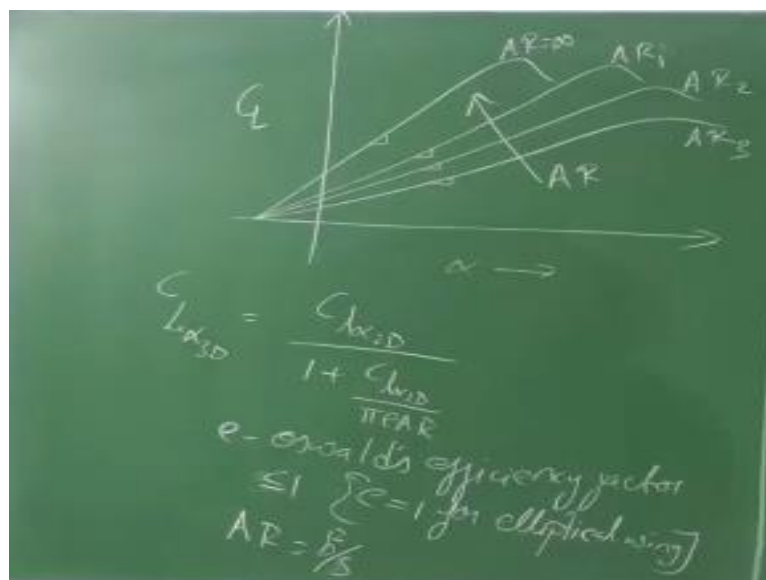


**UAV Design - Part II**  
**Dr. Subrahmanyam Saderla**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology-Kanpur**

**Lecture - 11**  
**Numericals Cont.**

Dear friends, welcome back. In our previous lecture, we discussed about the characteristics of infinite wings as well as finite wings and we related the lifting characteristics of an infinite wing with that of a finite wing, right.

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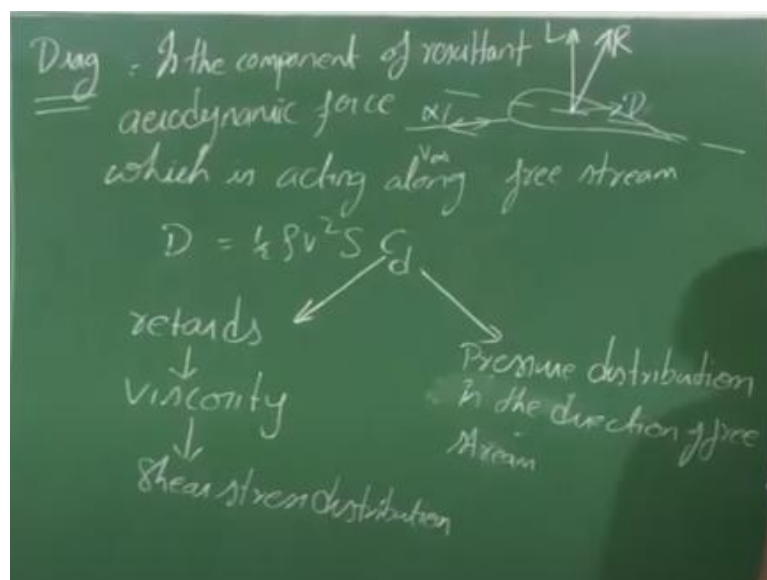
So we considered a lifting line theory from which we figured out that as the aspect ratio increases right. So this is the increasing order of aspect ratio, so the  $C_L \alpha$  decreases, right. The lift curve slope decreases. So this is a wing of infinite aspect ratio or for aerofoil. So this lift curve slope represents a lesser aspect ratio compared to that of an aerofoil.

So this is where the finite wing starts, right. This plot, from this plot we are talking about finite wing. So this is the variation of  $C_L$  with angle of attack. So if you can observe so as the aspect ratio increases  $C_L \alpha$  increases here, right. So and we have related  $C_L \alpha_{3D}$  is equals to  $C_L \alpha_{2D}$  upon  $1 + C_L \alpha_{2D}$  upon  $\pi e A.R.$

Where  $e$  is the Oswald's efficiency factor and is equals to 1 for and the limits are less than or equal to 1 where  $e$  corresponds to. So  $e$  is equals to 1 corresponds to a elliptical wing, right. So we know A.R is the aspect ratio of the wing, of the finite wing, okay. Right? So now let us look at what how can we relate what are the drag characteristics. We have not discussed much about drag here in the previous lectures.

