UAV Design-Part II
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Lecture-19
Examples on Performance Analysis of UAV

Dear friends, welcome back. So, in our previous lecture we discussed about the conditions for trim, where what should be the delta e deflection in order to achieve a particular angle of attack? right, so for which we have solved the 2 equations.

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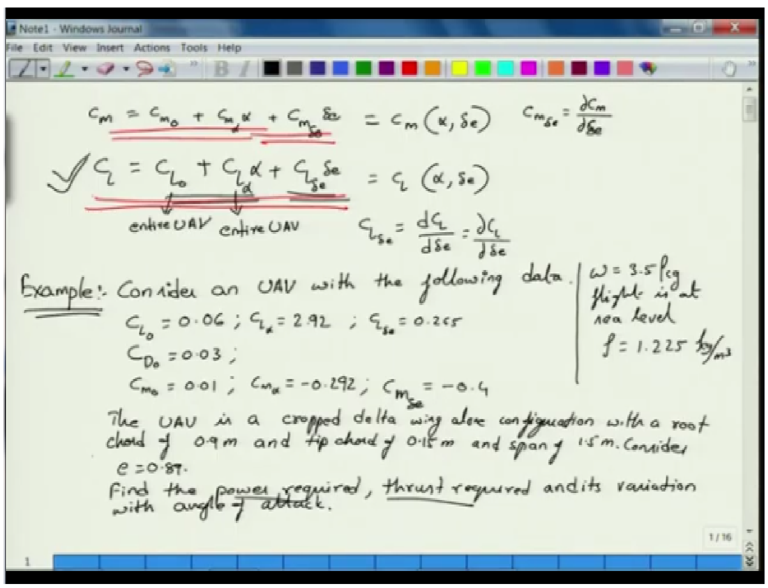

The pitching moment equation which is C m = C m 0 + C m alpha into alpha + elevator control power C m delta e times delta e. And then in general also the total lift coefficient of this aircraft or the UAV must be equals to the 0 lift coefficient this is the aerodynamic model that we are considering CL alpha times alpha + CL delta e times delta e. So, we have modeled the pitching moment as a function of angle of attack and the control surface deflection.

Similarly the total lift coefficient of the UAV as a function of angle of attack and elevator control surface deflection. So, this we need to be careful here this CL alpha is for the entire UAV just to remind you and CL 0 is also for the entire UAV, where CL delta e as we know is equals to dCL upon d delta e. So, the change in the lift coefficient of the aircraft due to elevated deflection that, ok.

So, you can closely write this as dou CL upon dou delta e, right, ok, similarly the C m delta e here which we have discussed in our previous lecture is the elevator control power which is dC m upon d delta e ok. Now what are we going to do in this lecture? We will consider a UAV right, with it is all the coefficients and the parameters given. And then we will see if you want to trim it at a particular angle of attack, what will be the corresponding delta e trim?

And what will be the resulting flight velocity? And then the thrust required as well as the power required to achieve such flight condition, right. So, that means given a UAV we are now trying to analyze the performance of the UAV for the level flight, where we will see what should be the power required to fly at different velocities which results in definitely a particular angle of attack for trim as well as delta e for trim, right.

So, for that let us consider an example, consider an UAV with the following data where you will just try to note down the parameters, where CL 0 of the UAV = 0.06 in C D 0 is 0.03 and C m 0 is 0.01 CL alpha of the entire UAV is 2.92 CL delta e is 0.265 and C m alpha is - 0.292 C m alpha. And then C m delta e of this UAV is - 0.4 yeah. So the value of k that can be figured out from the geometry, right.

So, the UAV is a cropped delta wing alone configuration with a root chord of 0.9 meters and tip chord of 0.15 meters in the span of 1.5 meters, ok. So, with this we will be able to figure out with an Oswald's efficiency of and consider e = 0.89, ok. So, with this we will be able to figure out what is the induced drag correction factor? So, first of all let us complete this question where we need to find out?

Then find the power required, thrust required and it is variation with angle of attack, ok. So, that is a question we have the data of a UAV which is a crop delta reflex wing configuration and we were given about the geometry where it is yeah we have solved this particular example many times in terms of the geometry. We figured out what is the planform area based upon, root chord tip chord yeah from which you can calculate taper ratio.

