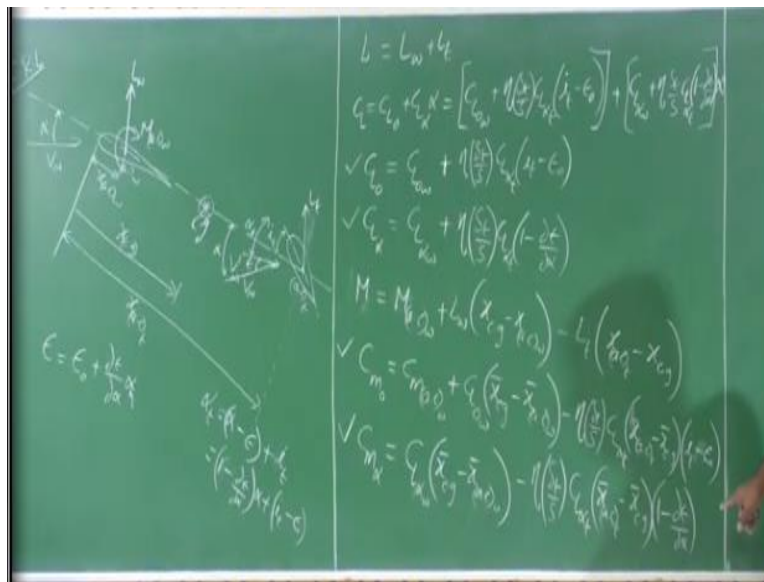


**UAV Design-Part II**  
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**Lecture-14**  
**Example Problems for Wing and Tail Combination**

Dear friends, welcome back. In our previous lecture we discussed about a wing and tail combination and the stability of that wing and tail combination. We derived neutral point for wing and tail combination which figured out to be the weighted average of lifting characteristics of wing and the tail, right.

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So, we started with the wing whose aerodynamic center is located at  $X_{ac}$  with respect to the leading edge of the wing. Say this is my location of the aerodynamic center  $ac$  of wing which is  $X_{ac}$  of wing. So, this wing and tail are positioned with respect to fuselage reference line, this is our fuselage reference line. And this aircraft is flying at a velocity  $V_\infty$  and the aircraft angle of attack is considered as wing angle of attack.

And we assume the wing aerodynamic center coincides with this fuselage reference line. And then we have a tail combination say this is my tail which is symmetric why because I need to produce equal amount of force when deflected either upwards or downwards. So, for that case I

want to have a symmetric tail here. So, which is inclined at an angle  $i$  of  $t$  with respect to fuselage reference line.

And then due to the downwards of the wing and first of all it figure out the aerodynamic center of the tail  $ac$  of tail right which is located at a distance  $X_{ac}$  of tail. So,  $X_{ac}$  of tail with respect to leading edge of the root chord and now the  $C_g$  of this combination is assume to lie somewhere in between wing and tail. And the corresponding location is  $X_{Cg}$  of this combination. Now due to the down motion of the wing, we know because of the presence of the wing there is a downwards near the tail, right, that is because of the wing.

So, due to which the resultant flow will be  $V_{\infty}'$  which is different compared to that of  $V_{\infty}$  here, right. So, what the aircraft is facing, so the initial or the wing  $V_{\infty}$  is parallel to this, right these 2 is let us assume these 2 are parallel. And then there is a modified flow at the tail due to the down wash which is a effect of  $\epsilon$ , right. Now the total angle of attack of tail  $\alpha_t$  in terms of known quantities, like what I know here is  $\alpha$ ,  $\alpha$  I know.

So, this is  $\alpha$  ideally, this is  $\alpha$  and what I know is  $\alpha$  subtracted by  $\epsilon$  will give me the  $V_{\infty}$  or the flow at the tail with respect to the fuselage reference line, right. So,  $\alpha$  minus  $\epsilon$  is this particular quantity added with this inclination of the tail will get me the total angle of attack at the tail, right,  $\alpha_t$  is equals to  $+ i$  of  $t$ . And we also witness that  $\epsilon$  can be modeled as a function of  $C_{L_0}$  and  $C_{L_{\alpha}}$  is not it, right.

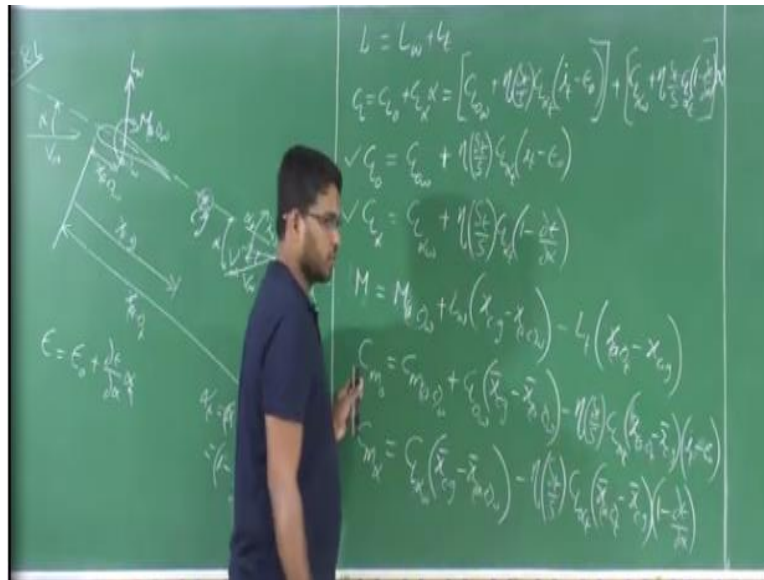
So, this is like 2 times of  $\alpha$  and this can be expressed as  $\epsilon_0 + \text{dou } \epsilon \text{ upon } \text{dou } \alpha$  times  $\alpha$  is not it. So, this  $\epsilon$  is a function of angle of attack at the wing, right downwash at the tail varies due to the variation in the angle of attack at the wing. So, this is not  $\alpha_t$ , this is  $\alpha$ , so you need to remember that. And then if you write the total lift equation from here, say now the lift at the tail will be acting perpendicular to  $V_{\infty}'$  and we neglected the effect of say horizontal component why because there is no offset here.

