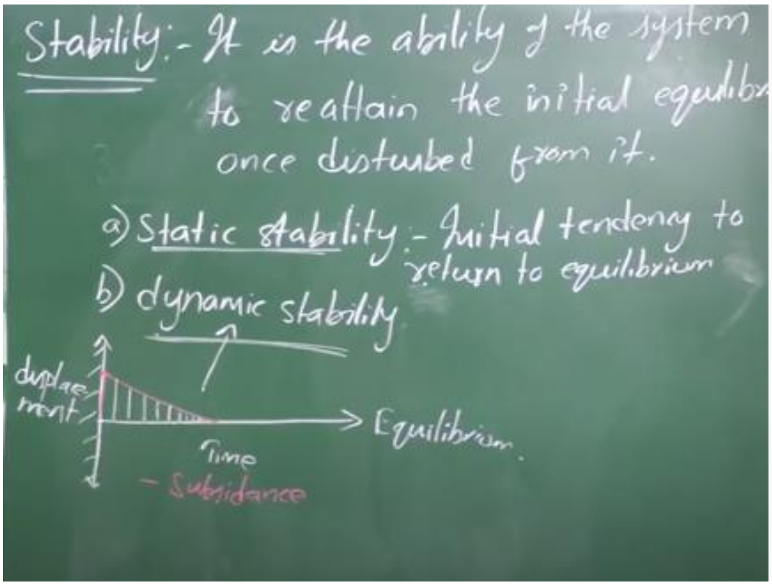

So with this let us define what is stability. So it is a ability of the system to reattain the initial equilibrium. So once you disturb it, disturb the system from that initial equilibrium, it should have the ability to retain that equilibrium state, right. Such a system if the system have that property then we say it is a stable system, right. So for us to understand about this stability we need to talk about.

So for in order to define the stability of the system, we need to talk about two cases called static stability and dynamic stability. So in this case in the first case, see the system is stable here, right. That is why the equilibrium we said stable equilibrium. So the ball tries to oscillate about this O and finally reaches O here, right. The same so in the other case, we call it as unstable equilibrium, right.

And the third one is neutrally stable equilibrium, right. So where in the second case, it is not able to get back to that initial equilibrium state. So it is not stable, the system is not stable here, right. Here the system is neutrally stable. Now what is static stability? So whenever, in this particular example, whenever this ball reaches to a maximum or say zero velocity location or the maximum location or the zero velocity location here, it will try to return back towards the initial equilibrium point.

So whether it stops there or not, we are not worried but at each, if you look at the extreme points at each and every time it has a tendency to move back towards this equilibrium location, initial equilibrium, right. So that initial tendency to come back towards this equilibrium is known as static stability. It is a initial tendency to return to equilibrium, right. What is dynamic stability?

So generally dynamic stability talks about the time history of motion of the system once disturbed from the equilibrium, right. It talks about how long it takes for the body to reach the equilibrium or whether it reaches equilibrium or not. So in order to understand this better consider this example, right.

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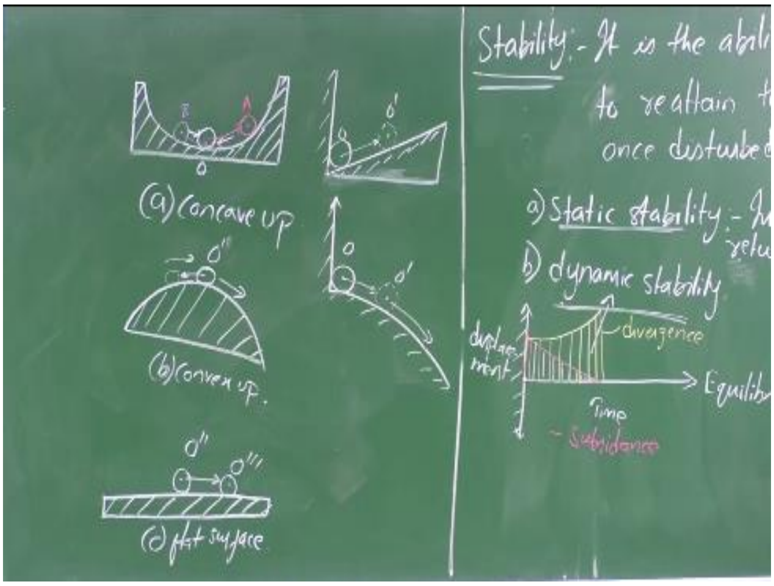