Using taper ratio and the span and the root chord we will be able to find out the planform reference area. And with the help of yeah this data we will be able to figure out what is the aspect ratio? And we were given the information about e which is 0.89 Oswald's efficient factor is 0.89. So, we can use that to figure out what is the induced drag correction factor k, right. So, why because why do we require K is because we are talking power required and thrust required.

So, we know power required and thrust required, power required for a propeller engine let us say is varies with a factors CL power 3 by 2 by C D right and for thrust required it varies with the factor CL upon C D, right. So, now we need to find out what is the if I trim the aircraft at a given angle of attack, what should be the corresponding delta e? What will be the resulting velocity? And then what should be the CL for that particular alpha? And as well as CD.

From there you can figure out what is the corresponding thrust required and power required? So, let us write down the steps that are required to solve this example.

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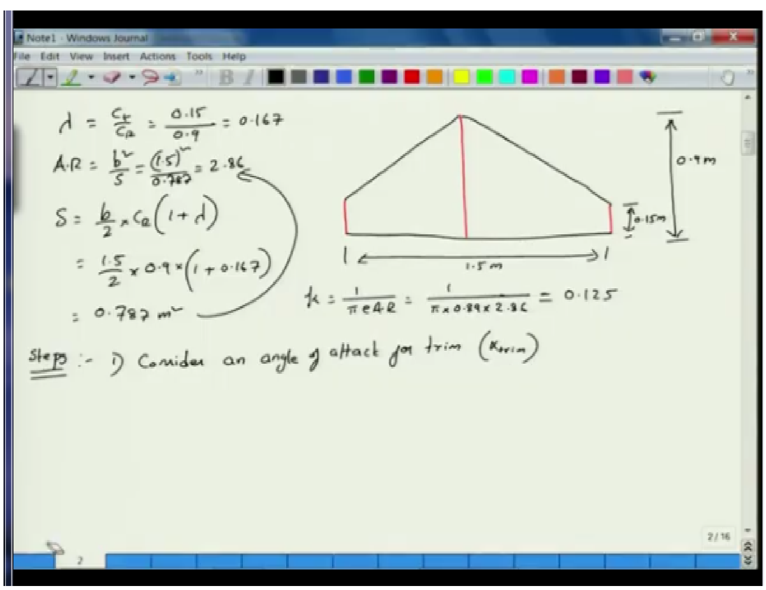

So, steps are, so before getting into the steps let us figure out what are these non dimension parameters? right. Some of this geometric non dimension parameters, so first we have root chord, right, and then we have tip chord information of span is also given, right. So, this is the UAV that we are talking about. So, where the root chord is about 0.9 meters and the tip chord is about 0.15 meters, right, and then the span of this UAV is about 1.5 meters, right.

So, let us quickly figure out what is the taper ratio lambda is C t upon C R. So, I am intentionally repeating the steps though we have calculated lambda many times in our previous example. But I still want you to get used to this particular parameter, which is about 0.167 and then the aspect ratio is b square upon S. So, we need to know what is S here? So, S we can figure it out by b by 2 times C R times 1 + lambda where b by 2 is 1.5 meters upon 2 times C R is 0.9 meters multiplied by 1 + 0.167, so it turns out to be 0.787 meter square, ok.

So, with the help of this yeah planform area we will be able to figure out the aspect ratio which is 1.5 meter square upon 0.787 which is 2.86, right. So, 2.86 is the aspect ratio, so with the help of this data we will be able to find out what is the induced drag correction factor? Which is 1 upon by pi e AR, so 1 upon pi, e is 0.89 which was given and then what is the aspect ratio is 2.86. So, the value turns out to be 0.125, we have now k right.

So, now we know what all we require in order to find out what is the power required and thrust required here? So, let us follow the steps here, so the first step I would like to consider an alpha for trim, right. So, you consider alpha trim using our these 2 equations, so this one right. So, using these 2 equations, we will be able to find out what is the corresponding delta e trim? Right, how can we do that?

