UAV Design-Part II
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Lecture-15
Example Problems of Wing and Tail Combination-continued

So, welcome back let us solve the second example with the same concepts what we discussed earlier.

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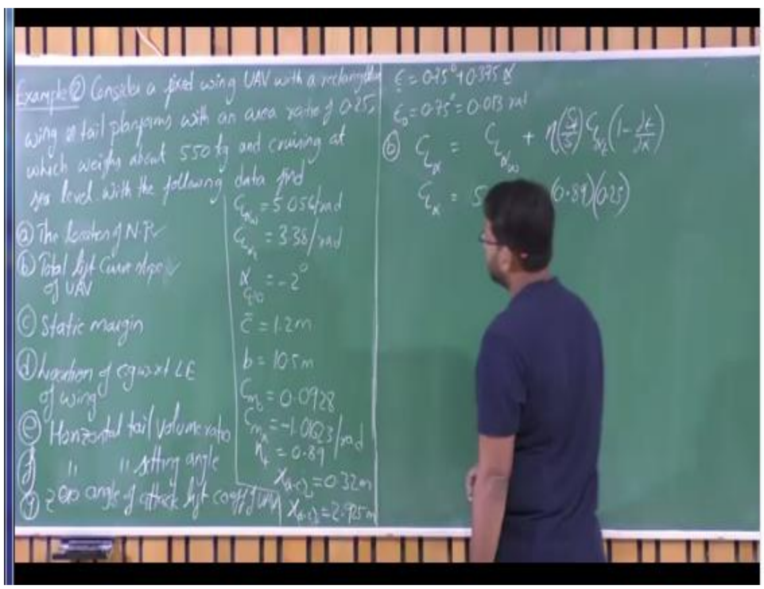

So, example 2, so consider a fixed wing UAV with a rectangular wing and tail plan form area rectangular wing and tail planforms with an area ratio of 0.25 which weighs about 550 kg ok and cruising at sea level. So, with the following data find a, so find the neutral point or the location of neutral point I say NP, right, neutral point b. So, total lift curve slope of UAV starting margin, location of Cg with respect to leading edge of wing, horizontal tail volume ratio and tail setting angle, so horizontal tail volume ratio.

So, with the stability analysis we are trying to fix the geometric parameters what should be the wing planform area, what should be their location, what should be the tail volume ratios, so that is the main aim here. Horizontal tail setting as well as horizontal tail setting angle zero left angle or zero angle of attack lift coefficient of UAV right. So, we are asked to find out zero angle of attack lift coefficient of UAV, so this is what we need to find out with the given data.

So, let us now look at what is the given data here, so C L alpha wing is given us 5.056 per radian So, C L alpha tail is given as 3.38 per radian alpha it which C L 0 is - 2 degrees, C bar is 1.2 meters and the span is 10.5 meters C m 0 of the UAV is 0.0928, right. So, in our previous example, we figured out what is C m 0. But right now, we are trying to find all these parameters assuming this particular C m 0 is given ok, so that is a difference.

So, C m alpha is - 1.016234 per radian 0.016 that is good enough, tail efficiency factor eta of t is 0.89 which is 90% approximately. And then X ac of the wing is 0.32 meters behind the leading edge and X ac of tail is 2.93 meters approximately behind the leading edge of the way.

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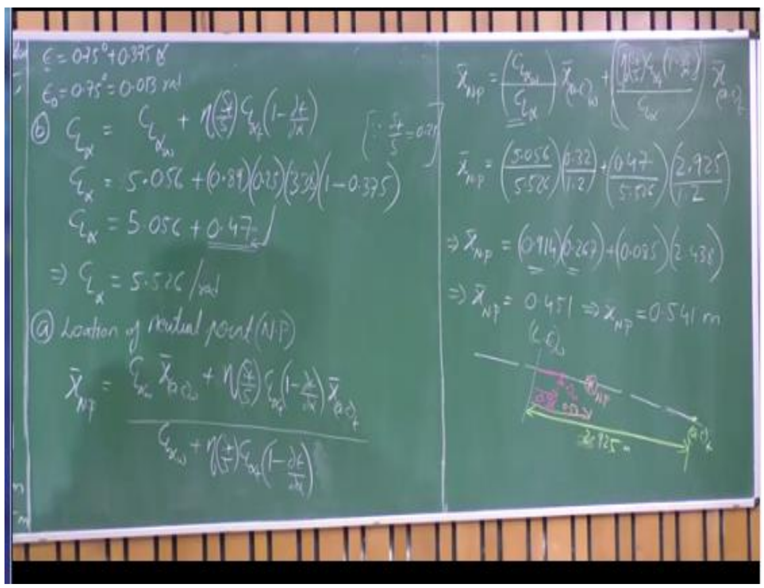

And we are also given the data about epsilon downwash is equal to 0.75 degree + 0.375 times alpha, ok. So, that means first thing that I would like to do is epsilon 0 is 0.75 degrees I need to convert into radians which is approximately 0.013 radian, right. So, dou epsilon by dou epsilon alpha it is same whether because degree upon degree or radians upon radian right. So, it will have an increment in a similar way over when you consider epsilon as a degree or epsilon as, yeah.