So, and effectively we assume the small angle of attack and hence the lift of the wing directly contributes for the past pitching movement in the lift of the tail contributes towards negative

pitching movement about the  $C_g$  here, we are considering, right. So, all these distances are measured with respect to leading edge of the root chord of the wing and which are parallel to this fuselage reference line.

And again for wing we are not commenting with there is a cambered aerofoil or symmetric aerofoil or a reflex aerofoil. That means we need to consider the moment about aerodynamic center of the wing, right.

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So, with this we can proceed to figure out what is the total lift generated by this aircraft, the total lift of this aircraft with the principal of super coefficient which is one of the assumptions that we considered earlier, is a combination of or the summation of the lift generated by the wing in the lift generated by the tail which is  $L$  of the total aircraft is  $L$  of wing +  $L$  of tail. And then we figured out that  $C_L$  of total aircraft or which is equal to  $C_{L_0}$  of total aircraft +  $C_{L_\alpha}$  times  $\alpha$  of the total aircraft which is equals to  $C_{L_0}$  of wing +  $\eta S_t$  upon  $S$  times  $C_{L_\alpha}$  of tail times  $i$  of  $t - \epsilon$ , alright minus  $\epsilon$  here.

So, this plus so  $C_{L_\alpha}$  of wing +  $\eta S_t$  upon  $S C_{L_\alpha}$  of tail  $1 - \epsilon$  upon  $\alpha$  times  $\alpha$ , right. So, with this by comparing the coefficients and constants of  $\alpha$  what we have is the total  $C_L$  of the UAV is equals to is contributed from  $C_{L_0}$  lift coefficient

of the wing. And there is a correction factor to the tail setting, see due to the zero lift of wing is 0 or horizontal tail is 0.

Because zero lift at 0 angle of attack there is no lift coefficient, this is symmetric aerofoil. So, the contribution is because of the initial tail setting which is constant throughout the flight. That means such a amount of like the lift coefficient times  $C_L \alpha$  times the angle of attack which is nothing but the at 0 angle of attack  $V_\infty$  prime is aligned along with this flow is not it. Let us assume there is no epsilon,  $V_\infty$  is aligned with the fuselage reference line effectively there is no angle of attack.

Since we have given a tail setting which is  $i$  of  $t$  the aircraft will continuously experience some lift because of the tail, right even at 0 angle of attack though this is 0 but still there is a constant or  $C_L \alpha$  times  $i$  of  $t$  you know in a loose sense. Like this will be to contribution of tail lift towards the pitching movement, right. So, and throughout the flight it will be constant and increases will become a summation as there is a change in angle of attack or when you are changed to as a different angle of attack, there will be a additional component because of that particular angle of attack at the tail.

So,  $C_L 0$  of the wing times yeah plays so the modified lift coefficient  $C_L \alpha$  of tail times  $i$  of  $t - \epsilon 0$ , ok. So, this is see the  $C_L 0$  of the aircraft and  $C_L \alpha$  of the aircraft is  $C_L 0 C_L \alpha$  from the wing the contribution comes from  $C_L \alpha$  of wing straight away. And this modification, so  $\eta$  is a modification  $\eta$  of horizontal tail is nothing but  $\frac{1}{2} \rho V_\infty^2 V$  prime square upon  $\frac{1}{2} \rho V_\infty^2$ .

That can be 1, right when there is no downwash that mean epsilon can be 0, so if you have equal wings like same wings as wing and tail. That means what you have is just  $C_L \alpha$  times  $i$  of  $t$ , if  $i$  of  $t$  is also 0 let us say then what you will be having is  $C_L 0$  you know  $C_L 0$  of wing +  $C_L 0$  of tail directly, ok. Here similarly similar contribution comes from  $C_L \alpha$  of tail  $1 - \epsilon$  upon.

So, this is the total aircraft  $C_{L0}$  or the UAV  $C_{L0}$  and UAV  $C_{L\alpha}$ . Similarly we have pitching moment about  $C_g$ . So, it is moment about  $C_g$  or the moment about entire object of this UAV is equals to the positive pitching moment provided because of lift, lift generated by the wing. Apart from that the wing also have moment about aerodynamics center here. It has a component, the wing has a component which is moment about aerodynamic center about the wing.

And then there is a positive contribution of lift from the wing and times the  $X_{Cg} - X_{ac}$  of wing which is a momenta - eta or say lift of tail times there is no since it is symmetric aerofoil moment about aerodynamic center of the tail is 0. And this lift from the tail contributes towards negative moment, that is why - t. And the moment is due to the force at the tail which is  $L$  of t times the momenta between  $C_g$  and ac of tail. That is nothing but  $X_{ac}$  of tail right or  $X_{Cg}$  removed from  $X_{ac}$  of tail, this is  $X_{ac}$  of tail if I remove  $X_{Cg}$  this is what I end up with.

So, what I have is  $X_{ac}$  of tail -  $X_{Cg}$  that is a corresponding momentum. By doing this the  $C_{m0}$  of the total aircraft is given by  $C_{mac}$  of the wing +  $C_{L0}$  of the wing times  $X_{Cg}$  bar -  $X_{ac}$  bar which is  $X$  bar upon  $C$  bar - eta  $X$  t upon  $S$  times  $C_{L\alpha}$  of tail times  $1 - X_{ac}$  of tail bar -  $X$  bar  $C_g$  times  $i$  of t + epsilon 0, ok. So, you can express this way right is not it  $1 - \text{dou epsilon}$  by  $\text{dou alpha}$  times alpha +  $i$  of t - epsilon 0.

So, this particular term contributes towards negative moment, so I am taking minus common out, so what I have is this ok and  $C_{m\alpha}$  of the total UAV is  $C_{L\alpha}$  wing times  $X$  bar  $C_g$  -  $X$  bar ac of wing - eta  $S$  t upon  $S$  which in general this product  $S$  t upon  $S$  times  $X$  bar ac -  $X$  bar  $C_g$  is known as tail volume ratio where  $X$  bar ac -  $X$  bar  $C_g$  is called  $L$  t bar you know length of the tail in in non dimensional form.