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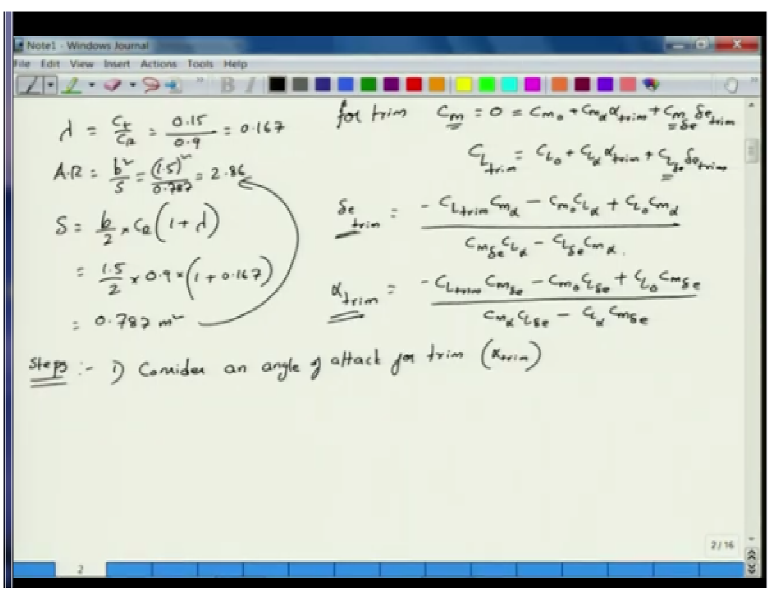

So, for trim we know that C m has to be 0. So, this we discussed in a previous lecture because it is a equilibrium condition about which the resultant moment and the forces has to balance and the resultant moment is should be 0 there, right. So, C m has to be 0 where which is equals to C m 0 + C m alpha times alpha trim. So, when you trim the resultant pitching moment C m is 0 and then the corresponding delta e is a delta e trim for that particular flight condition.

So, and then the second equation CL turns out to be CL trim which is equals to CL 0 + CL alpha times alpha trim + CL delta e times delta e trim. So, if you solve all these 2 equations, what we get is? So, delta e trim = - CL trim times C m alpha - C m 0 times CL alpha yeah this should be C m 0 times CL alpha and then CL 0 times C m alpha this should be plus and you take it on the other side this becomes CL 0 time C m alpha upon what you have is? C m delta e times CL alpha - CL delta e times C m alpha, right.

So, this is the delta e trim if you solve these 2 equations and then similarly you can solve for alpha trim, simply consider the top equation as equation 1 and the bottom one as equation 2. So, if you want to solve for delta e trim and then what you can do is equation 1 multiplied by CL alpha and equation 2 multiplied by C m alpha subtract both the equations. So, when you want to solve for alpha trim, then multiply equation 1 with CL delta e and equation 2 with C m delta e and subtract them, right.

So, this is simple, 2 equations 2 variable problem for similarly what we can get is if I have to solve for CL alpha trim then what I have is CL alpha trim time C m delta e - C m 0 CL delta e ok. So + CL 0 times C m delta e upon, now more or less the same denominator, what we have is? C m alpha times CL delta e - CL alpha times C m delta e ok. So, once you know all these parameters you can get delta e trim directly and alpha trim which can be used to solve a numerical problem.

But here we are trying to use any iterative approach, see we need to find out power required and thrust required for each and every angle of attack, right. So, for that first of all let us see before getting into the steps, let us see what is the power required and thrust required for a UAV in the level flight condition?

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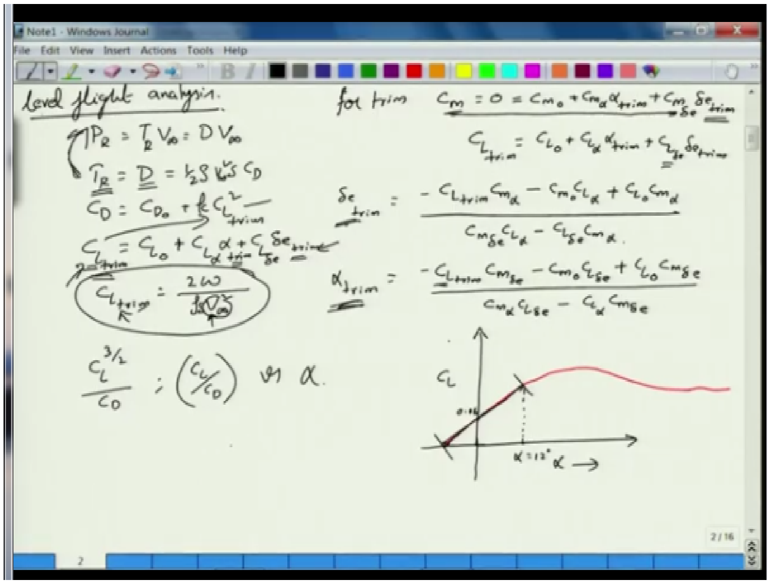

So, we are talking about level flight analysis ok, so power required is drag of thrust required times velocity. So, thrust required is equals to drag generated by the system times velocity of flight, right. And then similarly thrust required let us say when you are talking about and thrust required is equals to drag of the aircraft which is half rho V square S times C D right and C D = C D 0 + k CL square using drag polar, right.