So let us now relate what are the drag characteristics of a infinite wing as well as a finite wing. Is there any relation between this finite and infinite wings? So that is what we are going to do right now.

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So we know drag, right? So do you remember? So due to the resultant pressure distribution, so due to so when we consider an airfoil in the flow which is inclined at an angle of attack  $\alpha$  right. So there will be pressure distribution as well as shear stress distribution resultant of that over this surface will be an aerodynamic force called the resultant aerodynamic force.

And the component of this resultant aerodynamic force acting perpendicular to free stream is considered as lift. And a component of aerodynamic force, this resultant aerodynamic force which is acting along the free stream is considered as drag, right? So drag is the component of resultant aerodynamic force which is acting along or in the direction of  $V$  infinity, okay.

Fine, and we know drag is given as  $\frac{1}{2} \rho V^2 S C_d$ , right. So we are talking about infinite wings. We are considering an aerofoil in the first place. So let us consider this  $C_d$  as like for infinite wings, the drag coefficient is represented by small lowercase  $d$ , right  $C_{d}$ . So  $C_d$  is a drag coefficient. So what is the contribution? What do drag do in the first place to the flight vehicle?

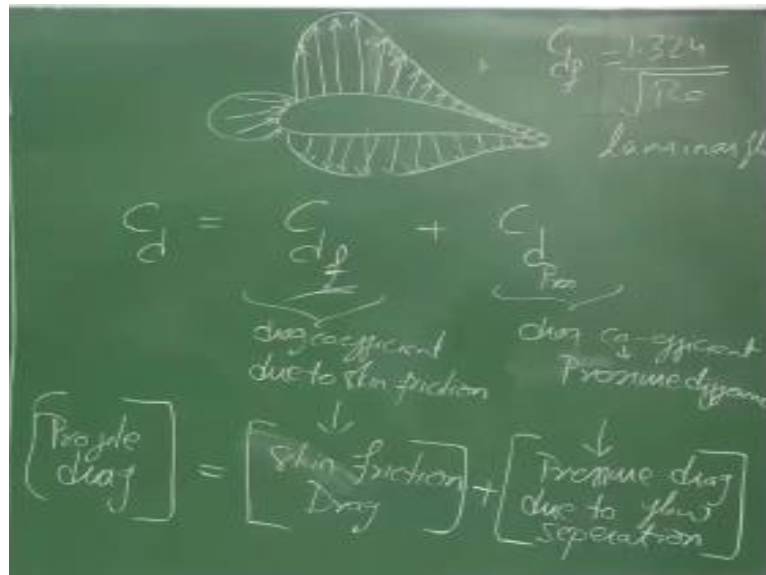
What does it do? It will retard the motion is it not? It is acting against the motion when we say it is the flow is in this direction at  $V$  infinity which means the body is also moving in the same direction with the same velocity under ideal conditions, right. That means the drag is acting opposite to your motion which is retarding your motion in the fluid, right. So there is so drag retards the motion.

So what do you remember as soon as somebody talks about retard retardation in the fluid, so what is the property that helps this to happen, property of the fluid that helps this to happen? Friction, fluid friction right? The fluid friction is called viscosity. So viscosity is the major contributor for the drag. Of course, there is pressure contribution as well. So viscosity creates shear stress is it not?

Shear stress distribution, okay. So we understood one component of this if not drag directly we can say drag coefficient. So is from the shear stress distribution right. So what is the other contribution? Let us say if there is a component of the pressure or the there is a pressure difference across the along the flow, there is a pressure difference across the body. That will also contribute towards drag.

If there is a higher pressure difference, higher pressure on the frontal part and the lower pressure on the back part. So that pressure difference along the flow will also create drag right, that pressure difference. So we also have pressure distribution of free stream.

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So do you remember this pressure distribution, a typical pressure distribution over an aerofoil, right. So where there is positive pressure acting in the frontal part and then negative pressure, right. So this is a typical pressure distribution. So higher the negative pressure what do you mean? Higher the negative pressure is a point that corresponds to higher velocity.

Am I correct or not? So that point corresponds to a lower pressure point. Higher negative pressure is nothing but lower positive pressure there, right. So there will be a component C, there is a pressure higher positive pressure here and lesser positive pressure on the backside. So this will also create, this pressure difference will also create certain force in the direction of free stream velocity.

