UAV Design-Part II

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Lecture-13
Wing and Tail Contribution, Neural Point

Dear friends, welcome back. Let us now proceed to analyze the stability of wing and tail combination, where in our previous lecture we talked about wing alone configuration for which the C m about Cg, we have estimated, and then we bifurcated that into 2 components C m 0 and C m alpha, where we figured out for C m 0 has to be greater than 0, right, we need to have a reflex aerofoil for a wing alone configuration, and then for C m alpha to be less than 0 for static stability condition. To satisfy that condition, we need Cg should be ahead of the aerodynamic center.

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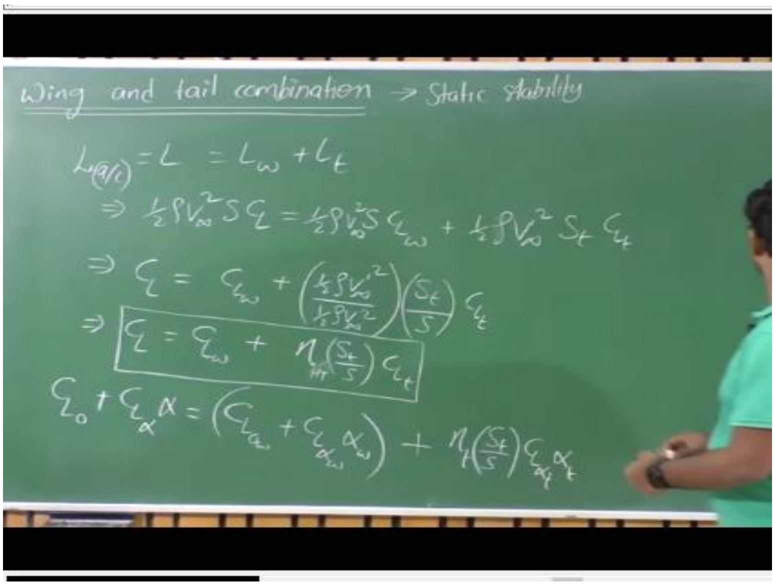

Now look at the contribution of wing and tail combination, a combination and its contribution towards static stability. So, I would like to divide this into 2 partitions. So, now let us have this wing and tail. So, say this is my fuselage reference line.

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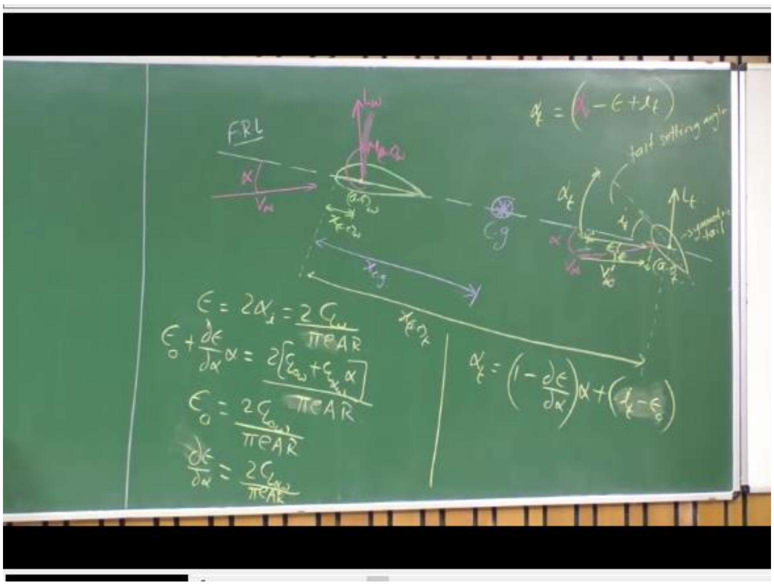

Say the chord of the wing coincides with the fuselage reference line here. So, this is the chord, the chord of the wing coincides with a fuselage reference line. And then we have a tail at after wing right and then we try and also let us not consider about the z offset of the Cg okay. So, we have fuselage reference line and for the wing, we know there is an aerodynamic center for this wing which is located at a distance x ac at the leading edge of the wing.

And the Cg of this aircraft with a tail we will draw the tail very soon. So, is located at a distance x Cg with respect to leading edge of the wing okay. And we have a tail here, we added a tail, let us say the chord line of the tail is oriented or inclined at certain angle called a i of t right, tail sitting angle, i of t is known as tail sitting angle with respect to the fuselage reference line.

So, this is a chord line of the plane horizontal tail and i of t is the inclination of this chord line with respect to the fuselage reference line. And we call this i of t as tail sitting angle. This tail setting angle and in general, this cross section of tail is symmetric aero for. Why because we know it has to produce equal amount of force on the either side of the deflection, the aim of tail is not to generate lift, to generate a moment right to control the orientation of the aircraft and also to provide static stability.

So, that we will see how to control the aircraft; that we will see later when we talk about elevator deflection. So, for the time being, we assumed like the tail is used to stabilize the aircraft and we will see how it is going to do that. And the tail is also located at a distance right we also have an aerodynamic center for this tail which is located at the leading edge here and this in the corresponding distance here is x ac of tail with respect to the leading edge of the root chord of the wing and is measured parallel to the fuselage reference line.

So this is my fuselage reference line FRL. Now when this is moving or in equilibrium at certain alpha. So, moving at certain V infinity, this combination is moving at V infinity. So, we have lift perpendicular to V infinity liftoff wing and the same time we have a moment about aerodynamic center of wing. Similarly, we should have lifted the tail as well right.

So, but the flow near the wing may not be same as flow near the tail, why because the wing we witness right from the lifting line theory, it creates a downwash behind the wing is not it. So, because of the downwash say this is my so V infinity, actual V infinity right. So, ideally this has to be my angle of attack, ideally it has to be there, but because of the downwash you know there is a downward component of velocity.

Downwash is nothing but the flow is pushed down right, it was pushed down by this wing right, it is heavy near the wing right in the vicinity of the wing as we go behind, because there is a forward component right of the velocity, there is a like component of velocity in this direction and there is a downward direction, in the component of velocity which is in the direction of flight or in the direction.