And then, so value of k we figured it out the CL value here is + CL 0 + CL alpha times alpha + CL delta e times delta e of the total aircraft ok. So, now we have to approach this now what is the CL? Corresponding we are talking about alpha trim and delta e trim. So, when you trim the aircraft at a particular alpha what you get is? delta e trim, so with that you are trimming this aircraft you need to generate a trim lift coefficient at that particular flight velocity, right.

So, how to figure it out CL trim? So, we know to 2w upon **v** rho S V infinity square, right. So, you need to know what is the corresponding flight velocity? If you know what is the corresponding flight velocity you will be able to find out CL trim. Once you know CL trim and using these 2 equations you can find out what is delta e trim as well as alpha trim by substituting CL trim in this equation with the given data. Once you have alpha trim and delta e trim, you know what is CL trim?

So, when you substitute CL trim here, so what you get is C D for that particular trim condition. So, the C D for that particular trim condition can be figured out like can be used to figure out what is the drag acting on the system which we need to satisfy in order to move at this particular velocity V infinity, right. So, our engine has to say if you are using a jet engine it has to produce so much thrust to move forward.

Otherwise when you talk about a propeller aircraft, if you wants to install a electric brushless motor with the propeller combination to propel this or to make this aircraft move forward. Then you need to satisfy this power requirement condition for the particular aircraft, right, which is thrust required times velocity. So, this is what we are going to solve, we will consider instead of approaching in this particular manner by assuming some velocity what we try to do is?

We try to iterate for alpha trim, substituting alpha trim in this equation what you get is delta e trim, right. Once you have delta e trim, what you can do is? You will be able to find out what is the corresponding yeah, CL trim. And then so once you have alpha trim using this particular equation C m = C m 0 + C m alpha + C m delta e times delta e. You can find out what is delta e trim? Use that delta e trim in this equation to find out what is CL trim?

So, use that CL trim right, so using that CL trim you can find out what is the velocity of flight? Do you understand? So, what we are not approaching it the way we solved a numerical problem rather we are trying to find out what will be the resulting velocity if I have to trim at this particular angle of attack, ok. So, and then with this CL trim I will be able to find out the drag coefficient and thereby drag which is thrust required by the system and the power required.

So, in this process we will also try to figure out what should be CL power 3 by 2 upon C D as well as what should be CL upon C D, ok and it is variation with yeah angle of attack. So, since we are varying alpha here, alpha is considered as an input variable. So, we will try to find out what is the variation with alpha, ok. So, we need to find out CL power 3 by 2 upon C D and CL by C D variation with alpha, right.

So, in order to complete this question, let us consider the following data, so which you must be having by this time? So, let us consider the weight of the UAV as 3.5 kg, right, and that the flight is at sea level ok. So, which means you are forcing the density should be 1.225 kg upon meter cube. So, I think you are now by this time you are comfortable with the MATLAB environment. So, we will try since it is an iterative approach, we need some program to solve this, let us build a small sub routine to help us in solving this example problem, ok.

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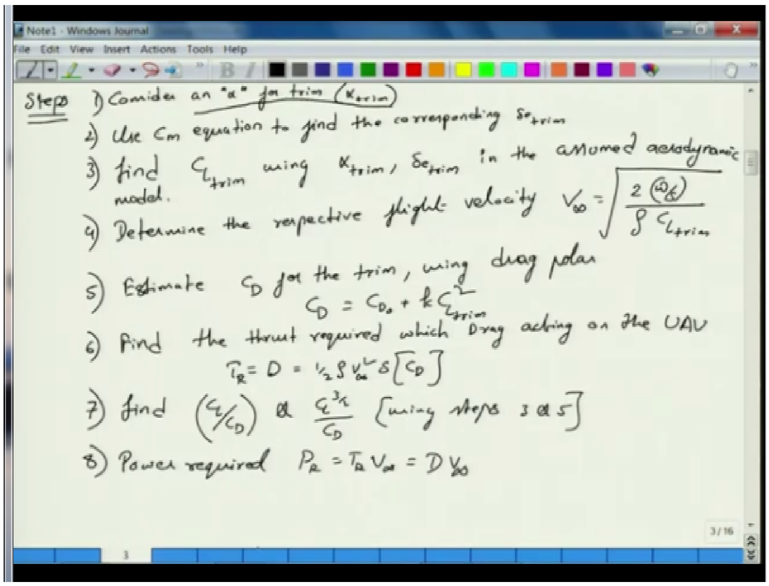