That is known as contribution of drag coefficient due to or the pressure contribution of this pressure difference along the free stream towards the drag coefficient, right. Now let us consider do you remember in the ideal flow theory, when you talked about a cylinder, we assumed that there are stagnation points S 1 and S 2. There is this is the typical pressure distribution that we discussed.

Do you remember that? So we have positive pressure on front and back and negative pressure on top and bottom because we and the highest velocity is equals to twice that of free stream velocity at point P at 90 degrees, right. Is it not? So it is same like the distribution on top and bottom is same right in case of a ideal flow theory.

And then on left and right or in the direction of flow the distribution of pressure is also symmetric about this object, right along the free stream. So this will not create any drag on the object, right. But intuitively we all know that okay when there is an object in flow, irrespective of that there is certain drag know. Whether irrespective, so we there is the we this body object or this body experiences certain drag is it not?

So this particular though it is in flow and there is no drag and no lift, right. Such a condition is known as d'Alembert paradox, right. So that is that corresponds to ideal fluid theory or ideal, yeah. Yes, why we are discussing this. So but in reality even at zero angle of attack, so this is this we are talking about ideal flow theory.

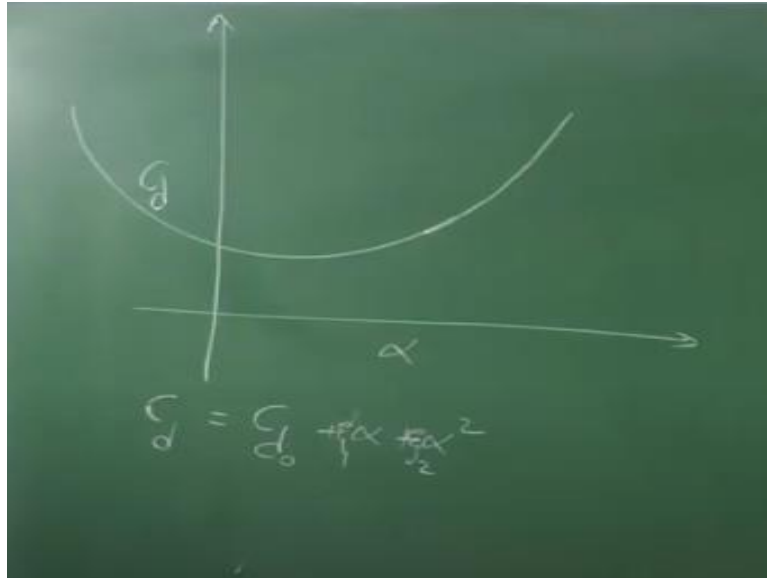
But in reality even at zero angle of attack, there is flow separation at the trailing edge, right that creates a pressure difference between the leading edge part and the trailing edge part, right. So the that pressure difference along the flow direction creates certain force on this body along the flow direction. So that contribution is towards this  $C_d$ , drag coefficient.

So this drag coefficient in the coefficient form can be the drag coefficient because of friction, that is skin friction and drag coefficient due to pressure difference, right. So what do you call this? It is in the coefficient form. We call it as drag coefficient due to skin friction we call it as drag coefficient due to pressure difference, right. Or so this first term is known as skin friction drag.

And the second term when multiplied by the other parameters here is known as pressure drag coefficient or pressure drag due to flow separation right due to flow separation okay. So these two together like skin friction drag and pressure drag due to flow separation is known as profile drag, right is known as profile drag here. So the  $C_d$  pressure okay. So for an aerofoil these are the only two contributions.

Like the two contributors for the drag for an aerofoil is or are the skin friction drag as well as the pressure drag due to flow separation okay.