So, is higher right which means the freestream velocity is in the opposite direction flight. So, which is higher. So, the resultant will remain flat, but still there is some deviation from the actual flow that the wing faces. So, deviation at the tail compared to that of what the wing faces. So, that is nothing but the downwash because of this induce because by this thing right at the tail right.

Let us say this is the downwash So, because of which the resultant freestream velocity here will be flattened. This is my velocity at the tail V t or V prime we call it, let us V prime here, okay. So, V infinity prime okay. So, this particular change in angle is epsilon okay. Is it clear. So, now, effectively I can this is equivalent to this say. So, this is equivalent to these 2 are parallel right. So, this is my V infinity and this is my epsilon.

So, now, the angle of attack that this tail C is with respect to this V infinity prime right, is not it. So, V infinity prime is a modified velocity at the tail because of the downwash induced by the wing. So, this downwards creates an angle epsilon with respect to the freestream velocity at the v infinity. So, now, the angle of attack that the tail C is with respect to this modified velocity which is V infinity prime and it is denoted by alpha at the tail right.

So, this alpha at the tail; I can express this in terms of known quantities for example. So, I know what is tails sitting angle here, I know what is wing angle of attack which I will be measuring using an angle of attack sensor there right. So, I know alpha, I know wing setting angle. So, can I express this in terms of these parameters and I can model this downwash you know it is similar to that of alpha i right, do you remember.

So, this epsilon can be modeled. So, it is twice that of alpha i at the downstream. So, this is from the lifting line theory, this is 2 times of C L upon C L of wing right, because this epsilon is because of the wing here. So, 2 pi e AR. So, this is a general estimate this. This can be expressed as a function of angle of attack. So, dou epsilon by the dou alpha times alpha is equals to 2 times C L 0 of wing + C L alpha wing times alpha upon 2 pi sorry pi e AR.

So, from this we can say that epsilon 0 is equal to 2 times C L 0 of wings upon pi e AR and epsilon or dou epsilon by dou alpha is equals to 2 C L alpha of wings upon pi e AR, where e is a aspect sufficiency factor, AR is the aspect ratio fine okay. So, once we know epsilon here, we will be able to. So, epsilon is also known, once we know C L alpha and the wing geometry we will be able to find out the induced downwards because of the wing at the tail right.

So, now we know if we know the all this quantity is like say i of t alpha and epsilon. Now, I would like to express this tail angle of attack in terms of these known variables. So, alpha t is equals to alpha right. So, this is alpha minus this particular angle will fetch me this particular angle made by V infinity with respect to a fuselage reference line right. That angle plus i of t will be my total angle of attack okay. So, alpha minus epsilon.

So, epsilon is because of the downwash here. So, indicated by this green line alpha minus epsilon plus i of t is the tail setting okay. So, what I can do further. So, alpha of t is equals to 1 - dou epsilon by dou alpha times alpha + epsilon 0 + i of t is not it am I correct or not. So, if I substitute this particular expression in that equation what is alpha t is equals to about epsilon can be minus okay.

So, this must be i of t - epsilon 0, so, this is i of t - epsilon 0. So, that epsilon is equals to - epsilon 0 - dou epsilon by dou alpha time alpha. So; if I substitute that and take alpha common out. So, separating the coefficients of alpha and constant I will have this particular expression okay. So, with this understanding we now can proceed with the modeling of moment for this particular configuration wing and tail configuration

And we will find out what is C m 0 and C m alpha for this configuration right, at the same time, we will also look at what is the total lift from this combination and then also the C L 0 and C L alpha of this configuration, wing and tail combination. So, the lift at the tail will be acting perpendicular to V infinity prime which is L of t right. There will be little drag, but we are neglecting that drag that horizontal component.

Or the vertical offset of Cg is not considered right and we proved that that is not affecting much we know with the flat plate flight we demonstrated that if that is small it is insignificant in for static stability contribution, is not it. So, I have L of t but I do not have a moment here, is not it. Do I have a pitching moment or moment about aerodynamic center. Why because we are using a symmetric tail right, in general it is symmetric tail.

So, made out of symmetric aero. So, we do not have C m ac there. So, now the wing lift. So and assuming a small angle of attack, so, L w cos alpha will be L L w lift of wing multiplied with this offset with respect to Cg contributes towards pitch up moment right. So, now taking moments about Cg and liftoff tail contributes towards pitch down moment am I correct or not.

Liftoff tail multiplied by the offset between this, offset is what this particular distance which is like the total distance of tail with respect to leading edge of root chord subtracted by the distance of Cg or the distance of Cg with respect to leading edge of the route chord. If I subtract this distance from this total distance, what I have is this particular distance I mean the momentum between Cg and the aerodynamics center of tail okay.

So, now what is the total lift of the aircraft a by c stands for aircraft here or say this is nothing but total lift of the aircraft L is equals to lift of wing + lift of tail from the principal. So do you remember those assumptions like one of the assumptions talks about principle of superposition right. So, the total lift here from that assumption it is equals to lift of wing plus lift of tail. So this is equals to so half rho V square.

So aircraft freestream velocity is equals to the wing freestream velocity times the reference area of the wing is reference area of the aircraft, C L is the C L of the aircraft is equals to half rho V square S times C L of wing + half rho V prime square, is not it. So, V infinity prime square infinity or say V infinity prime square which is at the tail, this is the velocity of the tail times S of the tail right.

So, the lift generated by the tail is due to the area of the tail is not it, planform area of the tail and the velocity at the tail, the velocity at the tail, we figured out it as V infinity prime which is modified due to the downwash and then the reference area of this tail is nothing but planform area of the tail, which is S of t right times the C L of tail here okay. So, now, if I simplify this further.

So, the C L of the total aircraft is C L of wing less half rho V infinity prime square upon half rho V infinity square right times S of t upon S times C L of tail okay. So, this particular factor know velocity at the tail upon velocity of like dynamic pressure with the tail upon dynamic pressure with the wing, this particular factor is known as tail efficiency factor okay. So, this can be further written as C L of wing plus efficiency factor which is eta of HT times S t upon S times C L of tail okay.