Consider this slope, right a surface with slope right and there is a abrupt obstruction towards the end of the slope, right. Now let us say initially we place this ball here on the surface at this junction, okay. Now it is in equilibrium definitely we know that. And once we displace this to a new location say let us say this is O and this is say O prime, right. Once you have displaced it, so the ball will try to automatically come back towards this equilibrium, is it not?

So if I plot displacement with time, say this is my equilibrium. What I can notice is, so this is my equilibrium. I am, initially this is at equilibrium. I am trying to take this to another location. I am trying to displace it, and then leave that object, right. So what it does is, it will try to achieve this equilibrium with time right. So such a system or such a process through which it achieves the equilibrium is known as subsidence.

So let us consider the other case. Again here we are talking about time history of motion once we disturb, right. So that pertains to dynamic stability, okay. Is it not?

We are talking about, so this is the initial amplitude. So at each and every time this initial amplitude of displacement is decreasing with time right. So this particular value this keep decreasing with time.

And consider the other case. Let us say we place this object here. Let us say somehow we made this ball stand at this particular location, right. Let us say the initial part may be a bit flat compared to this curve, okay. So this particular equilibrium may be O, right. So once your displace this to a new location here to O prime what happens is this ball will try to roll off this surface.

So again if you plot this particular, so say let us assume the magnitude of displacement is same, where we have displaced to this particular location. So what happens is with time the displacement increases, right. It keep, so what do you mean by displacement here? So it is a offset from the initial equilibrium location, right. So with time it is moving towards the equilibrium. That means, the displacement is keep decreasing. In this case, the displacement is keep increasing, is keep on increasing, right.

Why because it rolls off the surface and the distance between O and O prime is increasing, right. So this particular yeah so here you can see. So this particular initial displacement is nowhere reducing. In fact, as the time progress it is increasing, right. So this particular case is known as divergence. So with time it is not able to reach the equilibrium position. So we call it a divergence.

So it is dynamically unstable. So if with time if the object reaches the equilibrium then we say it is dynamically stable right. So for a stable system it is essential it should satisfy both dynamic stability as well as static stability as well as dynamic stability right. Now let us consider a different case. So this is these are non-oscillating motions right. But aircraft is a, so we can consider it as a second order system.

So in general have an oscillatory motion, right. Once we disturb the aircraft from its equilibrium location position, so it tries to oscillate about that equilibrium. It has certain oscillatory motion. So it is a second order system, okay. So I am trying to get back to that by erasing this example.

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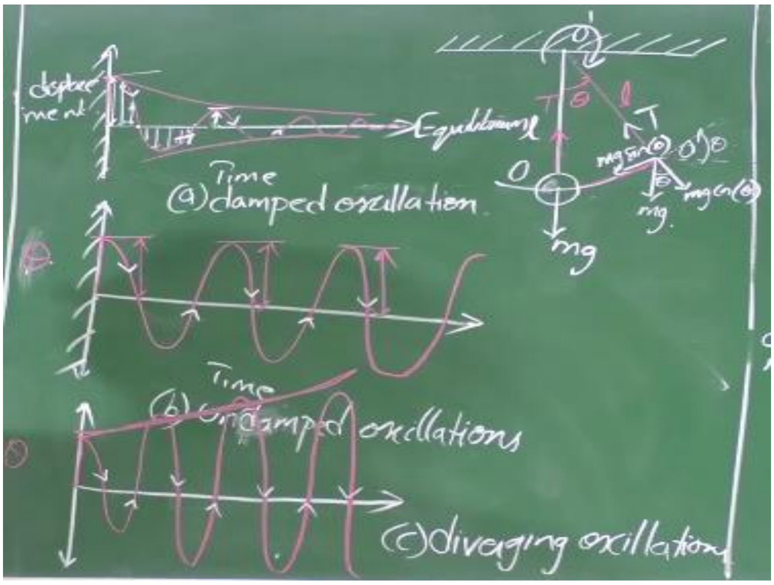