Or  $L$  of t which is  $X_{ac}$  of t -  $X_{Cg}$  in dimensional form  $C_{L0}$  of tail times  $X$  bar ac of tail -  $X$  bar  $c_g$  -  $1 - \text{dou epsilon}$  by  $\text{dou alpha}$  ok. Yeah please correct this, this should be minus epsilon 0, so, it contributes towards positive when  $i$  of t is negative this contributes towards positive here, right. So, this is  $C_{L0}$  is the same thing, right, it contribute it is from the  $C_{L0}$  itself  $i$  of t - epsilon 0 but it is giving pitch down moment that is minus of that right, so ok.

So, this is the total  $C_{m0}$  and  $C_{m\alpha}$  for talk about longitudinal static stability for this UAV we need to consider these two parameters  $C_{m0}$  and  $C_{m\alpha}$ . And  $C_{m0}$  has to be greater than 0 and  $C_{m\alpha}$  has to be less than 0 in order to have longitudinal static stability. And further we have proceeded to figure out what is the neutral point of this configuration which is  $X_{np}$ . It is a location of the  $C_g$ , right for which of the entire aircraft for which the  $C_{m\alpha}$  becomes 0 or the pitching moment is independent of angle of attack.

The variation in pitching moment is independent of angle of attack. That means about neutral point we have  $C_{m0}$  but not  $C_{m\alpha}$ , right  $C_{m\alpha}$  will be 0. So, by substituting that  $C_{m\alpha}$  0 to 0 and then replacing this  $X_{Cg}$  to the corresponding  $X_{mp}$ .

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What we can find out is  $X_{np}$  which we have derived in our previous lecture is equals to  $X_{ac}$  of wing times  $C_{L\alpha}$  wing. So, these are very important you need to you know practice this, so that you will be able to remember them or say without derivation you will be able to figure it out you know. If you practice multiple times and if you keep you know digesting the facts, what we have derived here you will be able to write down what I am trying to do right now, right.

So, again here it is a modified  $C_L \alpha$  it is a correction factor to the  $C_L \alpha$  of tail which is  $\frac{\eta S_t}{S} \times C_L \alpha_{\text{tail}} \times \frac{1 - \epsilon}{\alpha}$ . So, all this is the correction factor to the  $C_L \alpha$  here which we have discussed earlier. And then  $C_L \alpha$  of  $\bar{x}_{ac}$  of tail upon otherwise you can do it this way  $C_L \alpha$  of wing times. So, let us write  $C_L \alpha$  ahead of that  $\bar{x}_{ac}$  of wing.

There it is  $\bar{x}_{ac}$  of tail upon the total  $C_L \alpha$  at  $C_L \alpha$  of this aircraft, what is the total  $C_L \alpha$  of this aircraft your  $C_L \alpha$  right  $C_L \alpha$  of the wing +  $\frac{\eta S_t}{S}$  is a correction factor to the  $C_L \alpha$   $C_L \alpha_{\text{tail}} \times \frac{1 - \epsilon}{\alpha}$  ok, this is a neutral point. And now we also talked about static margin which is the difference between the neutral point  $\bar{x}_{NP} - \bar{x}_{cg}$ , right.

So, from here we have derived that the static margin is equals to, if I substitute this extra  $\bar{x}_{NP}$  here and then  $-\bar{x}_{cg}$  what I end up with is minus of  $C_m \alpha$  upon  $C_L \alpha$  of the entire aircraft which is equals to  $-\frac{dC_m}{dC_L}$ . So, that means if you have positive static margin right, you have negative slope. So, when for a statically stable aircraft the  $C_m \alpha$  when you have positive static margin, you will have negative slope, right  $C_m \alpha$  will be negative for an aircraft which is statically stable, ok.

So, I think with this we will now proceed to solve few example problems. Let us dedicate maybe this entire week to solve these problems why because this is what is going to help you for the design. Now, we talked about being and their contribution towards stability and tail and it is contribution towards stability. And then what is design, we need to know what should be the offset, what should be the location, what should be the area of the tail, what should be the area of the wing.

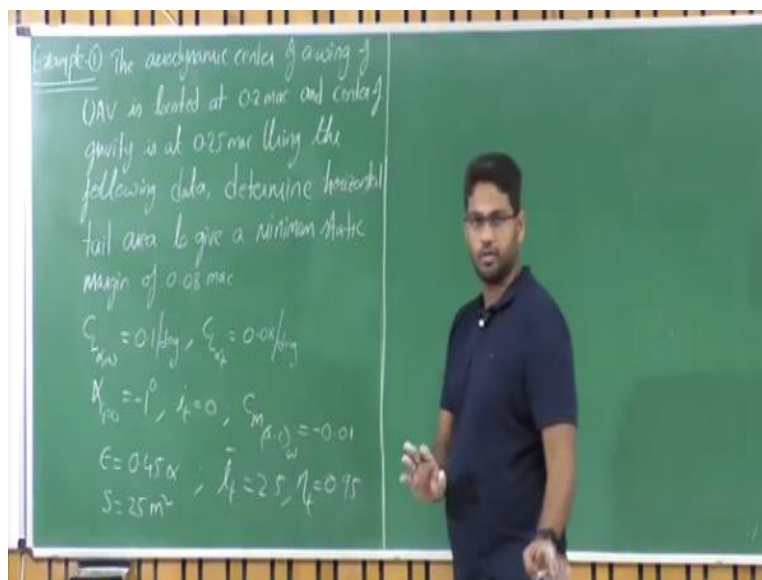
And what should be the location of wing and tail, what should be the  $C_L \alpha$  of wing and tail, right, is not it. So, we talked about all the major parameters that governs the design of wing and tail. For us to understand or digest these things better, we need to solve your example problems, right. So, while solving these example problems, we will consider the straightforward solution where we assume we know  $C_L \alpha$  of wing and tail and their locations.

And we will try to figure out what are its stability characteristics or we will also try to find out what will be the neutral point and all, these are the straightforward problems. Once we get used to it when we are approaching or solving that iterative method we will try to approach it in the reverse way where we will try to have these parameters and try to figure out what should be the wing location, what should be the  $C_L$  alpha of the wing or  $C_L$  alpha of the tail aerofoil, right.

So, we will try to get those parameters design parameters for you to select a particular aerofoil in order to build such a wing and tail combination, ok. So, let us proceed to solve few examples, so I am going to erase all these equations which you must be having already. So, once you practice these derivations you will be able to enjoy otherwise it becomes difficult, you may think that these equations are too lengthy you know to remember.