This is one important expression fine. So, further if I want to know what is the C L 0 of the total configuration and C L alpha of the total configuration, what I can do is I can express the linear regime of angle of attack the total aerodynamic C L is equals to as a function of alpha which is C L 0 + C L alpha of the total aircraft times alpha of the total aircraft which is equals to C L 0 of the wing + C L alpha of the wing times alpha of the wing is not it.

From the linear approximation; from the wing + eta of tail times S of t upon S times C L alpha of tail times alpha at tail right. This C L at tail is varied because of alpha at tail, is not it. And we know that since it is a symmetric tail, we have C L 0 of the tail is 0 okay. Okay so, this is alpha and V infinity right, this is poor tricky. So, by comparing, so, what is alpha of t.

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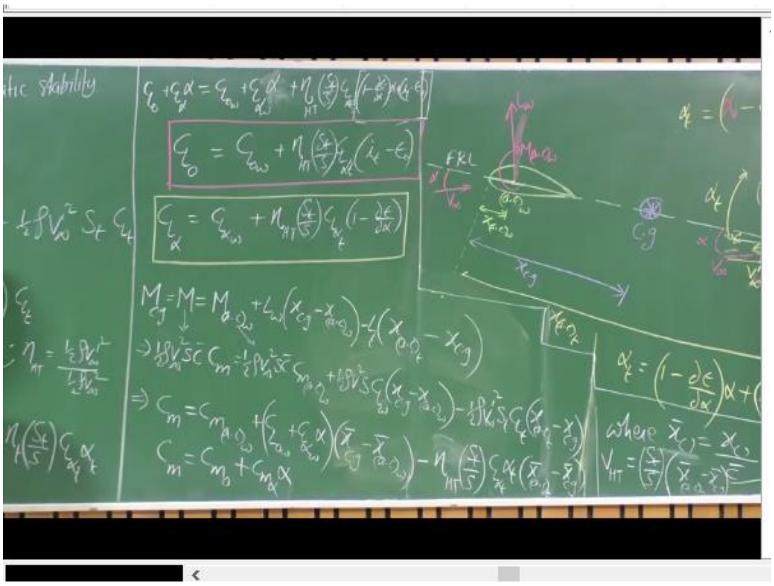

So, this is C L 0 + C L alpha into alpha of the entire aircraft is equals to C L 0 of wing plus C L alpha of wing multiplied by x bar Cg - sorry into alpha wing + eta, this is nothing but alpha, alpha of wing is nothing but alpha here is not it. So, eta of horizontal tail times S t upon S times C L of tail multiplied by alpha of tail, what is alpha of tail 1 - dou epsilon by dou alpha right, alpha into alpha + i of t - epsilon 0 okay.

So, by comparing the constants and coefficient the C L 0 of the entire aircraft is equals to C L 0 of wing + eta HT of horizontal time multiplied by S t upon S C L alpha of tail multiplied work i t - epsilon 0 for. So, you can assume epsilon 0 is small otherwise epsilon 0 in general is positive you saw that expression right. So, epsilon 0 is 2 times C L 0 upon 2 pi e R. So, that is a positive expression.

So, you can keep it as it is. So, this is one equation that talks about total lift of a wing and tail combination okay. So, and then we will talk about C L alpha by comparing the coefficients of alpha what I have is C L alpha of wing plus. So, I have the coefficient for alpha C L alpha of wing and there is a coefficient for alpha from the tail contribution which is horizontal tail and S t upon S times C L alpha of tail times 1 - dou epsilon upon dou alpha okay.

This is the contribution from tail here, okay sorry wing and tail combination. So, we have the C L alpha of wings. So, for wing alone these 2 terms is disappear, is not it. So, we just had this for wing alone contribution total aircrafts C L 0 is wing C L 0 and total aircraft C L alpha is wing C L alpha okay. Now we need to talk about moment right. So, this is the lift details about the C L 0 and CL alpha of the entire aircraft.

Now let us talk about the pitching moment about Cg. So, about Cg the y axis is into the board here, right. So, y axis is into the board. So, anything any moment that creates pitch up, nose up moment for this UAE nose up motion is considered as positive and nose down**,** the moment that creates nose down motion is considered as negative moment, right.

So, now the moment about Cg of this aircraft is nothing but moment is equals to a moment about aerodynamic center of wing right plus, lift off wing, right, because lift of wing is contributing towards nose up motion right, nose up motion. So, lift of wing times the corresponding distance between the Cg and the ac, right. So, this is the distance. So, I am subtracting x ac from x Cg, what I have is the momentum between ac and Cg here.

So, multiplied by x Cg, I am subtracting x ac from x Cg x ac of wing okay. So, this is the pitch up moment and then because of the wing, and then the tail has lift, it tries to there is no pitching moment coefficient about the aerodynamic center because of the symmetric nature. So, we have lift at the tail that contributes towards no zone moment. So that is a negative moment right.

So, which is equivalent to minus lifted tail times x bar x ac of tail I am subtracting the Cg distance from ac of tail, right this is ac of tail x ac of tail - x Cg okay. So, this equals to half rho infinity square S C bar times C m of the entire aircraft is equals to half rho V infinity square S times C bar C m ac of the wing right, this corresponds to this particular term right.

This corresponds to the moment about aerodynamic center whereas, the reference area is S and the reference characteristic length is the mean aerodynamic chord C bar, and the velocity of the wing is aircraft is equal to the velocity at the wing itself right. So, plus half rho V square S times C L of wings multiplied by x Cg - x ac of the wing minus. So, the lift at the tail is because the flow at the tail right.

The flow at the tail is half rho V prime square but half rho V infinity prime square times the S of t right reference areas should be S of t multiplied by C L of tail times the momentum x ac of tail - x Cg okay. So, if I divide this entire equation by half rho V square S C bar. So, I have it in the non dimensional form C m is equal to, so, C m ac of the wing + C L of wing which is

C L 0 of wing + C L alpha of wing times alpha multiplied by x bar or x bar C g - x bar ac of wing right.

Where x bar is x upon C bar okay any x bar x subscript something by C bar is nothing but x bar subscript okay. So, minus what I have is half rho V infinity prime square upon half rho V square is S of eta, eta of horizontal we just discussed. So, where here so where eta of horizontal tail is equals to half rho V infinity prime square upon half rho V infinity square okay.