So, same you need to have similar units here and there. So, this is a data first of all location of neutral point. So, before finding out the location of neutral point, I would like to address this second question. So, question b, I am trying to address it first total lift curve slope right, total lift curve slope C L alpha if the aircraft is, as we know it is from the contribution wing contribution and with the correction factor from the tail yeah as well as from the tail contribution with the correction factor here S t upon S times C L alpha of tail 1 - dou epsilon upon dou alpha straightforward, right.

Just substitute the values here the C L alpha of the total aircraft is what is C L alpha of wing which is given as 5.056 you can say 5.06 or 56 + eta is 0.89 89% and S t upon S is given by 0.25, right. So, the wing area ratio is 0.25 S t upon S from the given data, what we have is, since S t upon S is 0.25 and we were told that these are rectangular wings right, is not it, multiplied by C L alpha of tail is 3.38 1 - epsilon by dou alpha is so from here you know the epsilon by dou alpha is 0.375.

So, C L alpha contribution from wing is 0.056 plus tail with the correction factor we have 0.47, right. So, this implies C L alpha of the entire aircraft is 5.526 per radian, ok, simple straightforward question, no complexity. Now let us address this first question, what should be the location of neutral point, now you can appreciate this, right. So, how less the; contribution from the tail towards C L alpha? That is because that majorly affected by this S t upon S.

If this is 1 then it is almost close to 0.89 is like 90% of this and without downwash it is almost same, right is not it. So, because of this S t upon S which in fact you decide based upon your the required tail volume ratio. So, due to which your contribution of C L alpha tail towards the C L alpha aircraft is very small here, right and the location of neutral point here the, so the location of neutral point, right, ok, so location of neutral point NP, ok.

So, how can I find out I do not have the data about Cg here right or say if I have static margin then I will be able to find it out directly by adding static margin with the cg. But I neither have Cg data or the static margin data. All I have is a location of wing, location of aerodynamic center of the tail and then so I have the corresponding lift curve slopes of wing and tail, right, am I correct and I know what is the area ratio of wing and tail which is 0.25.

So, with this data I will be able to figure out not 0.2 right, weighted average. I use that weighted average formula which is X bar NP is equal to C L alpha wing times X bar ac of wing + C L alpha of tail ok with correction factor eta of tail S t upon S times C L alpha of tail times 1 - dou epsilon by dou alpha is a correction factor due to downwash multiplied by the location of aerodynamics center of tail with respect to the leading edge of the wing upon C L alpha of wing + eta S t upon S times C L alpha of tail times 1 - dou epsilon by dou alpha, yeah, ok.

So, can I say this is my total C L alpha of the UAV? So this particular denominator is nothing but the total C L alpha of my UAV, right, am I correct or not. So, I can directly substitute the value that I obtained here in the denominator, ok. So, what I will do is X bar NP is equals to C L alpha of wing upon total C L alpha of the UAV multiplied by X bar it is corresponding location. So, this is a weight to the location, right to the location of aerodynamic center of wing + eta S t upon S C L alpha of tail 1 - dou epsilon by dou alpha.

So, this is divided by total C L alpha of the aircraft, right. So, again this is the weightage to X bar ac of tail which is it is location, ok. So, X bar NP this equals to 5.056 C L alpha 5.056 divide by C L alpha is 5.526 multiplied by X ac, location of aerodynamics center is 0.32, right. So, 0.32 upon C bar is given as 1.2 meters, right. So, this is 1.2 meters plus, so this says approximately 0.47 right, is not it.

So, 0.47 upon C L alpha is 5.526 multiplied by X bar ac of tail which is again given by 2.925 upon 1.2 meters ok. So, from here I know what is X bar NP neutral point which is equals to 0.914 times 0.267 + 0.085 times 2.438 ok. So, what do you mean by that, so this particular ratio is the weight to the location right is a ratio of lift curve slopes you know of the wing to the entire aircraft, am I correct or not.

And lift curve slope of the tail to the entire aircraft. So, it is getting like the wing is having the major weight here you know that is why X NP tries to have major weight almost 1 towards it is location wing aerodynamic center location. That means this particular term will try to pull the aerodynamic center towards it because it has a more weight you know, yeah.