But still if you practice that this nothing from the first principle if you start deriving them by yourself a couple of times at least. So you will be able to appreciate what are all the terms and why there is a correction factor to the tail. So, all those terms you get comfortable with and then you can start using for your design process with more confidence. So let us take up the first example.

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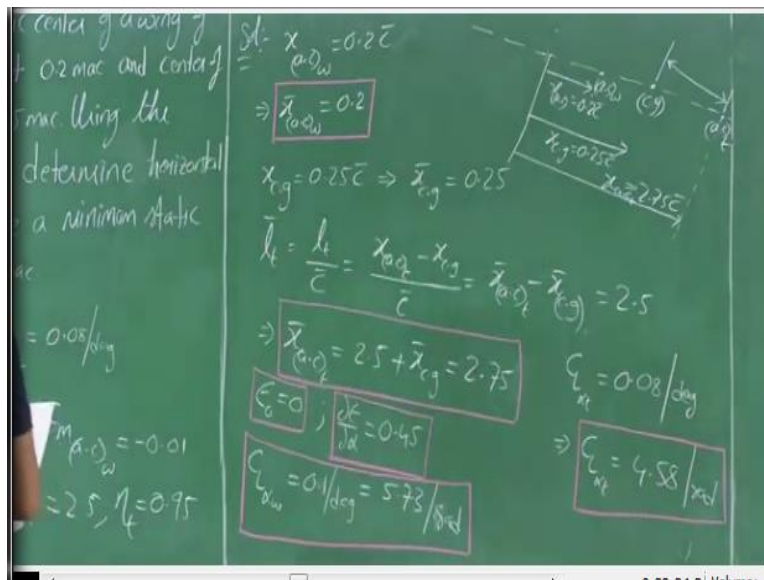


So, the aerodynamics center of a wing of UAV is located at 0.2 mac see and Cg or center of gravity is at 0.25 mac. Using the following data determine horizontal tail area to give a minimum static margin of 0.08 mac ok. So, the data given is C L alpha of wing is given as 0.1 per degree C L alpha of tail is given as 0.08 per degree. And 0 lift angle of attack alpha C L is equals to 0 as - 1 degree and it has a zero tail sitting angle i of t is given as 0, ok zero tail sitting angle.

And C m ac of wing is - 0.01 ok and epsilon is given us 0.45 alpha, ok and then l t bar is given as 2.5 with a tail efficiency factor of 0.95 ok. So, and the area of the wing, so the area of this wing is about 25 meter square. So, please expect these questions the similar questions in your examination, right. So, you need to get used to this particular data and all these coefficients that we are using here, so that can happen again only with practice.

So, let us what we need to do here we need to find out the like horizontal tail area when the static margin is given, right. See here we have static margin we got to know what is the location of aerodynamic center of the yeah wing as well as center of gravity location.

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So, what does it mean let us say this is my fuselage reference line, let us say this is my leading edge of the wing. And what I have is X or aerodynamic center of the wing which is X ac located at which is equal to ok 0.2 C bar. So, that means like they have non dimensionalize these

distance, right, that means  $\bar{X}_{ac}$  is 0.2, ok. So, given solution  $\bar{X}_{ac}$  of wing is  $0.2 \bar{C}$  which is again how much?  $0.2 \bar{C}$  is given right is not it,  $\bar{C}$  is not given.

So, let us keep it as it is, so this implies  $\bar{X}_{ac}$  of wing is 0.2 ok and  $\bar{X}_{cg}$ , see this is my  $\bar{C}_{g}$  location of  $\bar{C}_{g}$   $\bar{X}_{cg}$  is  $0.25 \bar{C}$ . And  $\bar{X}_{cg}$  is given  $\bar{X}_{cg}$  is  $0.25 \bar{C}$  which means this implies  $\bar{X}_{cg}$  is 0.25 ok and then the length of the tail is also given. Let us say this is my  $\bar{X}_{ac}$  of tail and it is located at a distance  $\bar{X}_{ac}$  of tail  $\bar{X}_{ac}$  of tail but this is not given straightforward, right.

We got a  $l_t$  bar, what is  $l_t$  bar which we just discussed, so  $l_t$  bar is equals to  $l_t$  upon  $\bar{C}$  bar which is equals to  $\bar{X}_{ac}$  time  $\bar{X}$  it is a distance between we call it as tail arm right. So, it is a distance between aerodynamic center of the wing of the tail and the center of gravity of the aircraft that  $\bar{X}_{ac}$  - of tail -  $\bar{X}_{cg}$  upon  $\bar{C}$  bar this is equals to  $\bar{X}_{ac}$  of tail -  $\bar{X}_{cg}$ , fine, so this value is given as 2.5, this is equals to 2.5.

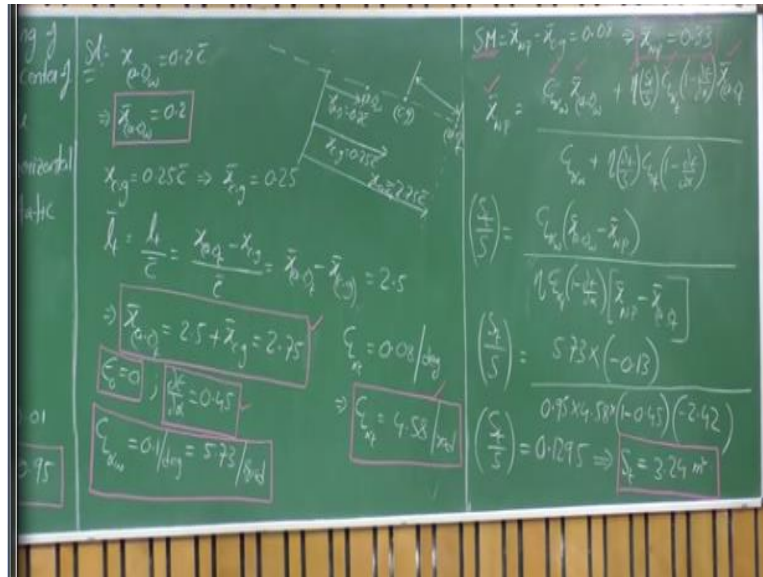
So, now  $\bar{X}_{ac}$  of tail from here  $\bar{X}_{ac}$  of tail is  $2.5 + \bar{X}_{cg}$ , so this is equals to 2.75 in the non dimensional form, so what you have is  $2.75 \bar{C}$  bar, this particular distance is  $2.75 \bar{C}$  bar ok. We are now with that given data of  $l_t$ , we are now able to figure out what is the location of aerodynamic center of the tail, ok. And then tail efficiency factor is given  $\eta$  is given 0.95, so epsilon it is given.