So, I just opened the editor page where I can start typing my code. So, just before this I will try to write down the steps which we just discussed. So let us consider phase 3, so step 1, steps otherwise. So, the first step what we will do is? Consider an alpha for trim which means alpha trim, right. So, use moment equation like C m trim, right, C m equation to find corresponding delta e trim, ok.

So, the third step is find CL trim using alpha trim, delta e trim in the assumed aerodynamic model TK. And then the fourth step should be, so determine respective flight velocity, so this should corresponds to V infinity sorry CL trim, right, is not it. So, the corresponding flight velocity is twice the wing loading upon the density at which the flight density or air at the particular altitude times the CL trim that we have figured out from step 3, ok.

Then once you have flight velocity then you can and you know, what is CL trim? So, also estimate C D for that particular trim C D for the trim using drag polar. So, what we need to do? C D = CD 0 + k times CL trim square ok. Now once you know this find the thrust required which is drag acting on the UAV, right. So, which is T thrust required or should be dragged which is half rho V square.

So, V we know it from step 4 and reference area times C D 0 or C D directly right C D we got it from step 5. So, now find CL upon C D and CL power 3 by 2 upon C D. So, this using steps yeah 3 and 5, ok, and now you can find out what is the power required? So, the power required is thrust required times V infinity which is drag times V infinity, right. So, now we need to plot this, so this is what typically the procedure that we are going to adopt to solve this problem ok.

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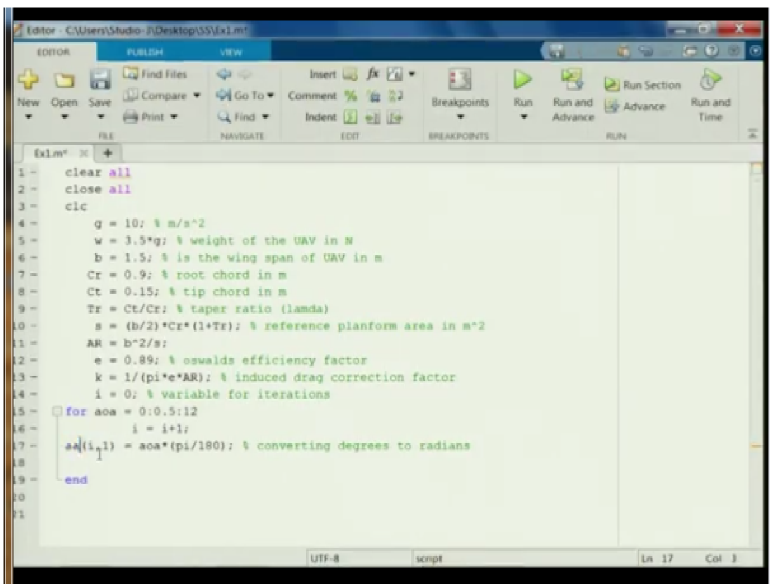

So, now I have quickly try to program this, clear all, close all clc. So, let us first write down the given data, so the weight of the aircraft is given as 3.5 kg, right. So, I am trying to multiply this by g ok, so this is our weight of the UAV, ok. So, I can do g I am trying to use it as 10 meter per second square, so which is in meter per second square. So, weight of the UAV in newtons, right.

So, and then the reference area we just figured it out, right, in our previous calculations which is 0.787 meter square and then the value of an aspect ratio is equals to b square upon s ok. So, b, where b is 1.5 meters wingspan of UAV in meters, so reference planform area in meter square, ok. So, instead of giving this directly, what we can do is? We can use the data which was given b by 2 Times C R times 1 + lambda, ok.

So, this b should be ahead of this because it requires, so to compute this equation at row 7 we need to have the information about b, ok. So, from the given data C r root chord is 0.9 meters, right, this is root chord which is 0.9 in meters and C t is the tip chord which is 0.15 in meters. So, what I find is lambda with this data which is taper ratio or say instead of lambda I will say T r right taper ratio which is C t upon C r which is taper ratio say bracket lambda ok instead of lambda what we will try to use is? T r.