So one easy way to visualize this is consider a second order system, a spring mass damper or a simple pendulum here, right okay. So let us say l be the length of this pendulum which is pivoted about point O prime and say and the weight of this bob which is hanging at the bottom of this length l, at the bottom of this length l is say mg right, which is which will be acting perpendicularly down say.

So this is perpendicular to this ceiling okay. And now, let us displace this object to a distance. So in this case, the tension acting in this string is balanced by mg and we can say this is an equilibrium state, right. And it is at rest, it is not moving. Now let us displace this bob by an angle theta. So say this is the displacement theta, right; l still remains same, is it not? So mg is acting downward, right.

So when we displace this and if you leave it we know it will try to move towards the initial equilibrium which is say O here. Let it be O be the initial equilibrium and say O prime is a displaced so location at which O yeah O prime is a geometric location that corresponds to this deflection theta, right. Now when we leave it from O prime the bob will try to come back, that mass will try to come back towards this equilibrium, right.

And then it will try to oscillate about this equilibrium. So what happens is, we can try to oscillate about this equilibrium with time, right. Is it not? Am I correct or not? So let us say if this is the displacement, right. So this is the initial, so this is again my equilibrium location, equilibrium right.

So when I displace this pendulum what happens is it will try to attain try to come back towards this equilibrium but will continue oscillating about that equilibrium until this disturbance damps out right. Am I correct? So such a motion where which helps the system to come back towards this equilibrium with time right is known as dynamically stable system, right.

So that system with this motion right that helps to helps it to come back towards this equilibrium we call it a dynamically stable system. So this is dynamic this promotes, this kind of motion promotes dynamic stability, right. Why because see here the amplitude this much. So with time it is changing, here it is decreasing, and here it is in opposite direction, right.

So if you see the maximum amplitude is significantly decreased compared to this particular location right, the initial amplitude. So with time if this happens if this amplitude keep decreasing with time then we can say this is a dynamically stable system. And we can also notice yeah, so let us move on to the second case as well.

Let us say if there is a motor here, which is otherwise, once I displace this here, if I do not allow this to damp. Otherwise, if you are conducting this experiment in vacuum, right, where there is no air, there is no resistance here. Why this is coming to rest is because of the air resistance. So when we do not, when we do this experiment in vacuum, it will try to oscillate. Am I correct?

What happens is the same system when we place it in vacuum, say in other words, there is no friction at the pivot and there is no damping because of surrounding air molecule right. So again, this is displacement and this is time, x axis is time. And we have displacement on y axis. Let us denote it by y, right. So if I displace it by an angle theta or say displacement here let us say theta, right.

So by an angle theta. So when there is no dissipation of energy in the system, it will try to oscillate right. So the magnitude of oscillation or the amplitude of oscillation will remain constant since there is no dissipation with time, right. So such a so there is no dynamic. So the dynamically it is unstable, right. Why because, it is not coming back towards its equilibrium with time.

Is it not? So it is trying to maintain the same energy or same amplitude all the time, right. There is no dissipation here. So what we call this as damped oscillations. So for dynamic stability, we look at damped oscillations for dynamically unstable system so is associated with undamped oscillations, right. Is that all? Can we expect something else or these are the only two possible cases?

So for a pendulum it will be quite a non-intuitive to imagine a diverging motion, right. Let us say if I program this at the pivot with a motor right where the motor's rpm or the angular displacement keep increasing or say if I am adding energy to the system after each and every oscillation, what happens is the amplitude of oscillation increases. Is it not?

So say I have initially displaced this by an angle. From there if the energy once displaced from it, if there is an addition in the energy. So the magnitude keep increasing it is like diverging right. So this is like divergence. So this is convergence. Is it not? Subsidence, similar to that of subsidence there. So the magnitude is decreasing in this case. It is increasing in this case.