So, see here you have only here it is 91% you know it is almost 1 here right 100% it will try to pull the wing try to pull the neutral point towards it because of this. But here you see it is only about 9% which is 8.5% but the length of the tail is what matters here right now, yeah. The location of the aerodynamic center of the tail is what making this to pull towards that, pull towards the tail.

So, the neutral point is now because of the weight to the location of the wing Cg and the weight of lift curve slope towards the location of wing aerodynamic center here, ok. So, what it turns out to be somewhere in between these 2 locations right X bar NP is 0.451, can you see this you know X bar NP is 0.451, if you multiply by C bar what is the value corresponding location is X NP is equals to 0.451 times 1.2, how much? 0.541 meters from the leading edge of the wing.

You see this is aerodynamic center of the wing is 0.3 meters, so shall we draw this, that will makes sense right is not it say. If this is my fuselage reference line, ok, let us say this is my reference point or leading edge of the wing ok, so the wing is located at a distance. So, wing ac aerodynamic center of the wing let us say is located at a distance here. So, let us say this is my ac of wing which is at a distance of 0.32 meters right this is 0.32 meters.

And the Cg or say the tail is located at a distance very far from here which is about 2.93 meters look at that, this is just 30 centimeters that is 292 centimeters from the leading edge. So, this is far away, this is ac of tail which is located at 2.93 meters approximately, ok, right. So, this is X ac of tail and the neutral point turned out to be somewhere here see approximately half a meter. So, this is your neutral point which is 0.54 approximately 54 centimeters, half a meter.

So, this is your NP you know, that means your Cg for a stable aircraft has to lie ahead of NP, that is what we discussed. NP makes the system if the Cg is beyond this NP it is a limit for Cg beyond which it becomes unstable C m alpha becomes positive. So, before that the same case for the wing alone we saw the wing starts flipping back as soon as the Cg shifted back, the aerodynamic center which is the neutral point for a wing alone configuration.

It start flipping back immediately right, so the Cg has to like close to this aerodynamic center itself towards this point, am I correct or not. And see thus strength of this C L alpha of the wing contribution, so it is pulling this towards it. Although the tail have a greater momentum which is about 2.95 meters with respect to leading edge of wing. So, but it has only a smaller momentum but the domination is from the wing contribution wing area, is not it, S t upon S of wing upon S is 1.

But here S of tail upon S is 0.25 one fourth of it. So, this is what is like making the contribution of C L alpha of the tail very less towards the neutral point comparatively, am I correct or not ok, so that is what is happening here. So, we figured out neutral point and the total lift curve let us talk about this static margin. So, let us talk about solution for this third part of this question which is find out the static margin with this current Cg location, ok, how to find out static margin if the current Cg location find the starting margin SM.

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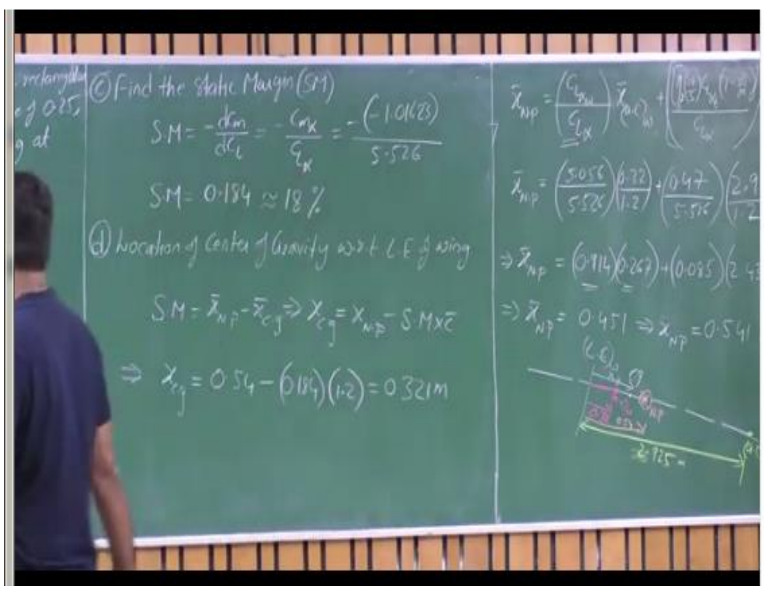

So, static margin is equals to - dC m upon dC L do we know that which is equals to - C m alpha upon C L of the total aircraft. In C L of the entire aircraft is known it is easy to find out static margin, right. So, C m alpha is - 1.01632 such a long number - 1.01623 upon C L alpha of the entire aircraft is about 6 if is in the order of 5.526 ok 5.526 per radian. So, the static margin turns out to be 0.184 which is approximately 18.5% or 18% I should say ok.