What is C L of tail here, S of t upon S okay. So, C L alpha of C L of tail is C L alpha of tail times alpha at the tail right, we go symmetric airfoil C L 0 is 0 of the tail multiplied by the momentum x bar ac of tail - x bar C g. So, this is x bar ac of tail. So, this is a momentum here right. So, where x bar is equals to x upon c bar. So, you can have anything in the subscript. Yeah, whether C g or ac or x ac of t of wing okay, it is a non dimensional length, is not it. So, I am dividing length upon C bar which is the characteristic length what I get is a non dimensional length here.

So, this particular term is sometimes also called as S t upon S multiplied by this particular term is also called as tail volume ratio. So, we HT is equals to S t upon S times L t L t upon c bar where L t is a distance between ac of tail and C g. So, what I will call it as x bar ac of tail - x bar C g, but I am not going to use it for the time being. So, when we talk about the iterations, then we will try to use it till then I would like to keep it as it is right.

So, I am not using this horizontal tail volume ratio, this is called tail volume ratio which is for the horizontal tail and we also have something called vertical tail volume ratio that we are going to soon discuss about it okay. So; now comparing the constants and coefficients of alpha. So, this C m can be exposed as the C m can be of the entire aircraft in the linear regime can be expressed as C m 0 + C m alpha into alpha.

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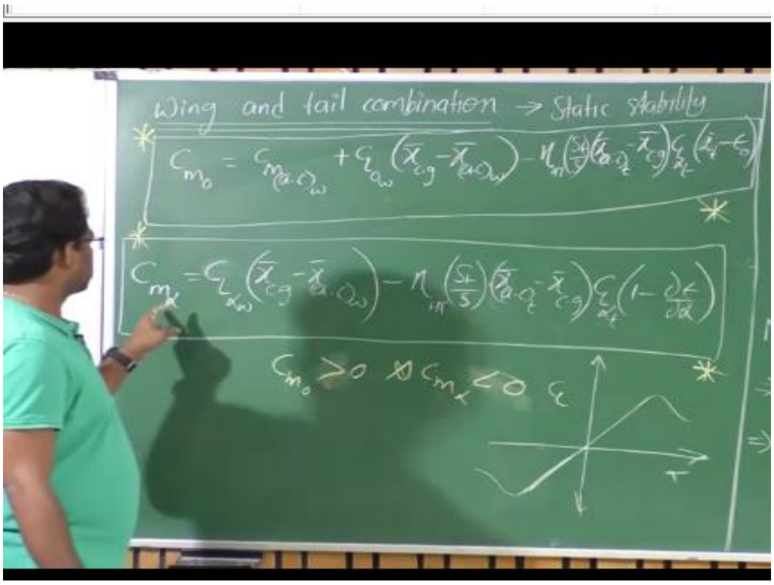

Now, from these 2 equations, comparing the constants and coefficient what I have is C m 0 of the aircraft is C m ac of the wings + C L 0 of wings times x bar C g - x bar ac of wing right. So, this is contributed the Cg behind ac. This is contributing for a positive C m 0 am I correct or not. And then this is minus eta of horizontal tail, S t upon S times x bar ac of tail - x bar Cg right.

So, this is x bar ac of tail - x bar C g multiplied by C l alpha of tail C L alpha of tail times 1 - so i of t - epsilon 0 okay. This is my expression for C m 0. So, you need to whenever you it is required you should be able to derive it quickly. Now, it happens with practice you need to practice this multiple times. And you need to understand the contribution from each and every term.

So, that you can easily get back to this equation. So, we will do it couple of times. So, that you will also be comfortable with the terms here right. So, this is C m 0 whereas C m alpha of the entire aircraft is from C L alpha of wing, multiplied by x bar C g - x bar ac of wing right and - of eta of horizontal tail and then S of t upon S times x bar ac of tail - x bar C g right times the C L alpha tail multiplied by 1 - dou epsilon by dou alpha right.

This is the C m alpha of the entire aircraft. So; for this aircraft to which is an important expression. So, please try to derive multiple times, so that you get used to this equation. So, in this equation, if you can see, what we need our aim is C m 0 has to be greater than 0 and C m alpha has to be less than 0. So, look at this particular expression. So for C m alpha has to be less than 0 say if the Cg is behind the aerodynamic center okay, the Cg is behind the aerodynamic, this contributes towards positive witching moment, right.

So this first term contributes towards positive witching moment because C L alpha of course, we know it is positive. So, eta is positive S t upon S is positive, and then the tail is behind the Cg, right this is positive, this contributes towards positive. And C L alpha is positive. So, 1 - dou epsilon by dou alpha is always less than 1 less than or equal to 1 at maximum, right. So, yeah, second on this negative have it, so, you need to choose this particular number strong enough to overcome this positive right.

So, the offset between these 2 should not be very high okay. So, the tail volume should overcome this particular positive term. So, if you have to design this, this particular term is known as tail volume ratio as we discussed. So, you need to choose a particular tail volume ratio in order to make this stable because, it is obvious that when you have wing and tail at an offset the Cg will of course, be behind right, is not it, you have to mount your engine and you have to place your batteries in such a way that you bring this particular Cg as close to this aerodynamics entire as possible okay.

That is what. So, now, looking at the C m 0 term the C m ac of the wing is let us say if you use a cambered airfoil this is negative right. So, in a previous lecture when we talked about reflects airfoil, you saw that you witness that the C L alpha is very less comparatively is not it C L alpha of the airfoil itself is less, why because you need to compromise with the camber towards the trailing edge for a reflex airfoil it will be upwards right to provide a pitch bend upwards.

So, bent upwards. So, that it will obstruct the flow and it pushes down it gives a couple at each and every angle of attack. Due to that we are compromising with the camber that means you are compromising with the C L alpha, which we have witnessed here. But in case of cambered airfoil, you have very high C L alpha right compared to the reflect airfoils in the normal airfoil.

But you have to compromise with that negative pitching moment here right. So, you have C m ac negative here and if this is positive, right this contributes towards positive and when you have this tail setting angle, so, if this distance is very less competitive, let us assume right. So, in order to overcome this moment about aerodynamic center this is negative right. So, say if this distance is very less, then you need to have a tail sitting angle here right.