Am I correct or not? So this is like can you see this? Like this is like e power t. This is e power minus t, similar to that is it not? And superimposed by harmonic motion. So the solution can be e power t times or e power minus t times cos of omega t or sine of omega t multiplied by. Is it not? If you look at this particular signal the solution can be yeah, you can guess the solution is it not?

So here we call this a diverging oscillations, correct? Case c is where we talked about diverging oscillations. So the last two cases corresponds to dynamically unstable system, right. Whereas the first case corresponds to dynamical stable system. So but if you carefully observe, in all these cases the system poses static stability, is it not?

If you observe this, so after the displacement say this is the equilibrium it tries to come back towards equilibrium is it not? So even here it is trying to come back towards equilibrium. Here it is trying to come back towards equilibrium, the same thing here, right. So the system is trying to attain that equilibrium once it is displaced from the equilibrium, right. So it has that initial tendency.

So even in this case it is true, right. So this is our equilibrium. So the system is trying to move towards equilibrium. Is it not? And even here, it is the same. So even here, even here. So though it is an undamped oscillation, but still we can see the system is trying to, system process that initial tendency to come back towards equilibrium. Am I correct or not?

So consider this case as well, you have that initial tendency to come back towards equilibrium, right. So though, the system is statically stable in all the three cases, we cannot claim it is stable system. Why because it is not dynamically stable. So static stability may not guarantee the stability of the system or may not guarantee the dynamic stability of the system.

But when the system is dynamically stable, it guarantees the static stability, right. That is what we noticed here, is it not? The system here is dynamically stable. That means it guarantees static stability. In other two cases, the system is not dynamically stable, but still the system poses static stability. So static stability may not guarantee dynamic stability, but dynamic stability guarantees static stability, okay.

So now, let us talk about aircraft, right. So first, when we are talking about aircraft, we will talk we will try to divide this stability concepts of aircraft to longitudinal stability and lateral directional stability. And to start with, we will talk about longitudinal static stability. In fact, we did this during our previous course.

So but still these concepts are important and it is worth refreshing them, right. So though we are trying to take some time, but still it is good if we visit through this.

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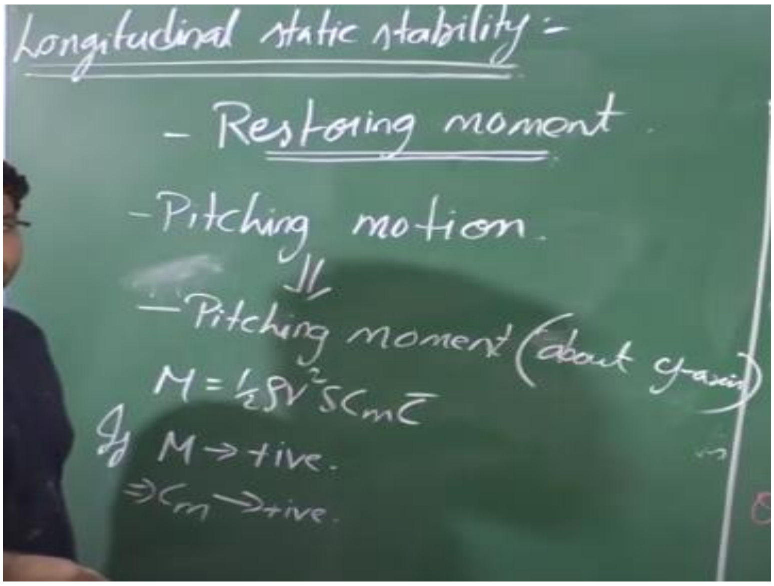

So what are we trying to start with is longitudinal static stability. So just before proceeding to this aircraft, let us now look at this setup carefully right. So what is that making the system to come back towards this equilibrium? Or what is helping this system to have stability? Can we say something about that? Who is helping here? Who is helping this pendulum to come back towards this initial equilibrium?