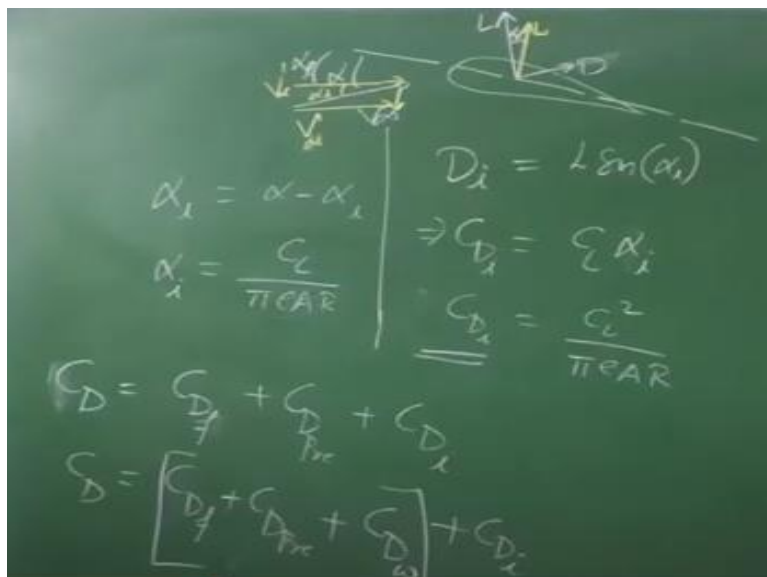
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So the  $C_D$  variation can be given as  $C_D$  at corresponds to  $\alpha$  zero plus  $\alpha$  plus  $\alpha$  square, right can be with some coefficients  $\zeta_1$ ,  $\zeta_2$ . So this is a typical pressure like drag coefficient variation with angle of attack in case of an aerofoil, okay. But what happens when we have a wing? Do you remember?

So we talked about lifting line theory which where there is a horseshoe vortex with a bound vortex and then associated with two trailing vortex closing by a starting vortex right characterized a high aspect ratio wings in terms of the lift curve slope, right. So we can use the theory to characterize that high aspect ratio. Am I correct or not? So while deriving the theory, we also witnessed that there is an induced angle of attack, is it not?

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For example, consider an aerofoil, which is oriented at certain angle of attack  $V$  infinity right at  $\alpha$ . But due to this downwash what happens is due to the downwash so the resultant vector will be different here. So this is your  $V$  resultant or  $V$  infinity prime right so which is equivalent to this. So this is  $V$  infinity prime, so induce a certain angle called  $\alpha_i$  right. So this particular angle is your local angle of attack, do you remember?

$\alpha_l$  is equals to  $\alpha$  minus  $\alpha_i$  right? So and which generates a lift. So say this is your lift, overall lift acting perpendicular to the overall free stream velocity, right but because of the induced angle of attack  $\alpha_i$  because of the upwash and downwash what we have is a change in local velocity due to which there will be a change in local lift generation. Am I correct or not?

So this the component of this lift that is generated at that particular location, right so because of the induced angle of attack, so we have a component of this acting in the direction of drag, right. So what we have is there is a induced drag because of the lift in the direction of drag, overall drag of the aircraft, which is  $L \sin$  of  $\alpha_i$  right. So the  $C D_i$  is equals to  $C L$  times  $\alpha_i$  assuming small angle of attack.

And then from thin aerofoil theory, sorry from lifting line theory what we have is  $\alpha_i$  is equals to  $C L$  upon by  $\pi e A.R$  right. This is at the wing. If you consider the downstream this is twice know two times of  $\alpha_i$ . That is the angle that, so we will talk about that when we are dealing what you call stability aspects during the second part of this course right.

Next during the second semester, when we are talking about flights to retain control. So the  $C D_i$  is equals to  $C L$  square upon  $\pi e A.R$ . So what we have in addition is a least induced drag. So which is due to the upwash under the downwash effects near the wing, right. Okay? So this apart from this see, again if you consider a finite wing it has the tip vertices from it. So it has tip vertices near the tip chords, right.

So those tip vertices will try to drive the air along the span wise direction. And then there is a wing, right we have wing which creates an upwash and downwash behind upwash ahead of upstream of the wing and downwash in the downstream of the wing.

So all three together will induce at an angle of attack or will alter the local flow and produce this induced drag, right.

So there is a contribution of lift to the drag. At the same time there is flow, right. When there is flow there is pressure distribution as well as shear stress distribution. So what we have is drag coefficient due to skin friction as well as pressure drag coefficient due to flow separation. Is it not? So the total drag here so drag coefficient  $C_D$  for a finite wing is equal to  $C_{D0}$  or let us say  $C_{Df}$  because of friction skin friction  $f$ , plus  $C_{Dp}$  because of pressure drag, right, and  $C_{Di}$  because of induced drag, okay.