So, why because this is positive, this is positive and this is positive we know that because aerodynamic center of tail is behind the Cg and then S of t these 2 are positive quantities C L alpha of course, we know it is positive. So, i of t - epsilon 0, epsilon 0 is positive. So, if i of t is negative this entire term will be negative times negative is like it contributes towards pitch up moment.

That means, i of t you need to have negative setting angle, what does it mean. So, i of t is positive above when incline above fuselage reference line. So, i of t is positive when inclined above the fuselage reference line right is negative when it is inclined or oriented below the fuselage reference line okay. So, that means you need to set your tail in with a negative orientation with respect to fuselage reference line.

So, by doing that what you are exactly doing when there is a flow, so, you are obstructing the flow there is a downward force. So, this downward force even at 0 angle of attack, right will create a pitch up moment about the Cg, is not it. So, that continues, that down that C m 0 the contribution of tail towards positive C m 0 continuous whatever the angle of attack it is right.

So, this becomes what, positive quantity here right. So, with the proper value of this it becomes more positive you can remit higher angles of attack as well okay. So, that means more value of C m 0 more the trim value that I can achieve, is not it. So, that this particular quantity alone can overcome this particular limitation, let us say in the case where you want to have i of t 0 right, you do not want to because you are again at the expense of track see.

For the symmetric wing whether is it a positive angle of attack negative angle of attack, it produces force, same force, am I correct or not, but in different directions, when it is in the positive angle of attack, it will produce upward force, when it is in the negative angle of attack, it produces downward force, that is why the C L variation with alpha for this symmetric aerofoil will be almost symmetric about angle of attack.

So, if this is C L variation with angle of attack right for symmetric airfoil. Even the stall characteristics should also be seen right, this is the C L versus it is just a mirror image here.

So, if you trim it at negative angles of attack here, it will produce negative lift negative C L that is in the downward direction, okay. So, now, when you put it at a negative i of t it produces a downward force at the same time it also increases drag because of that.

So in some cases, where you do not want to waste additional energy to overcome the drag, what you try to do is to put to maintain i of t 0, if I do that, this particular term vanishes, right, but if I maintain some adequate distance between Cg and aerodynamic center, so I have C L 0 for a camber wing positive, right. So what I can achieve is a positive value, if this is good enough, if the distance is good enough, you can overcome the C m ac negative.

So, you will still able to trim it positive angles of attack without i of t right for a cambered airfoil. If you take a symmetric airfoil C m 0 is 0 altogether, if your wing is symmetric C m 0 is 0. In that case, what you need to do, you need to give i of t for the tail always am I correct or not, if you give i of t you get positive C m 0 i of t negative for a symmetric airfoil. Am I clear.

Because for symmetric airfoil both these terms are 0, for the wing I am talking about the wing, if it is a cambered wing, right. So; irrespective of that, whether symmetric or cambered airfoil. So, this still remains negative am I correct or not. Because see C L alpha whether symmetric or cambered it does not matter, it has its own C L alpha right and all these terms are positive right.

So this becomes negative, even though this Cg is behind ac right. So, this term may be positive, but still if you choose a proper value of tail volume ratio, you will be able to anchovies the C m alpha negative okay. Whether it is cambered or symmetric wing it does not matter. But here for cambered wing you need to choose i of t negative is not it, why because or you need to choose this distance in such a way that this particular value becomes positive right.

So, either you choose i of t negative that makes C m 0 positive, because this is negative strong negative for Cambodia airfoils okay, so that is a conclusion from here. So, for a wing and tail combination, this is what, it has to be the C m alpha, you get it from the wing combination as well as tail combination right**.** Now let us find out a location. So you said this C m alpha, you see is about Cg is not it. Is it not about Cg, C m alpha is about Cg is not it.

Now vary the location of Cg in such a way that the C m alpha. So, find out the location of the Cg for which the C m alpha is 0 right. We will try to find out that location right. So, that particular location about which the entire aircraft pitching moment is independent of angle of attack is called neutral point, right. Let us consider a location, some location. It is also location of Cg right, say this is NP, call it as NP.

So, this is the location of Cg about which the pitching moment is independent of angle of attack of the entire aircraft. What does it mean?. So C 0 alpha is 0, you have C m 0 but there is no C m alpha about that particular Cg location okay. So let us figure out what is that okay. So, I am erasing this particular part right, and see if you can notice this particular portion C L 0 + C L.

This C L 0 of the entire aircraft for this wing and tail combination, your C L 0 of wing positive if it is a cambered airfoil, if it is symmetric airfoil this is 0 right and then if there is no i of t this term most likely vanishes without any epsilon 0. Assuming epsilon 0 is very small so this term vanishes the C L 0 of the entire aircraft is 0 in that case right. If it is a cambered airfoil, if it is symmetrical airfoil this is 0 right.

And then if there is no i of t, this term most likely vanishes without any epsilon 0 assuming epsilon 0 is very small. So, this term vanishes the C L 0 of the entire aircraft is 0 in that case right. If it is a cambered airfoil that C L 0 is positive right. And if you i of t negative, this term contributes negative, because you want i of to be negative for C m 0 to be positive, am I correct or not.

So, if that is the case, the overall lift of the aircraft decreases why because here near the tail you are producing a downward force right, near the wing you are producing an upward force. So, there is a resultant force, which is less than that of wing upward only upward force right. Am I correct?. So, again you have to compromise at C L 0. If you give it a negative tail setting angle you should also compromise that C L 0 here.

Tail setting angles does not going to influence this C L alpha, lift curve slope of this wing and tail of the total aircraft okay. So, let us figure out what is the corresponding location of Cg for which C m alpha is 0. So, this is that particular point is called neutral point denoted by X N P.

So, this particular if I substitute 0 in if I equate this equation to 0, right. Then, C m alpha is 0, which mean. So, at that particular, the Cg location, about which C m alpha is 0 is called neutral point. So, in that case x Cg becomes X NP okay.