So let at equilibrium O the mg is acting perpendicular to the local horizontal right and the tension in the string is perfectly balanced by mg here, right. So there is no upward force and there is no rotation, there is no moment is it not? Now when we as soon as you displace it to a new location, mg again acts perpendicular to the local horizontal right and a component of this weight which is mg cos theta.

So let us say if we draw a tangent to this curve at this particular location, so tangent to this and this is theta, which is this is theta, why because this is theta right? So this local horizontal and this string right at O location O are perpendicular, is it not? And this tangent is perpendicular to this particular length at this point. So it makes the same angle and yeah, the same case here.

This mg cos theta is perpendicular to this or say we have this mg perpendicular to this local horizontal. So a component of weight mg cos theta, which is balanced by the tension in the string, right. And the component of this weight, which is mg sin theta is what helping this object to move in the first place, right? But this motion is about point O right is it not?

So with respect point O this is now turns out to be a rotational motion, is it not? So there is a rotation or so this force multiplied by this momentum is what is rotating this particular pendulum back to its equilibrium, right. So there is a moment that is helping this pendulum to come back towards this equilibrium. We call this moment as restoring moment. So for a system to be stable, we need to talk about its restoring moment.

So restoring moment is what helps is what is helping the system to come back towards to a towards its equilibrium position, right. We call it as restoring moment, right. So we need to understand who is providing this restoring moment, right. Now, get back to this aircraft, right. So when you are talking about longitudinal case, what is the possible moment in this longitudinal case?

It is a pitch of motion is it not? Is a rotation about or the moment about y axis right, body y axis which is the pitching motion. So we need to talk about pitching emotion here, which is given by our pitching motion produced due to pitching moment which is rotation about or moment about y axis, right about body y axis that we just discussed. So pitching moment and is given by half rho V square SC m.

So pitch up is considered positive, pitch down is considered negative here, right. That is what we just discussed, times C bar, fine. Where C m is the coefficient of pitching moment. Now when M is positive, if M is positive, when pitching, M is positive when pitching moment is positive, right. When the aircraft perform a pitch up motion, right. So this implies C m is also positive and vice versa. If M is negative C m is negative.

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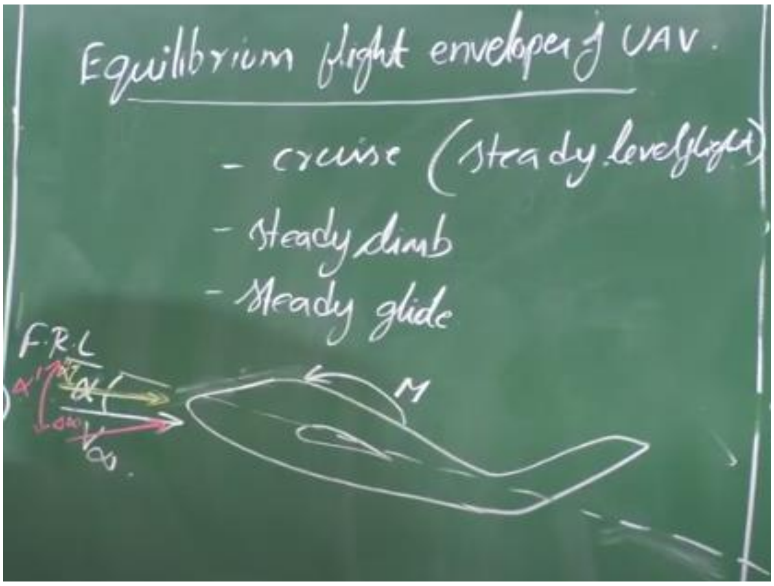

Now let us consider what are the possible equilibrium flight envelopes. So it is cruise. I can name I can immediately named cruise, right. It is a steady level flight. You can refer to our earlier lectures where we discussed about this. Steady level flight and then steady climb, right. And then steady glide yeah steady glide. So this flight envelopes correspond to equilibrium flight, right.