So, what is epsilon 0 is 0 since from this data we just have  $\epsilon$  by  $\alpha$  there is no epsilon here. So, epsilon 0 is equal to 0 and so I will try to highlight what we have derived till here. So, we have this particular factor and we also have this and also epsilon 0, right and  $\epsilon$  by  $\alpha$  is equals to 0.45, this is another parameter, variation of downwash with an angle of attack at the wing right.

So,  $C_L \alpha$  of wing is given let us convert it into radians right,  $C_L \alpha$  of wing is equals to 0.1 per degree which is equals to 5.73 per radian, right this is 5.73 per radian, we have another value. And then we can also talk about  $C_L \alpha$  of tail which is how much?  $C_L \alpha$  of tail is given as 0.08  $C_L \alpha$  of tail 0.08 per degree. So, this is equals to  $C_L \alpha$  of tail is 4.58 per

radian ok. So, I mean this is one of the parameter that I required to solve this problem. So, this is what I can see alpha at which  $C_{L0}$  is -1 and  $i$  of  $t_0 C_m$  ac of the wing is also given, so this is what I have and again static margin is given here, right.

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So, static margin SM is equals to static margin is equals to  $X_{NP} - X_{CG}$  which is 0.08, right see it is already in  $C$  bar in  $m$  ac right. So, if we just take  $X_{NP} - X_{CG}$  then it is 0.08  $C$  bar you know. So, it is already we are talking in terms of non dimensional numbers, so let it be 0.08 then. So, if you substitute the  $C_g$  location here, you will be able to find out what is the neutral point is not it, so can we do that.

Otherwise this implies  $X_{NP}$  neutral point is equals to so  $0.08 + X_{CG}$ . So,  $0.25 + 0.08$  which is 0.33, so I now got the location of neutral point. Now I need to figure out with this data I need to figure out what is the tail area you know. We also have the tail setting angle data about tail setting from the definition of neutral point  $X_{NP}$ , can we find out something. So, I immediately can see what is  $X_{NP}$  from the definition of neutral point, right.

So, I will try to figure out whether I will be having adequate data to find out what is  $S_t$  in order to have this particular static margin, right. So, this is equals to  $C_L \alpha$  of wing times  $X_{ac}$  of wing plus see I am intentionally writing these equations again and again, so that you will get used to it, ok. So, I wish you should also take down the notes for this  $\eta S_t$  upon  $S C_L$

alpha of tail times  $1 - \epsilon$  by  $\alpha$  multiplied by  $X_{ac}$  of tail  $\bar{X}_{ac}$  of tail upon  $C_L \alpha$  of wing and the correction factor lift curve slope of the wing due to the downwash and the area of the wing.

So, you are normalizing the area of the wing here  $1 - \epsilon$  by  $\alpha$ . So, let us look at the parameters that we got, so we have  $C_L \alpha$  of the wing, right, is not it. So, which is highlighted here and we have  $X_{ac}$  of the wing,  $X_{ac}$  of the wing is given and we know what is  $\eta$ ,  $\eta$  of the tail is given as 0.95. So, we have the data  $\theta$  and we do not know what is  $S_t$  upon  $S$ , this is what we need to find out,  $S$  is given 25 meter square.

But we do not know what is  $S_t$  which is need to be figured out from the data. So,  $C_L \alpha$  of tail is given, so we just now converted into per radian. So,  $C_L \alpha$  of tail is available and I have  $\epsilon$  upon  $\alpha$ , right which is 0.45. So, even I have this and also we just figured out what is  $X_{ac}$  of tail, am I correct. So, we just got what is  $X_{ac}$  of tail, so  $X_{ac}$  of tail is here which is 2.75 in non dimensional form.

So, that means, we got all the data and neutral point we have figured out from the given static margin, we have estimated what is the neutral point with the given  $C_g$  location. With the given static margin and  $C_g$  location, we got where should be the neutral point for me to have neutral point at this particular location, what should be the area of the tail. So, that is what we can solve it from this particular equation.

So, you can simply substitute this or say  $S_t$  upon  $S$  if I rearrange this equation, what I can have is  $C_L \alpha$  of wing times  $\bar{X}_{ac}$  of wing  $- \bar{X}_{NP}$  upon  $\eta C_L \alpha$  of tail  $1 - 2 \epsilon$  by  $\alpha$  ok, this is in the denominator. I am taking this common out, so multiplied by  $\bar{X}_{NP} - \bar{X}_{ac}$  of tail, am I correct, yeah. Aerodynamic center of wing is ahead of the neutral point, so that is what we will be ahead because you know 0.33 is a neutral point and 0.2 is a aerodynamic center, so this will be a negative term.

So, negative upon negative because neutral point lies in between aerodynamics center of tail and aerodynamics center of wing. That means it is lesser than the  $X_{ac}$  of tail, so this is a negative

quantity again. So, negative upon negative what you have the ratio is positive here. So, if you can substitute these values here, so  $C_L \alpha$  wing is 5.73 per radian multiplied by  $X_{ac}$  of wing -  $X_{np}$  neutral point is about, so this is like + so  $0.33 - 0.2$  which is  $0.13$  ok. So, this equals to  $-0.13$  upon, so  $\eta$  is  $0.95$  from the given data and  $C_L \alpha$  of tail is  $4.58$ .

We convert it into per radian multiplied by this correction factor due to induced downwash from the wing, right. So, this is  $1 - 0.45$  which is  $0.55$  I guess multiplied by the distance between neutral point and the aerodynamic center of tail which is again  $2.75 - 0.33$  is like  $2.42$ , right -  $2.42$  here. So, can you quickly solve this, so what I have is  $S_t$  upon  $S$  is equals to  $0.1295$  which means  $S_t$  is close to  $13\%$  of  $S$ , right. So, the tail area is  $13\%$  of that of wing area just the  $13\%$  of wing area. So, the wing area you have is  $25$  meter square, so what it will be.