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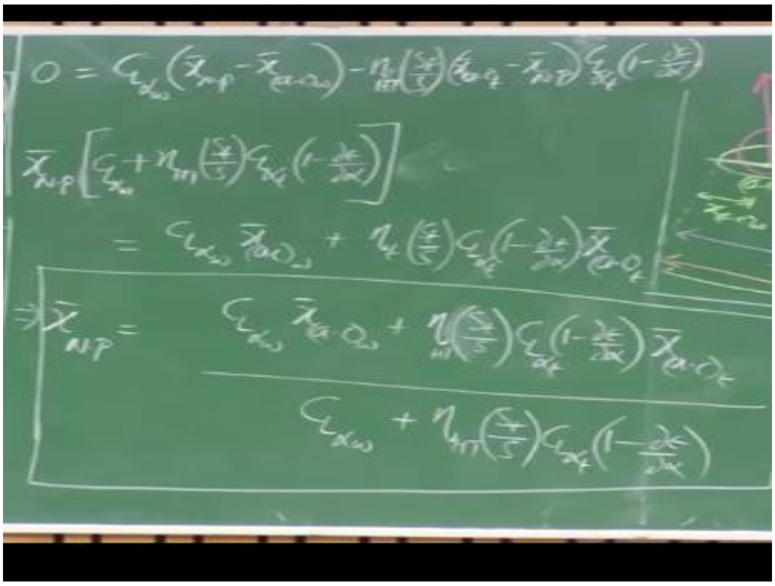

So, what I have is 0 is equal to C L alpha of wing times x bar NP - x bar ac of wing right - eta of tail horizontal tail S t upon S times x bar ac of tail - x bar NP neutral point right, multiplied by C L alpha of tail - multiplied by 1 - dou epsilon upon dou okay. So, can I find out the particular NP, can I solve it for x bar N P from this equation?. So, x bar NP multiplied by this is C L alpha wing okay. So, C L alpha wing + eta HT of horizontal tail and S t upon S C L alpha of tail multiplied by 1 - dou epsilon dou alpha.

This is equals to C L alpha of wing, times x bar ac of wing + eta S t upon S times C L alpha of tail multiplied by 1 - dou epsilon by dou alpha times x bar ac of tail right okay. So, this implies x bar N P neutral point is equals to C L alpha of wing, times x bar ac of wing less eta of horizontal tail S t upon S C L alpha of tail or say yeah C L alpha tail - multiplied by - dou epsilon by dou alpha times x bar ac of tail upon C L of wing + eta of horizontal tail times S t upon S times C L alpha of tail multiplied by - dou epsilon by dou alpha right.

This is a corresponding location of neutral point for which C m alpha is 0. If you go beyond this neutral point location, if you look at the Cg of the configuration behind the neutral point, what happens is, this will become positive okay. So, this becomes positive, which means the system becomes unstable. Do you want me to prove that, okay, I can prove that. So you have a neutral point here.

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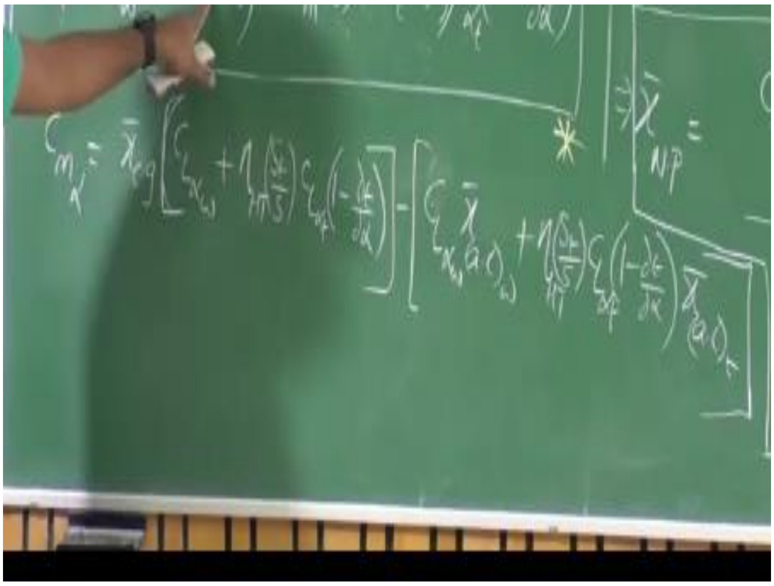

Now so this C m alpha is equals to. So, take out the Cg terms here x bar Cg right x bar Cg times C L alpha of wing okay. So, this is minus into minus becomes plus, + eta of horizontal tail S t upon S multiplied by C L alpha of tail okay. And then the correction factor, 1 - 2 dou epsilon by dou alpha right. So, I am just separating Cg terms in the options here. So, this will be like, okay. So, I have the Cg multiplied by C L alpha is here.

And then the Cg multiplied by the entire term minus of minus, it becomes plus, I have eta S t upon S C L alpha 1 - dou epsilon multiple by Cg is here. And then I have taken Cg outside Cg okay. So, and then what I have is - of C L alpha and then this minus term. So, I am taking minus out here. So, this - C L - of; so, this is C L alpha wing times x ac of wing + eta S t upon S eta of horizontal tail times C L alpha of tail times 1 - dou epsilon by dou alpha times x bar ac of tail okay, this is a.

So, constant terms with respect to x Cg okay. So, just multiply this I have taken minus out so that C L alpha wing times x bar ac of wing is here. And then I have taken this minus out, so it becomes plus eta S t upon S C L alpha of tail times 1 - dou epsilon by dou alpha multiplied by x bar ac of tail. Am I correct. So, can I write this expression right, say can I write this expression as x bar N P, multiplied by this particular factor okay, so from this expression.

So, this particular term is C L alpha times x ac of wing eta HT S t upon S C L alpha tail 1 - dou epsilon dou alpha multiplied by x ac of tail. So, this expression I can replace it by x N P times the denominator here right.