Now consider one such case where you have our aircraft. Let us say a steady level flight. This is our FRL, fuselage reference line. Again, this is a reference axis with which all the components are mounted and it is a imaginary axis again, right. And we have V infinity means, the flow is moving at a speed V infinity towards the aircraft which means the aircraft is moving at a speed V infinity into static air, right.

And this flow is making an angle. So this aircraft is maintaining an orientation alpha with respect to this V infinity called angle of attack alpha right. And now say if there is a small disturbance right, if there is a small gust that increase this angle of attack to a new alpha, alpha prime. So there is an incremental alpha here. So that means what? You have your initial equilibrium condition is now changed right is it not?

So this change in angle of attack will change the aerodynamic forces right, which in turn change the moment as well. So when we are talking about the longitudinal case here, we need to talk about the pitching moment right. So the change in the pitching moment should result in such a way that we should again go back to this alpha, right.

It is not the location or orientation, it is about we are talking about that state know equilibrium state when it is when the aircraft is maintaining certain alpha. So our aircraft has to return to that particular alpha once it encounters a disturbance right. So that is when we can say the aircraft is stable, at least longitudinally statically stable, longitudinally stable, right. Now how this aircraft can return to this alpha?

If this aircraft can rotate in such a way that the increased angle of attack is nullified, right. Say now we have the change in angle of attack is nullified, right. So we the aircraft now sees a new angle of attack alpha prime, if I rotate this aircraft in such a way say if I rotate this pitch down, right nose down moment, then I can reduce this particular alpha prime to alpha by nullifying this delta alpha, right.

So that means for there should be a restoring moment by means of this pitching moment right. Is it not? So for an increase in alpha there should be a decrease in or there should be a negative moment, right. So similarly, when there is a decrease in angle of attack, let us say. So due to some disturbance, there is a decrease in angle of attack. Now this particular angle of attack, so there is a decrease in angle of attack.

So this is alpha, alpha double prime right. So now my aircraft has to rotate in such a way that the decrease in angle of attack now should or say the change in angle of attack should return to the initial angle of attack, right. So from alpha prime if I have to maintain alpha where alpha prime is less than alpha, what I need to do? I need to rotate the aircraft up, right. I need to give a nose up motion.

That means for a negative change in alpha or decrease in alpha I need to increase the moment. For an increase in alpha I need to decrease the moment, right. That is when we can say aircraft is possess longitudinal static stability.

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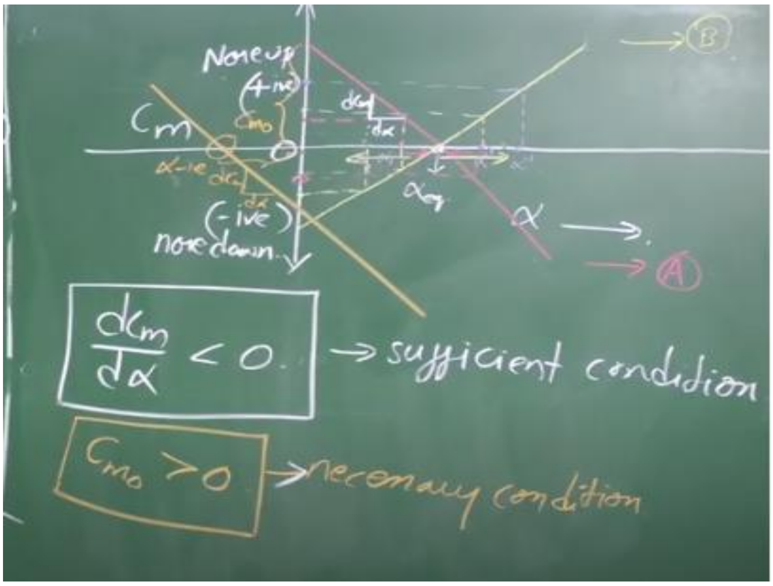

Now let us consider so let us say C m is the coefficient of pitching moment variation with angle of attack, right. So this is positive and this is negative. What is positive? We say this is nose up motion and this is nose down, right. Let us consider the following two curves. Let us say the aircraft is having this sort of variation of pitching moment coefficient, in fact, the pitching moment with angle of attack, right.