So apart from this, so this is okay as long as we are talking about subsonic speeds. So as we enter the higher subsonic regimes or supersonic regimes, yeah, we should talk, yeah, very high subsonic or supersonic regimes, then there is an additional component due to which there is additional drag, right. So there is an additional component of drag because of shockwaves, right.

We will talk why there is a shockwave. We will very soon discuss about this. But for the time being assume that there are shockwaves at higher Mach number. So which will contribute towards this drag coefficient, right. So  $C_{Di}$ . So these three put together are known as parasite drag coefficient, right. So this and again see profile drag varies with  $\alpha$ , right.

Because see skin friction has very less effect with angle of attack but the pressure distribution has definitely has a higher influence, right. So the angle of attack may not influence this skin friction much, but it definitely has a higher influence on this pressure drag due to flow separation, right. So because the pressure distribution changes with angle of attack, is it not?

So and so this particular  $C_{Df}$  pressure distribution  $C_{Df}$ , because of the pressure distribution, correct. So that is why we had that variation of  $C_{Df}$  with  $\alpha$ , there is it not? So the contribution is mainly from this particular parameter, right. And again for a very thin aerofoil, the  $C_{Df}$  can be estimated using analytical relations of flat plate where  $1.324 \sqrt{\text{Re}}$  upon root over Reynolds number.



This is in case of laminar flow, right. So if you use this relation, you will be able to estimate the corresponding drag coefficient due to skin friction, right. So that is an approximation that we are using. So that is an analytical relation for flat plate pressure drag coefficient.

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Parasite drag coeff = [Skin friction] + [Pressure drag due to thin air] + [Wave drag coeff]

$$C_{D_{\text{Parasite}}} = C_{D_f} + C_{D_{\text{Profile}}} + C_{D_{\text{Wave}}}$$

$$C_D = \left[ \text{Lift independent drag coeff} \right] + \left[ \text{Induced drag coeff due to lift} \right]$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e A R} \rightarrow C_D = C_{D_0} + k C_L^2$$

→ Drag Polar  
 $k = \frac{1}{\pi e A R}$

So these three put together called parasite drag coefficient, which is equals to due to skin friction, drag coefficient due to skin friction and drag coefficient due to pressure drag coefficient or pressure difference due to flow separation. And what we have is wave drag, right. Drag coefficient due to shock waves. So all three put together called parasite drag coefficient, right.

So at lower subsonic speeds this will disappear, wave drag coefficient will disappear. What we have is  $C_{D_0}$  which is equals to  $C_{D_f}$  plus  $C_{D_{\text{Profile}}}$  or what we can say  $C_{D_0}$  due to so all the drag coefficient, parasite drag coefficient is due to skin friction and profile drag coefficient plus yeah you can say  $C_{D_{\text{Wave}}}$  drag coefficient. But at lower subsonic speeds this will disappear.

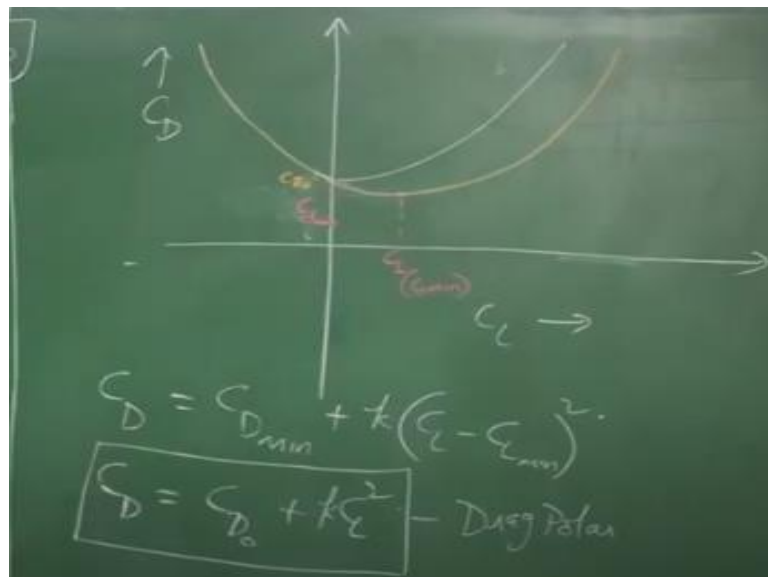
So now we can write this total  $C_D$  as lift independent drag and lift dependent drag. So all these are lift independent drag is it not? So  $C_{D_{\text{Wave}}}$  is because of the Mach number not because of the lift. So  $C_{D_{\text{Profile}}}$  is because of the pressure difference along the free stream velocity not perpendicular to free stream, right. And  $C_{D_f}$  is due

to the skin friction which is a major contributor towards the in the direction of free stream velocity right.