So, approximately 18% is the static margin of this configuration with this current Cg location ok. So, let us move ahead to find out what is d right, what is the d location of cg. Now location of center of gravity with respect to leading edge of wing, what do you mean by that, so I need to find out. So, we know what is NP, you now know the static margin as well, you find what is the location of the Cg ok, so say somewhere here.

Say this is my Cg location and this is my X Cg ok you know by the definition of static margin we have X bar NP - X bar Cg, right. This implies X bar Cg or X Cg is equals to X NP - static margin times C bar ok, so since we want a dimensional quantity with respect to the leading edge. So, I am trying to multiply that C bar for this entire equation, so X Cg is X NP which is 0.54 meters 54 centimeters from the leading edge of the wing minus static margin is 18% right.

So, say 0.184 times C bar is 1.2 meters, so 18% of 0.2 or close 20 it is like yeah 20% of 1.2 meters. So, this is equivalent to 0.32 meters and the Cg and the ac are almost same. So, very close Cg and the aerodynamic center of the wing are very close here almost close, yeah. If you have 20% static margin almost close to 20%, then the Cg of the UAV is almost close to the aerodynamic center of the wing here, right.

That means there will be a very weak contribution of C L of the wing towards C L 0 of the wing towards C m ac or C m 0 sorry C m 0 of the aircraft ok. So, let us go ahead and then solve this problem horizontal tail volume ratio, so what is horizontal tail volume ratio?

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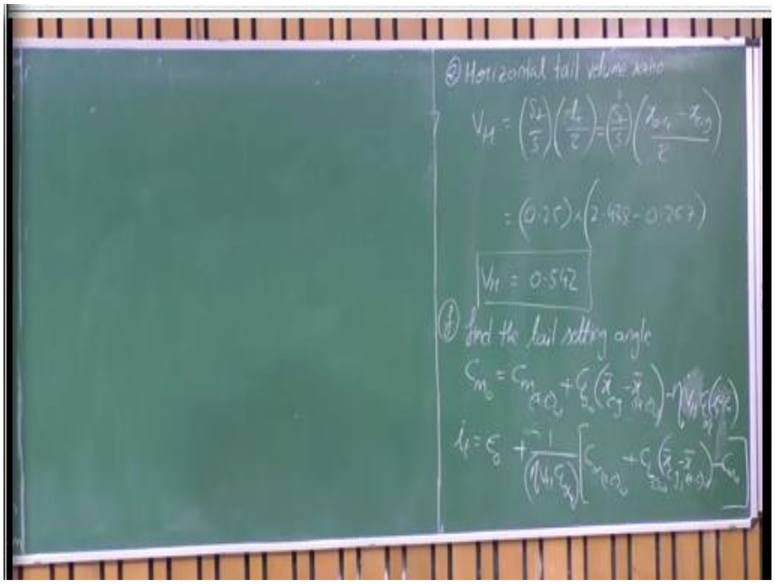

So, the d is a we need to solve for horizontal tail volume ratio. So, we know V H right S t upon S times X bar X ac of tail or l t upon C bar which is S t upon S l t is given. Otherwise you can say S t upon S l t is X ac of tail - X Cg upon C bar. So, straightforward, please substitute this and then this is 25% one fourth of the wing times. So, this is 2.438 - 0.267, so this turns out to be tail volume ratio as 0.542, right.

So, this is a very good number, you know very good tail volume ratio 0.542, ok. And the next question is about finding out the tail setting angle, right. So, find the tail setting angle, how do you find this tail setting angle, you have C m 0 given, right you know C m 0 we know tail setting appears in C m 0 right affects the C m 0. So, let us write down that equation C m 0 is equals to C m ac of wing + C L 0 of wing times X bar cg- X bar ac of wing - eta S t upon S C L alpha of tail 1 - sorry i of t - epsilon 0 multiplied by or you can directly say V H tail volume ratio I am writing in terms of V H here, right.

Since we have already calculated what is V H I am just substituting that there. So, i of t + epsilon right, is not it or - epsilon - epsilon yeah true, true,. So, i of t - epsilon 0, epsilon 0 is 0 for this or not, no. So, from here I can say i t + i t is equals to epsilon 0 + alright, i t is equal to epsilon 0 + 1 upon eta tail volume ratio, C L alpha of tail, ok multiplied by C m ac of wing + C L 0 of wing times X bar Cg- X bar ac of wing minus of C m 0 of the entire aircraft, is not it, ok. So, from the given data you can substitute this and figure out that value, what should be the i of t.