And now, so I am in a position to find out the aspect ratio as well here. So, with and we know what is a Oswald's efficiency factor which was given as 0.89, right. So, this is Oswald's efficiency factor. And then e is given, so I can calculate k which is induced drag correction factor. So, 1 upon pi e AR, ok, so this is our Oswald's efficient sorry induced drag correction factor, ok. So, now we need to do iterations, right, so for example I need to start our step 1, what is our step 1?

So, this is consider an alpha for trim which is alpha trim, so I need to consider this ok. So, let us say I vary this alpha aoa let us say is a angle of attack ok, so, I would like to vary this from, ok. So, this is like starting with 0 angle of attack, right, 0 with an increment of 0.5 and up to let us assume this linear variation is up to 12 degrees, ok. So, this is what we are assuming here, what is the meaning of this step? so, the variation.

So, let us say this is my variation of lift coefficient CL with angle of attack alpha, right. So, what I am considering here is a linear variation. So, this is typically for this delta wing UAV I intentionally try to you know make this plot this is what V observed you know even up to 48 degrees angle of attack, this is how the CL variation for this configuration. So, what I would like to consider is?

This particular regime which is up to alpha = 12 degrees, right, is almost linear. So and also assume CL 0 of the configuration is given from the given data we have what is CL 0? CL 0 is

0.06 ok, so CL 0 what we have here is? 0.06, ok. So, in this particular regime we are using this linear approximation where CL = CL 0 + CL alpha into alpha right, CL = CL is CL 0 + CL alpha into alpha and then superimposed by CL delta times delta e, right.

So, this particular aoa from 0 to 12 degrees is linear, ok, so that we can use those equations. And the step here is 0.5 degrees angle of attack that I would like to consider, ok. So for these iterations to happen if I have to save the data I am using a variable i starting with 0, right, just variable for iterations. So, what I i do is? I will try to increment this i during the variations, so, i = i + 1. So, the current value of i during the first iteration is 1, right, where the angle of attack is starting from 0 degrees.

So, now I need to convert this aoa, so here we consider degrees right but in MATLAB we may not be able to use degrees as such. So, MATLAB do not understand degrees we need to converted into radians here, so what I try to do is? So, alpha right let it be complete is equals to aoa times pi by 180, ok, converting it leads to radians, ok. So, I would like to store this data as well, so whatever we are going to get, I would like to store this data.

So this can be like alpha of i, 1, so this will become a column vector, is not it. So I am trying to save this data, ok, so alpha is increasing inside the loop that is a reason why there is it is highlighted either like you can try to assign the size of this alpha outside this loop itself. So, once we have alpha then we need to find out using the step 2 we need to use C m equation to find out the corresponding delta e trim, right.

So, what is that C m equation to find out delta e trim? ok, so this C m equation, right, C m trim = 0, right, is not it. For C m trim = 0 which is C m 0 + C m alpha alpha trim + C m delta e delta e trim, ok.

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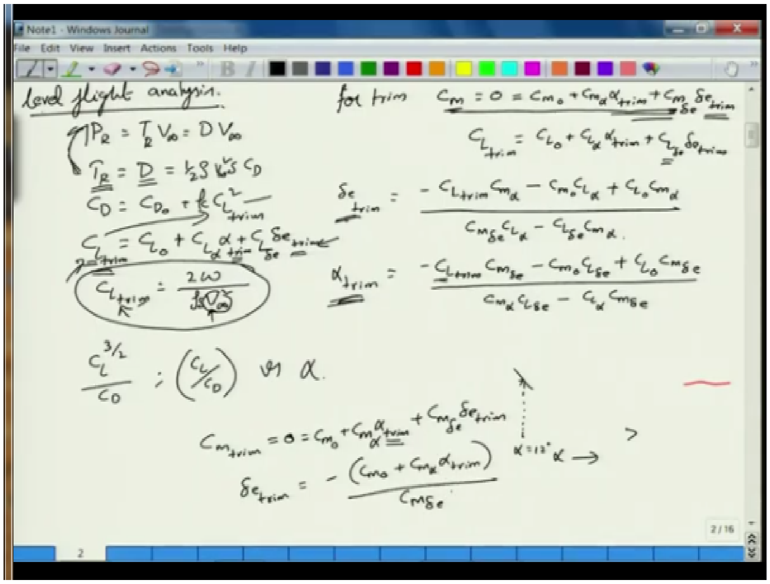

Now substituting this particular alpha trim what we have assumed earlier? right. So, substitute that in this particular equation to find out delta e trim which is equals to minus of C m 0 + C m alpha times alpha trim upon C m delta e, substitute this, ok. **(Video Starts: 34:56)** Instead of alpha let us say aoa trim, right, that means we are talking about aoa underscore trim, ok tr talks about converting trim angle of attack from degrees to radians, ok.