Major contribution of drag toward in the direction of free stream velocity. So all three put together we can say it is a lift independent drag coefficient. And what we have is lift dependent or induced drag coefficient due to lift, right. So let me name it as  $C_{D\text{ naught}}$  plus. So what is this induced drag coefficient?  $C_{D\text{ i}}$ , okay. So this is equals  $C_{D\text{ naught}}$  plus  $C_{D\text{ i}}$  is given by  $k C_L^2$  square right where  $K$  or  $C_L^2$  square upon  $\pi e AR$ .

This is your drag coefficient. So this particular equation is known as drag polar, which is also written as  $C_D$  is equals to  $C_{D\text{ naught}}$  plus  $k C_L^2$  square where  $k$  is the induced drag correction factor which is equals to  $1$  upon  $\pi e AR$ . So  $k$  is known as induced drag correction factor, okay. So this particular relation for a finite wing is known as drag polar, right. Represents the variation of  $C_D$  with  $C_L$ , okay.

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So if we look at the variation here if I plot  $C_D$  with  $C_L$ . So how  $C_D$  changes now in the lower subsonic speeds? So here you can see  $C_{D\text{ naught}}$  is lift independent drag, is it not? So as the  $C_L$  changes, we have  $C_D$ , variation in  $C_D$  is it not? So what we can say is so if have  $C_D$  on the y axis and  $C_L$  on the x axis, right. What I can say is. So for example, let us consider this particular case, right.

So it is a parabolic variation, is it not? It is a second order equation in terms of  $C_L$ . So what you have is when  $C_L$  is equal to zero, the corresponding  $C_D$  is  $C_{D0}$ , right? This is the lift independent drag coefficient where when  $C_L$  is zero what you have is  $C_D$ . So this is the corresponding point. And then yeah, so for the other cases where let us say if there is yeah  $C_{D0}$ , right?

So you can see the minimum point is not  $C_{D0}$ , it is something else. Am I correct or not? So if I postulate that with this particular equation, the minimum that I can expect because  $C_L$  can never be negative here. So the minimum that I expect is  $C_{D0}$ . Am I correct or not here, is it not? Because even  $C_L$  if it is negative, the contribution will be positive here right.

Now let us assume there is a case where there is a minimum point compared to that of  $C_{D0}$  here, right. So let us say this is that particular minimum point which corresponds to so this corresponds to  $C_{Dmin}$ , right  $C_{Dmin}$  and the top point corresponds to  $C_{D0}$ , right. And the corresponding  $C_L$  for  $C_{Dmin}$  is  $C_{Lmin}$ , right okay.

So if we have that then how can I how so how should I model it? I will change this equation to  $C_D$  is equal to  $C_{Dmin}$  plus  $k$  times  $C_L$  minus  $C_{Lmin}$  square, right. So when  $C_L$  is  $C_{Lmin}$  what you have the contribution from this induced drag is zero in this particular equation, it becomes  $C_{Dmin}$ . What you have is  $C_{Dmin}$ , right.

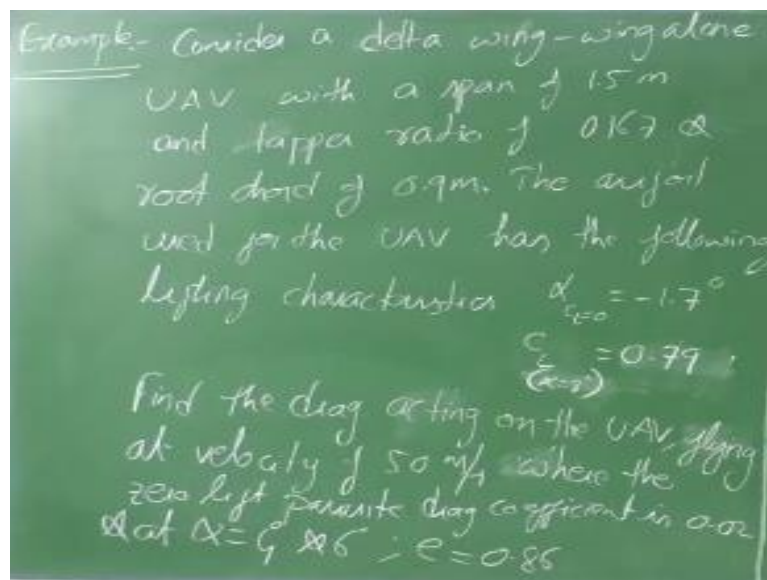
So but for modern aircraft with moderately cambered aerofoils the difference between  $C_{D0}$  and  $C_{Dmin}$  is very less okay. So and the difference can be almost negligible and we will neglect for the entire course. So from now we will use this particular equation  $C_{D0}$  plus lift independent drag coefficient and lift induced drag coefficient to represent  $C_D$ , okay.