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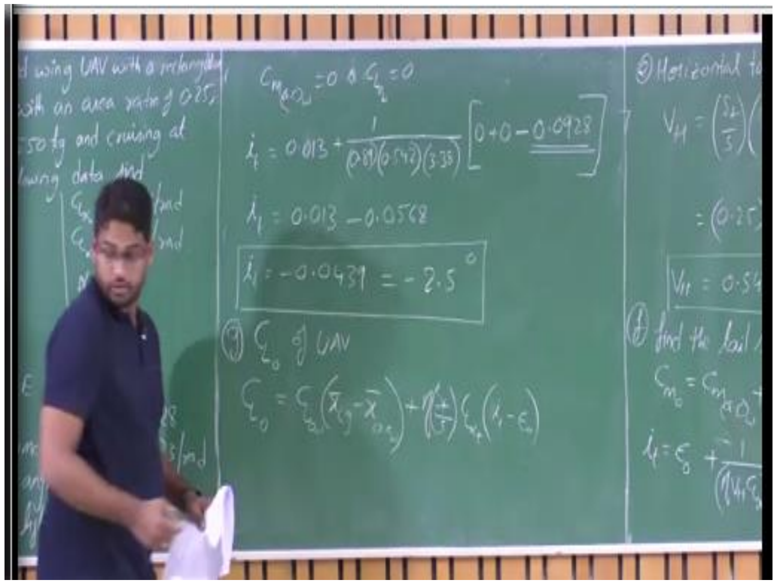

So, consider a rectangular symmetric wing and tail it should be a symmetric you know rectangular symmetric wing and tail ok. So, please add this part here consider a rectangular symmetric wing and tail which makes C m ac is 0 and C L 0 of the wing is 0, right. So, what you end up having is epsilon 0 which is i of t is 0.75 was given right, 0.013 + 1 upon 0.89 times tail volume ratio is 0.542 multiplied by C L alpha of tail which is 3.38.

So, this is equals to 0 + 0 - 0.0928, ok, so what do we end up having is, so i of t is equals to 0.013 - 0.0568, alright. So, what is the value i t is - 0.0439 which is approximately how many degrees this is in radians, so this is - 2.5 degree, ok. So, this is a tail setting angle you need to maintain in order to have this C m 0, right, this is the C m 0 in order to have this C m 0 positive, you need to maintain this particular tail setting angle, right, which is - 2.5 degrees.

So, let us move on to the next question 0 angle of attack lift coefficient, right. So, we need to find C L 0 of UAV, so it is straightforward, is not it, C L 0 of UAV is equals to C L 0 of wing times X Cg - X ac bar of wing + eta S t upon S times C L alpha of tail times i of t - epsilon 0, ok. So, but C L 0 of wing is 0, is not it I am sorry.

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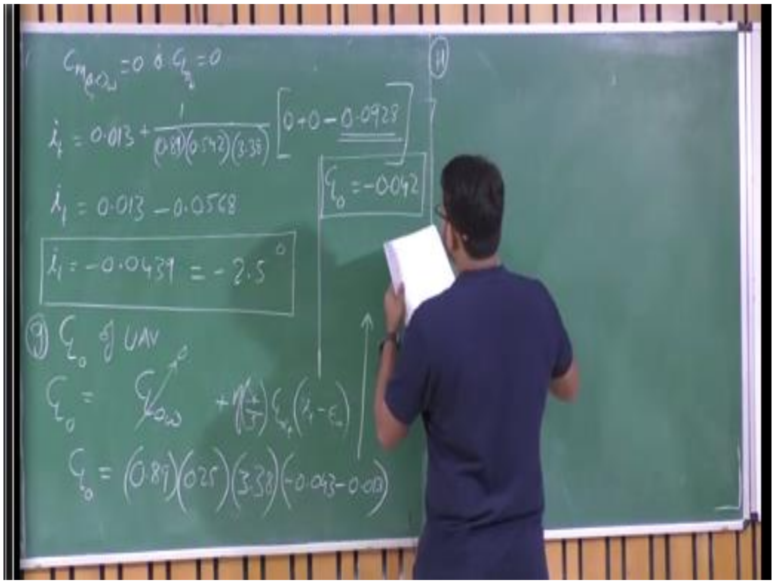

So, please make this correction I am just trying to write about C m 0, now this is C L 0 of wing + eta S t upon S C L alpha of tail i of t - epsilon 0. So, C L 0 of wing is 0 because we have a symmetric wing and tail, right because of which C L 0 of wing is 0. So, the C L 0 of the total aircraft is from this tail contribution, ok. So which is - 2.5 degrees, right which is approximately - 0.0439 - 0.013, right.