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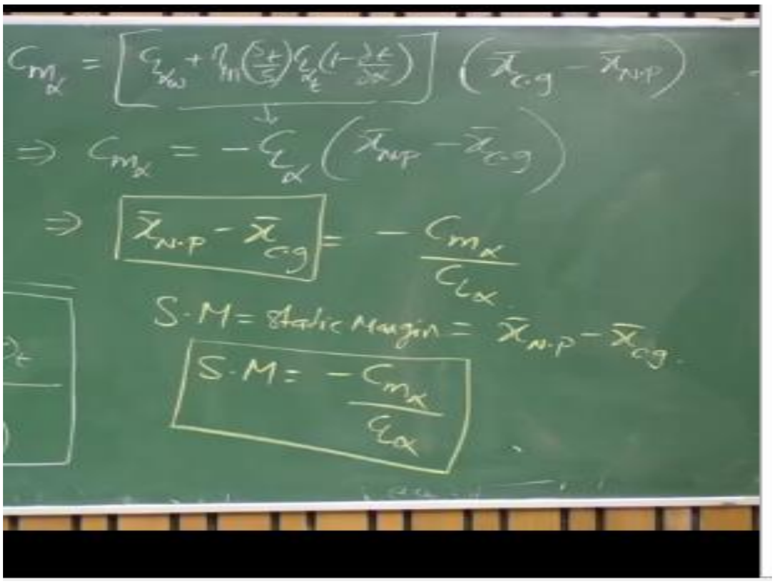

So, what I have is C m alpha of the aircraft is equal to. So, C L alpha wing + eta of horizontal tail S t upon S times C L alpha of horizontal tail right C L alpha of tail times 1 - dou epsilon by dou alpha here, is that what this denominator is right. And you have the same terms here. We have the same terms here okay I am taking the common multiplied by x bar Cg - x bar N P okay.

So, otherwise in other words, what I can say is, so do you remember this particular expression is nothing but total C L alpha of the aircraft. So, this is C m alpha for the aircraft, just go back to your previous expression, and see what is a total lift curve slope of wing and tail combination?. This is the left curve slope of the total aircraft, is not it, this is nothing but C L alpha of the total aircraft.

This implies so, C m alpha is equals to - of C L alpha of the entire aircraft here C L alpha of this entire aircraft. This particular term multiplied by x bar NP - x bar Cg okay. So, now will you appreciate my initial statement. So, if this N P; if your Cg crosses this NP. That means this becomes larger than this. So, this particular term is negative. So, the expression becomes positive.

So, C m alpha becomes positive. So, Cg of the configuration to in order to have longitudinal static stability your Cg should be ahead of this neutral point okay. So, this is the thing. So, this particular thing, x bar NP - x bar Cg is equals to - of C m alpha upon C L alpha of the entire term. So, this particular the distance between this neutral point and center of gravity is called static margin right.

Static margin SM is static margin. So, positive static margin means what, neutral point is behind the Cg or Cg is ahead of the neutral point of wing and tail combination right. So, static margin is equals to x bar NP - x bar Cg okay. So, positive static margin is behind the neutral point, this quantity is less than this neutral point x bar NP right. Generally it is given in percentage 10%, 5%, 15% starting.

That will solve, while solving examples, you will be more comfortable with the terms right. So, this is equal to - C m alpha upon C L alpha for a statically stable aircraft C m alpha is negative, right, negative of negative is positive. So, you will have positive static margin, which means the Cg should be ahead of the neutral point. So, neutral point is nothing but the aerodynamic similar to that of aerodynamic center of entire aircraft okay.

So, for a statically stable aircraft C m alpha is negative and a negative of negative is positive. So, C L alpha we know for the entire aircraft is positive, of course. So, if this is positive, which means x NP is this positive terms plus C L x Cg right, which lies behind the neutral point, lies behind the Cg location. So it is a limiting condition for static stable flight and static unstable flat okay.

And this is called static margin, so static margin as, again, is equals to - of C m alpha upon C L alpha. Can you get something out of it, but exactly is this material part. Let us look about it in more detail. This is my perspective again right. So, you understood right, it is like a limiting condition similar for a wing alone configuration this neutral point is nothing but there is no tail here, right, this is nothing but x ac of wing.

Am I correct, for a wing alone configuration, if you substitute C L alpha t is 0 x ac of t is 0 right. So, this becomes 0 and C L alpha of t does not make sense, there is no C L alpha of t. But these 2 terms disappear, this term and this term disappears. What I have is x NP is equals to x ac for wing alone wing**.**

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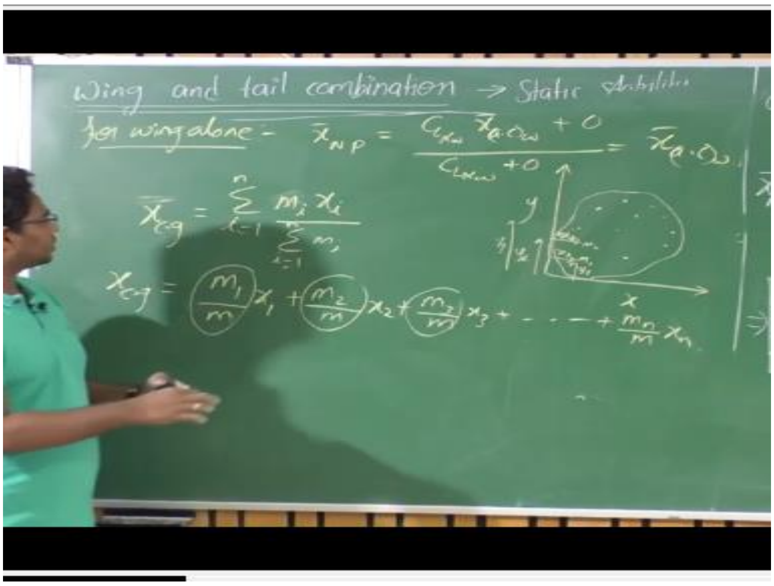

X bar N P is equal to C L alpha of wing times x bar ac of wing + 0 upon C L alpha of wing + 0, which is equals x bar ac of wing. So, for wing along configuration neutral point is nothing but aerodynamics center of wing okay. So, in that case, in order to have positive static margin right for a statically stable case, the Cg should be head of this neutral point, which is nothing but the Cg should be head of the aerodynamic center of the wing okay.