So, once you have this we know what is delta e trim? So, say del e underscore tr is equals to of i, 1 I am trying to store this value as well is equals to **see** minus of C m 0 - no + C m alpha + C m underscore aoa otherwise C m alpha times alpha trim which is aoa tr trim of i, 1. So, this is a variable here, right, upon C m underscore del e right, so what do we require here? We need C m 0, C m alpha and C m delta e, so let us give those inputs here.

So, let me consider them as, so these are the input geometric parameters, ok. So, these are input geometric parameters. So, we should also talk about input aerodynamic meters, right. So, what we have is CL 0 is 0.06 from the given data and CL underscore a talks about alpha ok otherwise, alpha CL underscore alpha which is 2.92 per radian and CL underscore delta e CL delta e is 0.265, ok CL delta.

Otherwise let us round it up to 2 digits here CL delta 26 and then CD 0 is equals to 0.03. So, this is again from the given data CD 0 is 0 C m 0 C m alpha and C m delta e. So, C m 0 is 0.01 for it is positive for this UAV, right. So, C m alpha is - 0.29 and then C m underscore del e - 0.41 C m delta e. So, k is already figured out from here which is 1 upon per year. So, we can if you want we can shift this to aerodynamic parameters, ok.

So and now we are proceeding ahead to figure out over requirement and the thrust requirement with for different trim angle of attacks, right, ok. So, CL del e underscore trim, right is equals to this and then now the third step here is find CL trim using alpha trim and delta e trim using the assumed aerodynamic model, ok. So, CL underscore tr CL trim again let us store this CL trim of i, 1 CL trim is equals to CL 0 + CL alpha underscore al alpha times alpha trim aoa underscore tr of i, 1, ok C m alpha sorry CL times alpha trim + CL underscore del e.

So, we have this C m underscore del e, so let us make this one also capital del E, so C m del E, ok. So, CL del E times delta e trim, del e let me write this one also del E capital underscore tr of i, 1, ok. So, and the third step right now is determine the respective flight velocity, right, so we need to find out what is V infinity. So, flight velocity V for this trim condition is equals to square root of twice the wing loading 2 times.

So, let us find out the wing loading error w for say, that is a good practice let us w underscore s talks about wing loading which is equals to w which is in newtons here, you need to observe that. Even mass is let us say m is a mass of the aircraft which is given as 3.5 kg, so it is given mass of the aircraft, ok in kg. so, we converted it to weight in Newtons and then now we now changed the wing loading in Newton upon meter square, ok.

So, the same wing loading I am using here underscore s divided by density, so density is equals to 0 sorry it is a tree level, so it is 1.225 kg upon meter square. So, density of air at sea level in kg upon meter cube ok. So, we now got the density times CL trim, right, CL underscore tr of i, 1, ok. So, this is in the denominator and this entire thing is inside this square root, you can see once I select the end bracket.

So, the bracket which is immediately squrt is the starting of that bracket, ok. That means this entire expression is within this square root, so after finding velocity we need to find C D that is the step here estimate C D for the trim using drag polar. So, what I can do is C D of i, 1 is equals to C D 0 + k times k is the induced drag correction factor which we have figured out earlier, right of the given data times CL trim square.

So, this drag is corresponds to the trim flight curve i, 1 square, ok. So, once you note drag you can find out the thrust required T let us say thrust required T r which is half density rho times V infinity square. so, V of i, 1 square of rho V square times the reference area S half rho V square S times C D of i, 1. So, this will this equation will try to figure out what is the thrust required of the drag acting on the aircraft based upon the flight velocity at that yeah at V of that particular.

i, 1 corresponds to this particular iteration at this particular angle of attack, right. So, initially at 0 alpha what is the corresponding V i and the C D i, so it should corresponds to this particular angle of attack. So, we are trying to use this indexing C D of i, 1, ok and V of i, 1, so that we will get the corresponding thrust required for that particular angle of attack. and then the power required of i, 1 is equals to thrust required i, 1 times velocity of flight, V of i, 1.