So, eta is 89% and 0.25 is S t upon S, this is 3.38 is the slope of tail multiplied by - 0.069, right - 0.043 - 0.013, - 0.056 yeah, so - 0.056, right. So, this equals to, so if you multiply all of this what you are going to get. So, C L 0 is equals to, so the C L 0 that I am going to have because of this is how much - 0.042. So, this is the negative C L 0 contribution because of the tail setting you to tell.

So, when wing is symmetric, so at 0 angle of attack you have negative lift ok, if you have a - 2 degrees tail setting angle. So, you may not be able to fly at 0 angle of attack; of course we have a symmetric wing there. So, you need to trim always at a certain positive angle of attack, right or employ wing setting angle, right continuous wing setting angle when you consider a symmetric aerofoil for the wing then you have to maintain a continuous wing setting angle ok.

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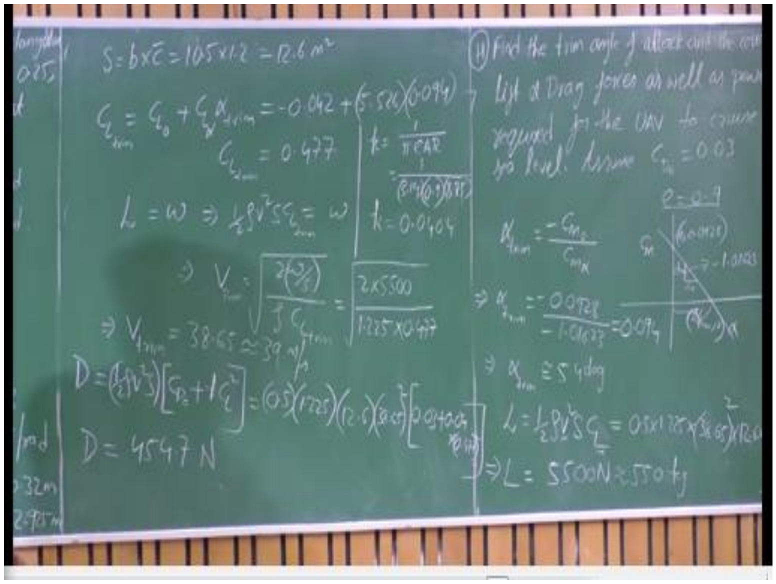

And let us take up the final question, so it is all together a performance question kind of you know. So, find the trim angle of attack and the corresponding lift and drag forces as well as power required for the UAV to cruise at sea level ok. So, assume C D0 is 0 lift, drag coefficient is 0.03 and Oswald's efficiency factor of 0.9 ok. So, we need to figure out first the trim angle of attack you know from the given data for the same UAV.

Again we know we will go back to the C m versus alpha for trim angle of attack, right. So, we have C m 0 we know C m 0 what is the C m 0 value for this aircraft 0.09 it is given 0.0928. So, the coordinates at this point is 0, 0.0928, yeah. And we know alpha trim here, so this corresponds to alpha trim, 0 at trim angle of attack the moment coefficient is 0, am I correct ok. So, I know the slope here D C m upon D alpha C m alpha is known, what is the slope - 1.01623 yeah per radian ok.

So, by using the definition of slope I will simply calculate what is the corresponding trim angle of attack here? right. So, alpha trim is equals to C m 0 upon minus of C m 0 upon C m alpha, is not it. So, what is alpha trim? C m 0 is 0.0928 upon C m alpha is - 1.01623, this is equals to 0.094 radians. So, this implies alpha trim is approximately 5.4 degrees, ok. So, this is the trim angle of attack you know 5.4 degrees, I was able to find out the value.

So, have this trim angle of attack and with this data can I be able to find out what is the lift and drag coefficient, what do I require to find out those. So, first of all lift is half rho V square is times C L is not it, this is half rho V square S times C L. And we need to know what is the cruise velocity as well as C L here. Because density at sea level is 1.225 kg upon meter cube and we have wing area because V is given and C is given right.

So, I know S is b times C bar, C bar is nothing but so c throughout you know it is a rectangular wing, so c bar does not make any sense, it is nothing but root chord, tip chord everything at every at a given cross section this will be the corresponding chord length. So, for a rectangular wing this is b time C bar which is nothing but b time C, so this is 10.5 multiplied by 1.2 which is 12.6 meter square, ok.