That is the reason why even during our flat plate flight demonstration we have shifted by adding a small way we shifted our Cg head of the neutral point there, which is aerodynamic center in that case right okay. So, now let us look at this neutral point in more detail. So, what, so we have calculated Cg earlier right. How have we calculated? So, x bar Cg in general is equals to, in general fine, if you consider a mass right, which is made up of m 1 m 2 right.

So, you have y axis and x axis. So, this is at a distance m 1, is at a distance of x 1, m 2,is at a distance of x 2 right. And similarly m 1 is at a distance of y 1 and m 2 is at a distance of y 2. So, m 3 is at a distance of y 3, so on right. So in this case, sigma I stands from 1 to n. So, m i x i upon sigma m i right. So, which I can say, so x Cg of a given body is like m 1 upon m, where m is sigma m i right, times x 1 + m 2 upon m times x 2 + m 3 upon m times x 3 + 1, up to m n upon m times x n.

So, is it not a weighted average. So, you have weights here to each and every x 1, x 2, you have weights here, weighted average. So, Cg is nothing but the weighted average of the locations, weighted average of masses, is not it, here weighted average by the masses right. Similar to that can we look at this expression x NP.

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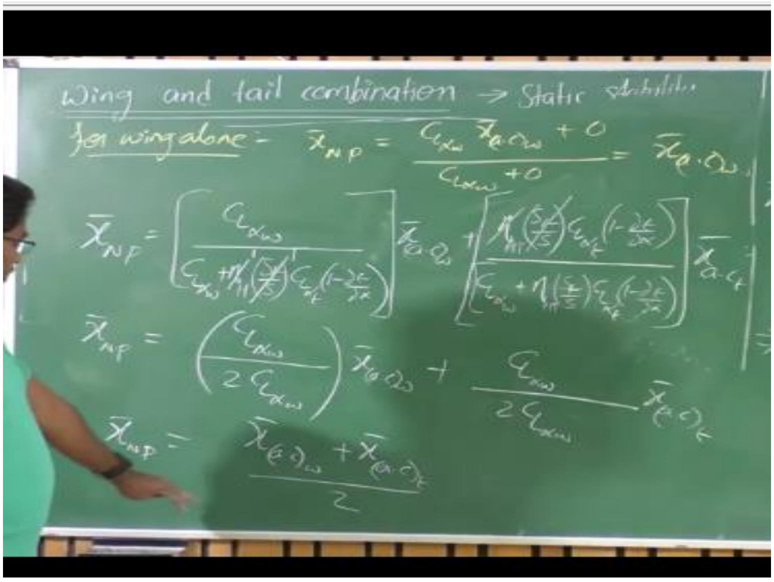

So, x N P is equals to C L alpha of wing upon the total C L alpha of the aircraft, which is like total mass of the aircraft, is not it. So, total mass of a body is like C L alpha wing + eta of horizontal tail S t upon S times C L alpha of tail time 1 - dou epsilon by dou alpha am I correct. So, is it not that weight total C L alpha of the aircraft. So, can I express this as the total C L alpha of the aircraft right.

So, this multiplied by x bar ac of wing right + eta of tail horizontal tail S t upon S times C L alpha of tail multiplied by 1 - dou epsilon by dou alpha divided by again this total C L of the aircraft, which is C L alpha of wing + eta HT S t upon S times C L alpha of tail multiplied by 1 - dou epsilon by dou alpha right. So, this entire thing multiplied by x ac of tail x bar ac of tail. Am I correct or not.

So, neutral point is the weighted average of C L alpha. Lift curve slope of wing and lift curve slope of tail. Do you accept this?. Dou you appreciate that. So, let us understand it in a bit more detail right. I think you are not happy with this. So, let us understand it with a bit more detail right. What exactly is this or these terms now. So they are disturbing a bit, is not it. So, let us assume a case where I have a wing here right, there is 1 wing.

Let us assume an identical wing, which is at a far away right. I have 1 wing right. I have the same thing, similar wing, which is located at a distance in the downstream, I have 2 such wings right. Do you follow that. So, the area, which means there is planform area same and the C L alpha of that wings are same okay. And assume that there is very minimal downwash, there is 0 downwash okay. In that case what happens.

So, I have a main wing with C L alpha right. And then, this particular downwash is 0 that means eta is equals to 1. So, in that case, eta is equals to 1 and S t upon S is 1, because I have 2 identical wings, 1 as wing and 1 as tail. So, this becomes 1. In that case, am I correct, and C L alpha of tail is nothing but C L alpha of win. Both are same, and there is no downwash, that is 0.

So, this entire expression will turns out to be C L alpha of wing times 2 times the C L alpha of wing. Am I correct or not multiplied by x bar ac of wing x okay. Similarly plus. So, this x neutral point in that particular case where you have 2 identical wings as wing and tail combination okay. So, and then here eta of tail is 1, right, because there is no downwash. So, S t upon S is again 1. So, C L alpha of tail is nothing but C L alpha of wing am I correct.

So, downwash is 0, epsilon is 0 divided by again, C L alpha 2 times of C L alpha of wing times x bar ac of tail. So, this is nothing like this, in this case, the neutral point is nothing but x bar ac of wing + x bar ac of tail, that is nothing but wing 2 upon 2. It is just the midpoint of separation. It lies at the midpoint of the distances between x aerodynamic center of wing and x aerodynamic center of tail.

How many of you appreciate this okay. So, that is nothing but the neutral point right. So, what are these terms here. So, now you can understand right, these are nothing but correction factors to the C L alpha. Am I correct or not. So, when there is no downwash that means, this is gone and this is 1. And this is nothing but the correction factor for or normalizing factor for with respect to the wing right.

When you have 2 similar wings, this becomes 1. This is just a normalizing factor. So, this will be equal to the wing C L alpha itself. So, this entire term is nothing but a correction factor to the CL alpha of that particular geometry. If it is a tail, if it is just an identical wing that is nothing but one and the same. If the downwash is minimal, if the downwash is 0 and this efficiency is 1 right. Do you appreciate that.

So, in that case, it is nothing but the midpoint of the distances separating these 2 wings right. So, if you do not accept this result we will try to demonstrate it right. So, in the next lecture, we will have a demonstration of 2 identical wings, and I will prove that the neutral point is the distance like midway between their aerodynamic centers okay. Thank you.