Let us say this is my aircraft A or UAV A, right. Now let us say there is another aircraft whose variation of pitching moment with angle of attack is given by this curve B, right. So this is for UAV 2 and this is for UAV 1. Now so what is this junction? Can you guess what is this? So what is this location, C m is? So this is positive negative then this must be zero right. So when C m is zero m is zero, moments are zero here.

So can we say this as a trim condition or in equilibrium condition? So what we have is an equilibrium here, let us say alpha eq, right. Now as we mentioned earlier, so the disturbance that can happen for a trim flight in the longitudinal case is in terms of angle of attack, right. So there in terms of like for a pendulum, it is in terms of angular deflection there, right. Is it not, angular displacement. So here in terms of angle of attack.

For a longitudinal case, the disturbance that can happen is in angle of attack here, right. So now, when there is such disturbance here in angle of attack, which has increased it right, increased that particular yeah let us say increase the initial trim angle of attack, right. So there is a disturbance now. Because of this disturbance, the aircraft now seeing alpha prime instead of alpha eq, equilibrium angle of attack, right.

So for an aircraft with this particular variation C m that this particular curvy B variation, what happens with the increasing angle of attack there is an increase in or positive pitching moment right, which means it will try to rotate the aircraft upwards right. Is it not? Am I correct or not? So because C m is positive it rotates the aircraft up. So what happens in that case?

So let us say this is my aircraft. This is my nose, right. So this is my initial condition. So this particular yeah paper cutter points out the initial condition. So when there is so otherwise you can say this is my free stream velocity, paper cutter is free stream velocity and my aircraft is oriented in this particular fashion and moving at a level flight condition, right in a level flight condition.

Now when there is increase in angle of attack because of the disturbance so for the curve if the if this particular aircraft possess the C m variation as that of curve B, what happens is this will produce a positive pitching moment. So what is positive pitching moment? Rotation like nose up, right is it not? That positive means nose up. So it will increase the it will try to rotate up which further increases the angle of attack here.

So this further increase in angle of attack creates furthermore pitching moment and you will try to move from this particular equilibrium point that similar to the one that we discussed, right. So on convex up condition, right. So that means, this increase in angle of attack, increase in angle of attack for aircraft with curve B will increase will create a positive pitching moment.

So that positive pitching moment further increases the angle of attack, right. Or sorry, yeah further increases the angle of attack. Let us say this is alpha double prime, and this increase in angle of attack will further increase the pitch up moment, right. So the you are trying to move away from this particular so the aircraft with this yellow graph right yellow variation will try to move away from this equilibrium position, right.

So you are trying to move away from this equilibrium position, right. That means, it is the aircraft is said to be unsteady, statically unstable here. If the aircraft process this particular variation, we say the aircraft is statically unstable. Does not process longitudinal static stability, right.

And on the other hand, if the variation otherwise if the disturbance reduces the angle of attack, say subscript 1 right alpha subscript 1. So if it if the variation reduces the angle of attack for example, if this is my free stream velocity right, this paper cutter represents free stream velocity and my orientation with respect to this represents angle of attack of the duster with respect to this free stream velocity is angle of attack.

Now this has reduced the angle of attack. So the disturbance now reduced the angle of attack here. What happens is so this yellow curve will produce due to that it will produce a negative pitching moment right, is it not? This is the negative axis here right. So this so it will try to pitch down the aircraft. So say there is this is the initial angle of attack. Now there is a decrease in angle of attack.

So because of the external disturbance and now the aircraft will try to further rotate it down which means, it will further decrease the angle of attack. So you will try to move away from this equilibrium location. So you will try to reach another angle of attack or reduce the angle of attack right. So angle of attack is decreasing in this direction, increasing in the opposite direction, right.

This is increasing axis of alpha, right. So this pitch down moment will further reduces the angle of attack, which will also produce a higher pitch down moment, is it not? Higher nose down moment that will take you away from this particular equilibrium location. So this alpha is moving away from this alpha eq. So for the aircraft with this yellow curve you are trying to move away from this equilibrium location right.