And then we have the area, now we need to find out C L alright, this C L is C L of the total aircraft is not it CL of the total aircraft corresponding to alpha 5.4 degrees straight am I correct or not. So, I can express this C L as of the total aircraft as C L 0 of the total aircraft times C L alpha of the total aircraft times alpha trim, right. So, this is C L trim, so this is equals to C L 0 of the total aircraft we have calculated it as - 0.05 how much is that.

C L 0 of the aircraft -0.042, right plus C L alpha of the total aircraft is 5.526, right multiplied by 0.94 it is in radians you know this is in radians. So, let us continue working in radians alpha trim in radians is 0.094, so what is the value 0.477 so C L trim is 0.477. So, if you substitute C L trim there you will be able to find out the lift but we do not know what is the corresponding velocity? right.

So, since we have cruising at sea level during cruise we know lift is equals to weight of the configuration, this implies half rho S times C L trim is equals to w this implies V for C L trim or V trim corresponding V trim is equals to root over 2w or twice the wing loading upon rho C L for trim. So, this equals to Pravijith 2 times w 550, so 550 kgs it was given, so we need to convert it to newtons and considering g as 10 meters per second square.

So, this is 5500 newtons upon 1.225 since we have flying at C level, so density at C level is 1.225 kg upon meter cube times C L trim. So, C L trim what we have is 0.477, so quickly calculate the value of V trim. V trim is equals to 38.65, so close to sorry 39 meters per second right close to 39 meter. So, if you substitute V trim in this particular equation what I have is half 1.225 times 38.65 square, ok half rho V square times the area which is 12.7 we just calculated it 12.6 times 0.477, what is a value.

The lift force acting is what should be the value, without even calculating that we should say, is not it, it should be 550 kg or 5500 newtons close to that maybe 5493 or something, how much is that 5500 newtons which is 550 kg, we need to calculate that. Because we are in cruise lift should be equal to weight, there is no need to calculate that lift is equals to weight there, you can directly say it.

And what is the drag, so drag is equals to again half rho V square S times C D 0 from the drag polar equation times K C L square. So, you substitute the value of C L here, your given C D 0 and find out what is, so if you do that what you are going to have is 0.5 times 1.225 times the reference area here is 12.6 half rho yes V square is 38.65 square ok half rho V square times C D 0 a 0.03 + K we need to find out, right, is not it.

K we do not have the value of k, k is equals to 1 upon pi e AR, right, so 1 upon 3.14 times what is e sorry e is considered as 0.9 sorry 90% efficient. And then the aspect ratio is b square upon S or b by C directly, so b is 10.5, C is 1.2 b by C is approximately 8.75**.** So, the value of k turns out to be 0.04 right. So, 0.04 multiplied by C L square is 0.5 approximately or 0.477 square, right.

So, if you solve this the drag required is approximately 454 newtons or k yeah 454 newtons 454.74 you know newtons. So, this is the amount of drag that is offered by the system when you are flying at this particular velocity in that particular trim condition, ok. So, you need to overcome this drag by producing so must thrust if you are using a jet engine. So, if you want to use a proprietary engine then what we need to find out is power required.

So, can we quickly find out what is the power required for this condition, right, so if it is a jet engine yeah we can say we can talk in terms of thrust that need to be generated. So, let us say if you employ the proprietary engine then we need to know what should be the power generated by the engine for this particular flight condition.

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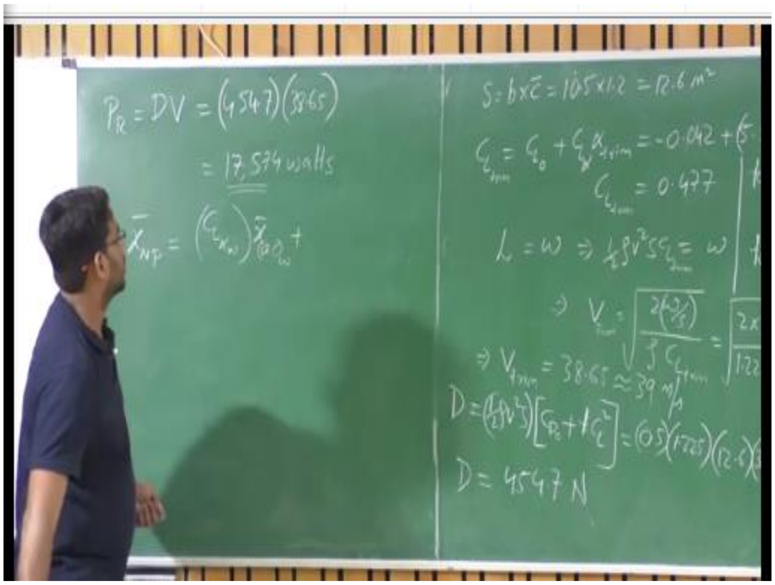

So, what is that means we need to figure out what is the power required by the system which is drag or thrust required times the velocity of flight. So, the drag is 454 or 455 newtons closely which is 45 kg, this is an amount of force you need to produce in the forward direction of flight. So, 0.7 newtons multiplied by 38.65, so this is approximately 17734 watts, right, this is the amount of power that you need to produce to make the system, is it correct 17574 574, ok, just check.