So, in the process we can also figure out what is CL to C D CL underscore C D of how L by D is varying, right. So, CL underscore C D of i, 1 which is equals to so CL underscore trim of i, 1 upon this is t r ok, 1 upon C D of i, 1 when CL 3 underscore 2 let us say CL 3 by 2 underscore C D, ok. Let us say CL 32 talks about CL power 3 by 2 let us assume that let us consider this as the nomenclature.

So, i, 1 for CL power 3 by 2 by C D CL rise to the power of 3 by 2 upon C D, right. So, **so** this entire variable is varying, right, CL power 3 by 2 by C D with alpha which is equals to again. So, the CL which also CL rise to the power of 3 by 2 upon C D, ok. So, that is it, this is the end of the code, we will see whether this code runs or not, so in order to see this output we need to plot is not it.

So, let us start a figure here, ok, so figure 1 that talks about now let us have 6 subplots here, ok, for all the variables, subplot of what all we required alpha delta e C D thrust required CL and power required about 6 subplots right. Let me make it 8, for those other variables as well, ok, plot. So, what I am trying to plot is on the x axis I would like plot velocity because that is how we are use, right.

The variation of velocity and the corresponding power required and thrust required and let us adopt to the same convention. So, I will try to plot velocity on the x axis which is V it will in the corresponding first plot should be. So, the first plot can be from yeah aoa angle of attack for trim aoa underscore tr, ok, colon k star, ok. So, this is for the line that I would like to plot you know we can.

So, y label is alpha underscore trim y label, ok, so to first plot we have angle of attack variation with velocity. And the second one will be about delta e trim, right, velocity in del E underscore T r right. So, which is delta underscore E underscore trim, ok. And then the third subplot is about variation of CL underscore trim with alpha. So, this I can write CL C underscore L underscore trim, ok. So, I will not write the complete T, T stands for trim here.

And the fourth plot should be about C D variation with alpha which is C D, right, so this is C underscore D, ok. And the fifth plot should be thrust required T r right T r variation with on y axis we need to talk about T r and it is variation with alpha, ok. Let us go fifth plot, so the corresponding y label for this is T underscore r thrust required in Newtons again, ok. So, the sixth plot is about CL upon C D, ok, so this is C underscore L divided by C underscore D CL upon C D.

And the seventh subplot is the variation of power required Pr with velocity and the final plot is about variation of CL underscore 32 which is 3 by 2 upon C D with alpha. So, what C underscore L rise to the power of 3 upon 2, ok, divided by C D, ok. So, in the x label is velocity or say V underscore in meters per second. So, let us just run this code and see, so you can see CL power 3 by 2 C D variation with ok. So, there are small like errors in the symbols I will just try to correct them. Here , ok, so can you see the variation here which so the alpha trim which V varied from 0 degrees to say yeah which V varied from 0 degrees to .

So, it is radiance let me just plot it in degrees here, so delta E and alpha are in radians, so it is better I convert them to write degrees. So, this is multiplied by 180 upon pi I am just converting from radians. See, so alpha is varied from 0 to 10 degrees, right at 0 degrees angle of attack almost see y is 0 here. So, you need to fly this UAV at say about close to 33 meters per second, so, ok.

So, you have to fly this UAV at 33 meters per second because the total lift is depend upon CL 0 there and alpha S 0. So, in the corresponding trim delta e is almost close to yeah, ok, so it is close to -1 degrees here, so trim delta e is. And then CL trim, so at 0 yeah it is almost 0.06 which is nothing but CL 0 close CL 0 + CL delta e times delta e here , right. So, this plot typically talks about the performance analysis here for the level flight.

So, when I have to trim an aircraft at different angles of attack what will be the corresponding variation or thrust requirement. So, when I have to fly at o angle of attacks, so I need to produce more thrust here which is 15 in order to produce the same lift, I need to fly at higher velocities when I am trimming it at 12 degrees angle of attack I can fly at low velocities maybe 12 meters per second here, close to.

So, x is 10.6 or 11 meters per second here and the corresponding power required here is say 47 watts. And then the thrust required is about just 4 newtons you know, so close to what? 0.4 kg, 400 grams of thrust is required, fine. I wish you should repeat this exercise, you should practice this, so that you get comfortable as we progressed to the other sub routines that we are going to develop, thank you. **(Video Ends: 57:33)**