So on the other hand, when you talk about this aircraft with this red graph, right. If the variation of pitching moment follows the curve A, the UAV or the aircraft which follows this curve A, so when there is a change in angle of attack, which is an increasing angle of attack because of the external disturbance. So this will produce a negative moment, right.

So when there is an increase in angle of attack, so the aircraft with curve A will try to produce a negative moment right, negative pitching moment which is pitch down moment. So we have initially this particular orientation and due to external dissonance, there is an increase in angle of attack. So this produces a pitch down moment that decreases that right.

So the increase in angle of attack with this from this curve will produce a negative moment. So that means, the aircraft will try to pitch down right. So this pitch down will decrease this angle of attack. So it will decrease the angle of attack which helps you to move towards the equilibrium.

On the other hand, when you when the external disturbance decreases the angle of attack, so the aircraft with curve A will produce a positive pitching moment that will help the aircraft to rotate nose up right, which increases the change in angle of attack. Is it not? So it will try to increase the angle of attack. What is the increasing direction of angle of attack? Towards my right.

So from here you are trying to move towards this point alpha eq. So you are trying to move towards equilibrium location right. So one condition that we can observe is the slope here if you look at the slope the change in pitching moment due to change in alpha right. So this corresponds to alpha 1, alpha 2, the change in alpha. So the change in pitching moment or this change in angle of attack should be negative, right or less than zero here.

You can see so at this alpha 1 right alpha double prime alpha double prime you have a higher pitching moment here right and alpha prime which is higher than alpha double prime you have lower pitching moment. So the change is negative. There is a negative slope here. You can see there is a negative slope. So the C m versus alpha the C m variation with respect to angle of attack should be negative.

So this is one important like condition that we need to maintain for a static longitudinal static stable flight, right. And is that all? Is that sufficient enough? So let us consider another plot which is just parallel to this red line, right. Say the slope is negative. The slope is negative here, okay. So can so yeah we have satisfied this condition dC m upon the d alpha is negative, right.

But which curve do you prefer for the aircraft to process? Yeah A, okay. Why we need A? Say if you look at this right, so the alpha trim here, what is the trim condition for this particular aircraft when is C m is zero at this particular location, is it not? So where at this particular location you have negative angle of attack. Alpha is negative, right. So you will not be able to trim the aircraft at negative angle of attack, is it not?

You will not be able to fly the aircraft at negative angle of attack. So you should not trim it at negative angle of attack. So the necessary condition is that you should maintain this particular coefficient which is C m at alpha zero should be positive. So this C m at alpha zero is the y intercept here we call it as C m naught. C m naught has to be positive has to be greater than zero and C m alpha has to be less than zero right.

So these are the two conditions. So this is the necessary condition and we have this as sufficient condition, okay. So our aim is to look at what is the contribution of these various components of the aircraft. Say we looked at various major components, right like wing, propulsion system and tail and vertical tail. So how they are contributing towards this C m alpha C m naught of the aircraft, right.

And we should never forget that these moments we are talking is about C g, center of gravity of the system, right? It is about the C g of our aircraft, right. So with respect to that how these components are contributing towards this stability, right. So what happens when I add a tail? What happens when I add a canard? What happens when I have double biplane wings?

So if I have to design a UAV, say if I have to design a biplane UAV, I need to understand whether the addition of these two wings is it contributing towards stability or destabilizing the system, right. That is the main aim here. So we will try to look into those in the coming lectures. So this is the necessary condition. I think in our previous discussion this was swapped right.

So try to correct this. Now this is the necessary condition to have C m naught positive and C m alpha, sufficient condition is C m alpha should be negative. So for those aircraft with C m alpha greater than zero, and C m naught greater than zero, so that can we can still fly those aircraft, right but with a onboard controller. But even if C m alpha is less than zero, if you have C m naught less than zero, you may not be able to fly the aircraft.

You have to trim at negative angles of attack. At negative angles of attack, you will try to reach towards C L 0. You do not have lift, right. So yeah, see you soon.