So this is the equation that we are going to use it for the entire course, okay. And the equation is known as drag polar as I told you, okay. So let us now talk a bit about yeah, before doing that, let us solve one example problem. So why because yeah, I

want to talk about this wave drag, right. Why there is a wave drag at higher subsonic or higher speeds. We will discuss about this very soon, maybe in next 15 minutes.

So meanwhile, we will solve one example problem related to the concept that we just discussed.

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So example, I lost track of the example numbers. So you please kindly continue with the same example. In my opinion it must be either example 8 or 9 something. Either must be 8 or must be 9, right. So consider or say consider a delta wing, wing alone UAV with span of 1.5 meters and a taper ratio of 0.167 and root chord of 0.9 meters okay. So the airfoil used for the UAV has the following lifting characteristics.

So alpha at C L is equals to zero is - 1.7 degrees right. Or C L at alpha is equals to 8 degrees is equals to 0.48. This is the data that we have 4811 let us say at 8 degrees. So we need to find the drag acting on the UAV flying at a velocity of 50 meters per second, right where the drag or the drag coefficient independent of lift or zero lift parasite drag coefficient is 0.02, right.

So what are we asked? So flying at 50 meters per second and at an angle of attack at an alpha is equals to 4 degrees and 6 degrees, okay. So we need to find the drag acting on a UAV with the given data when it is flying at 50 meters per second, right at sea level. Let us consider let us assume the flight is at sea level and it is inclined at an angle of angle of attack of 4 degrees and 6 degrees, okay.

This is the information that we have right now. So how to proceed with this?

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$C_r = 0.9\text{m}, \lambda = 0.167, C_t = 0.15\text{m}$   
 $D = \frac{1}{2} \rho V^2 S C_D$   
 $S = 2 \left[ \frac{b}{2} \times \left( \frac{C_r + C_t}{2} \right) \right]$   
 $S = \frac{b}{2} \times C_r (1 + \lambda) = 0.787\text{m}^2$   
 $C_{D_0} = C_{D_0} + K C_L^2$  [from drag polar]  
 $C_L = C_{L_0} + C_{L_{\alpha}} \alpha = 0.086 + (2.92) \left( 4 \times \frac{\pi}{180} \right)$   
 $C_L = 0.289$  |  $C_D = 0.02 + 0.12 \times (0.289)^2 = 0.03$

What we got to know from the given data is  $C_r$  is 0.9,  $\lambda$  is 0.1667 and  $C_t$  will be approximately 0.15 meters 0.9 meters where the root chord is not given here? Yeah 0.9 meters it is given right. So  $C_r$  okay. So  $C_t$  will be 0.15 meter. So yeah taper ratio is given, right? You have taper ratio information  $\lambda$ . So from which you can, so what will be the area of this wing?

See in order to find the drag on this aircraft I need to know what is half  $\rho V^2 S$  times  $C_D$  is it not? So I know what is the velocity at which it is flying and  $C_L$  level density I know. I need to know what is the corresponding wing planform area as well as  $C_D$  at  $\alpha$  4 degrees and at  $\alpha$  6 degrees. So how can I find that? First I will try to find what is  $S$  for this configuration.

What is  $S$ ? So this is a delta wing right. So we know what is span. By this time you must be comfortable after solving those three examples during our in our earlier lecture. So it was about 1.5 meter, span is about 1.5. You have root chord  $C_r$  is 0.9 meters, right and  $C_t$  now you got otherwise you can say  $b$  upon 6 time sorry  $b$  times here  $b$  times  $C_t$  plus  $b$  by 2 times 2 times  $C_t$  plus  $C_r$  right, is it not, by 2.