This implies  $S_t$  is equals to  $0.1295$  multiplied by  $25$  which is  $3.24$  meter square ok, that is a wing area we need to figure out. So, from the given data we are able to find it out yeah, sorry that is a tail area I am sorry. So, given the wing area, we are able to figure out what is the tail area, what should be the like percentage of tail area in order to have this static margin. So, what will be the tail volume ratio, can we find the tail volume ratio of this.

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aerodynamic center of a wing is at  $0.25$  mac and center of gravity is at  $0.25$  mac using the above data, determine horizontal tail area to give a minimum static margin.

$$V_{HT} = \left(\frac{S_t}{S}\right) \left(\frac{l_t}{c}\right) = \left(\frac{S_t}{S}\right) \left(\frac{x_{ac,t} - x_{cg}}{c}\right)$$

$$V_{HT} = \left(\frac{S_t}{S}\right) (x_{ac,t} - x_{cg}) = (0.1295)(2.75 - 0.25)$$

$$V_{HT} = 0.323$$

(b) for the above data find  $C_{m_x}$  &  $C_{L_x}$  of the UAV

$$S.M. = -\frac{C_{m_x}}{C_{L_x}} \Rightarrow C_{m_x} = -S.M. C_{L_x}$$

$$C_{L_x} = C_{L_{x,w}} + \eta \left(\frac{S_t}{S}\right) C_{L_{x,t}} \left(-\frac{c}{l_t}\right) = 5.73 + (0.1295)(4.58) \left(-\frac{2.5}{2.5}\right)$$

$$C_{L_x} = 5.73 + 0.31 = 6.04/\text{rad}$$

So, the tail volume ratio  $V_H$ ,  $V$  of horizontal tail is  $S_t$  upon  $S$  times  $l_t$  upon  $C$  bar, this is numbered  $S_t$  upon  $S$  times what is the  $l_t X_{ac}$  of tail -  $X_{cg}$  upon  $C$  bar this can be written

as  $V_{HT}$  is  $S_t$  upon  $S$  times  $\bar{X}_{ac}$  of  $t - \bar{X}_{Cg}$  this is equals to, so 0.1295 that is approximately 0.13 multiplied by times  $\bar{X}_{ac}$  of tail -  $\bar{X}_{Cg}$  bar. So, this is this value what is that, so 2.75 - 0.25, right.

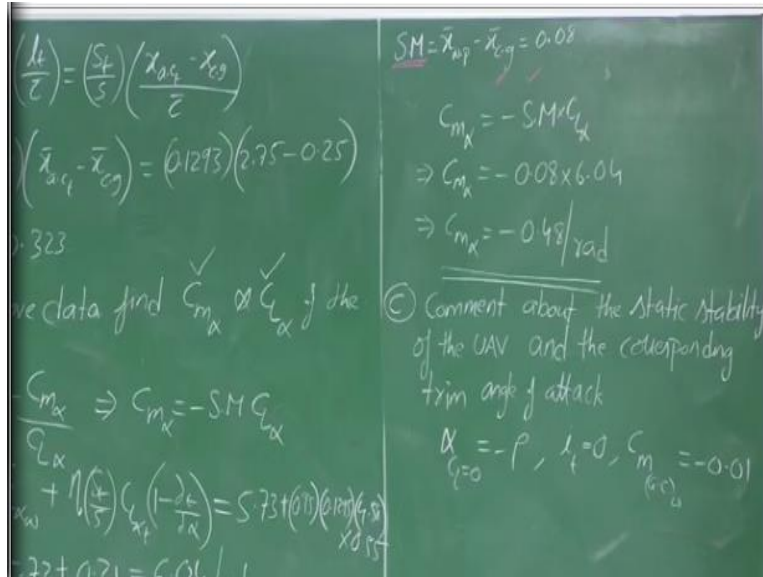
So, this is a corresponding value, what is the value here 0.323. Now also find the  $C_m$  alpha of this aircraft fine. So, let us say this is part a example 1a 1 part a and solving part b. For the above UAV find  $C_m$  0 sorry find  $C_m$  alpha and  $C_L$  alpha of the UAV from the above data find  $C_m$  alpha and  $C_L$  alpha of the UAV, right. So, we know from the static margin definition of static margin what we have in -  $C_m$  alpha upon  $C_L$  alpha of the entire UAV.

This implies if I want to know what is  $C_m$  alpha, this is minus static margin times  $C_L$  alpha of the UAV. So, how can I find  $C_L$  alpha of the UAV, so  $C_L$  of the entire aircraft is  $C_L$  alpha wing + eta  $S_t$  upon  $S$  times  $C_L$  alpha of tail multiplied by  $1 - \epsilon$  upon dou alpha, can you find this. So, it is  $5.73 + 0.067$  or say 0.95 multiplied by **1**. 0.1295 multiplied by 4.58 multiplied by 0.55 times 0.5.

So,  $C_L$  alpha of the entire aircraft is  $5.73 + 0.31$ , so see the major contribution of  $C_L$  lift curve slope for the entire aircraft is from the wing compare to that of tail, is not it, am I correct. So, that is because so the major what you call, so eta is close to 0.95 that is not going to affect the  $C_L$  alpha much just almost 95% of  $C_L$  alpha. And the downwash you know see it is almost may half of that  $C_L$  alpha and from that the tail volume ratio it just 13% you know.

So, in that half it is making 13% of that half that is why the contribution is very, very less here, is not it, so this is equals to yeah 6.04 per radian ok. Once you have  $C_L$  alpha it is easy to find out static margin, sorry we have static margin and  $C_L$  alpha, so we will be able to find out  $C_m$  alpha of this UAV with the current  $C_g$  location, right.