So, close to 17574 ok watts, so this is a amount of power that is required to propel the system forward if you are using a restless motor sorry proprietary engine, yeah, ok. So, well quickly solve another problem another example problem or we will give that as a assignment you know we ok, we will end up this lecture, we will that problem as an assignment. So, and we will be demonstrating that you know do you remember, can you recollect. So, this neutral point, so let us say X NP bar is equals to X ac of wing C L alpha of wing times X bar ac of wing, is not it plus try to erase this part.

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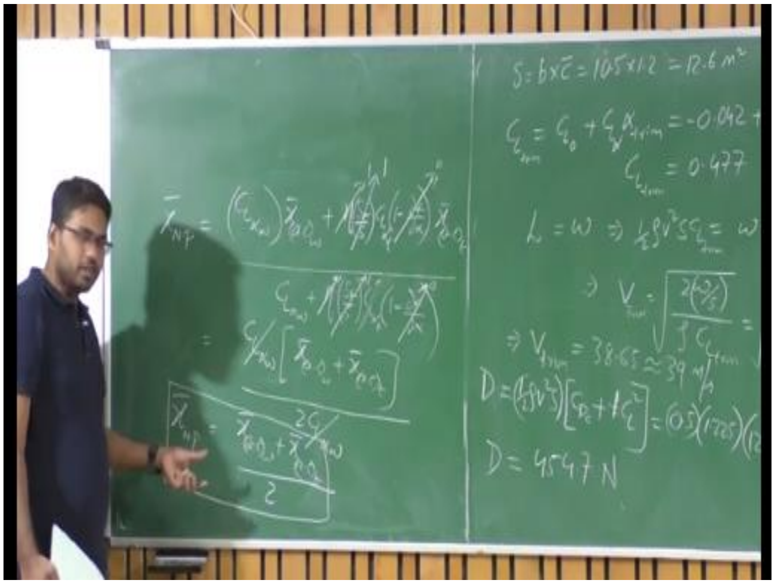

So, it is not connected with this question, so I am talking about a demonstration that we go into take up. So, C L alpha of wing times X ac of wing eta S t upon S C L alpha of tail 1 - dou epsilon by dou alpha times X bar ac of tail, right, upon C L alpha of wing + eta S t upon S C L alpha of tail 1 - dou epsilon upon dou alpha, right. This is a definition of neutral point, right, is not it, from the definition we have derived this as a neutral point.

So, if we consider to see this is about wing and a tail combination, right, let us now consider a case we have identical wing and tail, same wing as tail, ok. For example if this is my wing I will have this as a wing as well as tail separated by a adequate distance, right, ok, what do you mean by that, I have same planform area ok, am I correct. So, I have a same planform area for a wing as well as tail.

That means S t upon S will be 1, ok, so S t upon S is 1, further I assume there is no downwash you know, downwash effect is minimal, it is separated by very large this distance you know. That means eta is 1 because half rho V prime square will be effectively half rho V square itself and then this term disappears, this becomes 0, right, am I correct or not. So, that means this becomes 1 and this term becomes 0, ok I am sorry, yeah not this one that is epsilon term becomes 0.

So, and lift of wing is nothing but lift of tail because we have 2 identical wings. So, in that case what happens is, this is nothing but C L alpha wing taken common out X bar ac of wing + X bar ac of tail why because C L alpha of tail is nothing but C L alpha of a wing. And the correction factor is 1 right now, is not it, so there is no correction factor I will take C L alpha of tail which is equals to C L alpha of wing common out of this numerator.

And then what you have is twice that of C L alpha of wing 2 times of C L alpha of wing, so this cancels out what I have is X bar neutral point as the midpoint of the distance or the midpoint of separation of wing and X bar ac of tail upon 2, am I correct or not. It is the midpoint, can you see this, so this becomes neutral point becomes midpoint of these 2. And we know if the Cg is ahead of this neutral point then we have C m alpha negative, that static margin is positive, C m alpha is negative.

If it is behind the neutral point C m alpha is positive which means the aircraft behaves unstable. So, we will try to demonstrate that, ok, in the next lecture, thank you.