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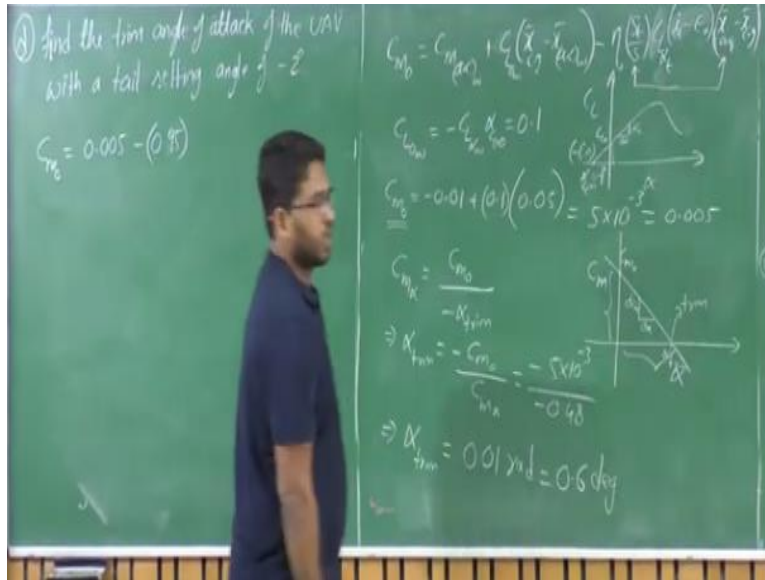


So, with the current Cg location C m alpha is equals to minus of static margin times C L alpha of the entire UAV. So, this is minus of 0.08, right static margin multiplied by 6.04 (()) (41:13) C m alpha this implies C m alpha is equals to minus of 0.48 per radian, ok. So, we are able to find out the C L alpha both the C L alpha is 6 per radian approximately. And then the C m alpha of this UAV is -0.48.

So, can we comment about stability right now, can we comment about longitudinal static stability, I do not think so, is not it. So, we just now what is C m alpha, we need to find out what is C m 0 of the aircraft as well. So, let us consider part c of this question, part c is so comment about the static stability of the UAV right and the corresponding trim angle of attack, we need to find that right.

So, for this we will have this data alpha at which C L is equals to 0 is - 1 degree and i of t is equals to 0 and C m ac of the wing is - 0.01, right. So, these are important, so to comment about static stability we need to find out C m 0 here.

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The  $C_{m0}$  of this wing and tail combination is given by  $C_{m ac}$  of wing you have to refer to this equation again, right,  $C_{m ac}$  of wing +  $C_{L0}$  of the wing times the momenta  $\bar{X}_{Cg} - \bar{X}_{ac}$  of wing, right minus of we have a negative contribution right or a negative moment due to tail. And the setting is what is going to make it positive for  $C_{m0}$ , so  $S_t$  upon  $S$  times  $C_{L\alpha}$  of tail, right.

This completes half  $\rho V^2 C_{L\alpha}$  of  $t$  completes the lift at the tail is not it, is in non dimensional lift curve slope at the tail times the corresponding downwash effect, right, so times the setting angle not the downwash effect here. So, what is the setting angle,  $i$  of  $t - \epsilon_0$  times  $\bar{X}_{ac}$  of tail -  $\bar{X}_{Cg}$ , am I correct. So,  $C_{m0}$  is equals to what is  $C_{m ac}$  of the wing, - 0.01 times  $C_{L0}$  of the wing.

So, do we have the information about  $C_{L0}$  of the wing now right, what we have is  $C_{L\alpha}$  of the wing and  $\alpha$  at which  $C_L$  is equals to 0, what is it mean. So, if you look at the variation of  $C_L$  with  $\alpha$ , right, of the wing here you are talking about wing. So, let us say it is a cambered aerofoil, so that is visible from  $C_{m ac}$  of the wing, right. Because it is negative I can say it is a positive cambered aerofoil.

So, what I have is linear resin  $C_L \alpha$ , I know the  $C_L \alpha$  d  $C_L$  upon d  $\alpha$  I know this. And I need to find out the  $C_{L0}$  given  $\alpha$  at which  $C_L$  is equals to 0 is which is - 1 degree.



So, I know this coordinates which are - 1 degree, 0 and I know this coordinates 0,  $C_L 0$ , so by using the definition of slope I can find out  $C_L 0$  of wing is equals to minus of  $C_L \alpha$  of wing times alpha at which  $C_L$  is equals to 0, ok.

So, this is approximately 0.1, right, so this is 0.1, ok. So, now yeah substitute this 0.1 here and we know  $i$  of  $t$  is 0  $\epsilon_0$  is 0 here. From the given data  $i$  of  $t$  is 0 and  $\epsilon_0$  is 0 this term vanishes, so what you have is  $C_L 0$  of the wing here and then the differences between  $X_{Cg}$  and  $X_{ac}$ . So,  $C_m 0$  will turn out to be  $C_m ac - 0.01 + 0.1$  times the distance between  $Cg$  and the aerodynamic center of wing which is 0.05, right.

So, what this turns out to be 0.005, right  $5 \times 10^{-3}$  yeah positive but very small, is not it 0.005. So, positive but very small very, very small number, so what we can say is  $C_m \alpha$  is negative and  $C_m 0$  is positive, we can say this is statically stable it posses longitudinal static stability this particular UAV with the current  $Cg$  location. So, how can I increases, so before are we satisfied with this  $C_m 0$ , how can I answer that question.

So, we have another part which is the corresponding trim angle of attack, right, so do you remember this plot the  $C_m$  variation with alpha. So,  $C_m$  variation with alpha, so  $C_m 0$  has to be positive and  $C_m \alpha$  has to be negative which makes this aircraft to trim at a particular angle attack called alpha trim. So, this corresponds to trim, is not it, alpha trim, am I correct, so this value is  $C_m$  at alpha 0 which is  $C_m 0$ .

So, you have  $C_m 0$  and you have  $C_m \alpha$  d  $C_m$  upon d alpha, so can we find out alpha trim from the definition of this slope, right.  $C_m \alpha$  is equals to  $C_m 0 - 0$  upon minus of alpha trim, right, am I correct. So, this implies alpha trim is equals to  $- C_m 0$  upon  $C_m \alpha$ , so this equals to minus of  $5 \times 10^{-3}$  upon  $C_m \alpha$  is - 0.48.

So, from here you can figure out alpha trim which is equals to 0.5 degrees approximately, right alpha is equals to alpha trim is equals to 0.6 or 0.01 radian, so this is approximately 0.5, right, can you cannot you consider this as 0.5. So, this is like  $10^{-2}$  upon yeah. So, this equals to

0.6 degrees approximately, right. So, that means this trim so this particular angle is 0.6 degrees you know not even 1 degree trim angle.

You are not able to trim the aircraft you know, so from the design it is trimming only at 0.6 degrees not even at 1 degree. And we know from the  $C_L$  versus  $\alpha$  at 0.6 degrees the value of  $C_L$  is quite less we may not flying at the desired  $C_L$ . The design  $C_L$  we call the corresponding  $C_L$  for this particular trim angle of attack is a design  $C_L$  may not be satisfied with this particular alpha trim.

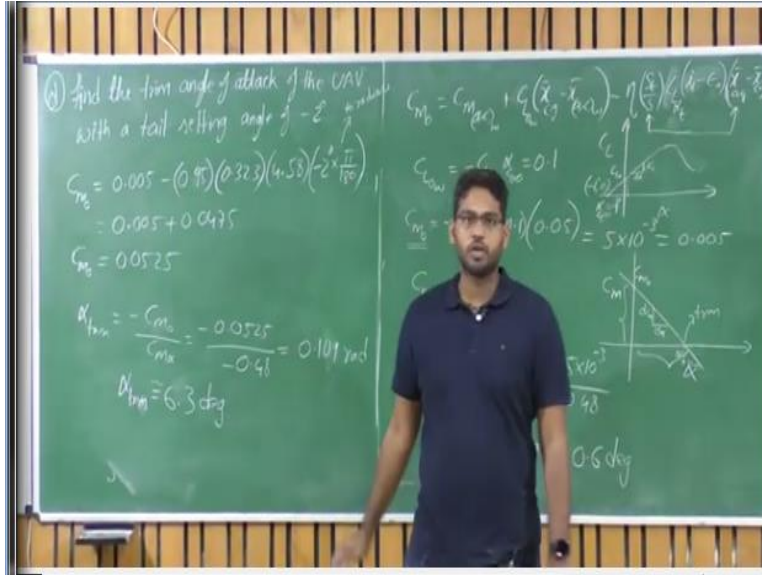
So, we have a horizontal tail but we made the tail setting angle 0, so what must be the reason, we cannot taken the contribution of the tail, we made it  $i$  of  $t$  0, ok. So, we have not included the contribution of tail though we have a tail but it is not contributing towards this trim, alright. So, yes as we discussed earlier there will be a additional drag due to the trim or that due to that setting called trim drag, right.

So, but still we cannot afford such a low trim angles for this aircraft you know, so that means if you have to trim at a higher angle of attack you need to give elevated deflection which again increases the drag a lot. So, which is not desirable, so what should we do right now, what as a designer do you think about this, ok. Let us to have a better feel let us now have a tail setting angle let us say about 2 degrees tail setting angle, right.

So, let us have a - 2 degrees tail setting angle and repeat the calculation, the same question we will have - 2 degrees tail setting angle and repeat the calculation, ok. So, then  $C_m$  we will see what will be the corresponding trim angle of attack, is it helping, is the tail helping us or not, ok. So,  $i$  of  $t$  is not going to affect the stability of the aircraft right, is not it,  $C_m$   $\alpha$  is not getting affected because of  $i$  of  $t$ .

If you look at the equation  $C_m$   $\alpha$  is you know like it is not getting affected because of  $i$  of  $t$ . So, what  $i$  of  $t$  is going to help is  $C_m$  0, if you look at this equation we have  $i$  of  $t$  here, right.

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So, part d of this question is, part d is find  $C_{m0}$  or say find the trim angle of attack of the UAV with a tail setting angle of the above UAV again. In the same UAV with the tail setting angle of -2 degrees, ok, let us solve this question again. So, we know what is this  $C_{m0}$ , now we have the contribution from tail, right i of t is positive there is a different value it is not 0. So, there is a contribution from here, ok, we cannot neglect this term just like what we did earlier.

So, the value of  $C_{m0}$  with i of t contribution is equals to, so this parameters are not going to change, right,  $C_{mac}$  will be same as and  $C_{L0}$  will be times the  $X_{cg} - X_{ac}$  is also same. So, what I have this as 0.005, right, so I am not again repeating that here - eta is 0.95 and what is the tail volume ratio. So, this is not affected because of i of t, right, is not it, sorry, S t by S multiplied by l t bar 0.323 multiplied by  $C_{L\alpha}$  of tail which is 4.85 what is  $C_{L}$  of tail.

I will just refer it again 4.58, so multiplied by i of t here epsilon 0 is 0 from the given data, so i of t is -2 degrees, so I need to convert it into radian. Because this  $C_{L\alpha}$  is in radians here, right, 2 degrees multiplied by pi by 180, so converting it to radians you know, so to radians, ok, degrees multiplied by pi by 180, I will convert it into radians. So, what I have is 0.05 plus a positive contribution, right minus of minus I have a positive contribution.

Because of negative tail setting angle which is 0.0475, so with this what I achieved is 0.0525. So,  $C_{m0}$  now is for a better value at least you know in the order of 10 right compared to earlier

case. So, what I have is 0.0525, now again by substituting that  $C_{m0}$  and alpha like  $C_{m\alpha}$  is going to remain same, so alpha trim I can figure out by  $-C_{m0}$  upon  $C_{m\alpha}$  by substituting that here.

So, alpha trim is equals to  $-C_{m0}$  upon  $C_{m\alpha}$ , so minus of 0.0525 upon -0.5 approximately I should 0.104. So, this is like - 0.5 it is like 0.134 radians sorry 0.109 or 0.11 which is approximately 5.7 degrees or 5.6 degrees 6.26 yeah. So, alpha trim is approximately 6.26 6.3 degrees approximately, 6.3 degrees. See now you may be flying close to that maybe aerodynamic efficiency, right, higher aerodynamic efficiency here.

So, you are now able to trim the aircraft automatically without any elevated deflection at 6.3 degrees. So, that is a significance of  $i$  of  $t$  you know that is how it can contribute towards static stability and helps you to trim at positive angles of attack, that we have discussed from the starting of this stability course, right, ok, thank you.