This is an area of this trapezium and twice the area. So  $b$  by 2 times  $C_t$ , average of  $C_t$   $C_r$  will be the area of this particular one half of the planform and twice of that will

be the entire planform area. What I have is  $b$  by 2 times  $\lambda$  times  $1 + C_R$  times  $1 + \lambda$ , right. Am I correct or not? So either you find  $C_t$  separately or then just simply use  $C_R$  or take  $C_R$  out of this equation.

So what you have so out of this bracket, so you have this reference planform area in terms of span which is given  $C_R$  is given at the same time  $\lambda$ , right. So if you substitute this what you have is 0.787 meter square, right. So I remember this configuration, we have used it for many experimental purpose. And then yeah, I got to know what is this. Now I need to know what is  $C_D$ .

$C_D$  is equals to you can express this as lift independent drag coefficient or zero lift parasite drag coefficient  $C_{D0}$  plus induced drag correction factor  $k$  times  $C_L$  square. I use drag polar right changed from drag polar equation. Am I correct? So if I have to find  $C_D$  at 6 degrees or 4 degrees at  $\alpha$  is equals to 4, I need to know what is the corresponding  $C_L$  at  $\alpha$  is 4 degrees.

Am I correct? So what is this  $C_L$ ?  $C_{L3D}$  is equals to  $C_{Lnaught3D}$  plus  $C_L \alpha$  times  $\alpha$ , right  $C_{L \alpha 3D}$ . So when I change this angle of attack the corresponding  $C_L$  value changes. So if I substitute 4 degree  $C_L$  I will get the corresponding  $C_L$  4 degree given  $C_{Lnaught}$  and  $C_{L \alpha 3D}$ . So how do I know  $C_{Lnaught}$  and  $C_{L \alpha 3D}$ ?

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The image shows a chalkboard with the following handwritten calculations and a diagram:

$$C_{L\alpha} = \frac{C_{L\alpha}}{1 + \frac{C_{D0}}{\pi e AR}} = \frac{4.7}{1 + \frac{4.7}{\pi \times 0.16 \times 2.86}} = 2.92$$

$$AR = \frac{b^2}{S} = \frac{(1.5)^2}{0.787} = 2.86$$

$$e = 0.86$$

A diagram shows a coordinate system with  $\alpha$  on the horizontal axis and  $C_L$  on the vertical axis. A curve is plotted, and a point is marked at  $(0.07)$ . The origin is labeled  $(0,0)$ .

$$C_{L\alpha} = \frac{0.79}{(8.17) \left(\frac{\pi}{180}\right)}$$

$$C_{L\alpha} = 4.66$$

$$C_{D0} = C_{L\alpha} (\alpha_{C=0}) = (2.92) \left(1.7 \times \frac{\pi}{180}\right) = 0.085$$

Okay we know  $C_L \alpha_{3D}$  is equal to  $C_L \alpha_{2D}$  upon  $1 + C_L \alpha_{2D}$  upon  $\pi e AR$ , right. So we okay we missed that Oswald's efficiency and also  $e$  is equal to 0.86. Consider  $e$  is equal to 0.86 is the Oswald's efficiency factor. Now we need to find out what is  $AR$  here right. So the aspect ratio of this wing is  $b^2$  upon  $S$  which is 1.5 square over 0.787.

So if you find it, it is 2.86, right. So 2.86 is the aspect ratio for this configuration. And we have  $e$  as 0.86. So please make a correction the  $C_L$  at  $\alpha = 8$  degrees is 0.79, right.  $C_L$  at  $\alpha = 8$  degrees. It is not  $C_L \alpha$ . It is  $C_L$  at  $\alpha = 8$  degrees is 0.79. So what we have is the following data, the aerofoil data here, right. Okay, so  $\alpha$  at which  $C_L$  is equal to zero.

So this is - 1.7 degrees, right. And then  $C_L$  at 8 degrees.  $\alpha$  is equal to 8 degrees. So the corresponding  $C_L$  value is 0.79, right. This is  $C_L$  2D case, right?  $C_L$  2D and  $\alpha$ . So the coordinates here are 8 degrees and 0.79 okay. So using this data, I will be able to find out  $C_L \alpha_{2D}$ , which is equal to 0.79 upon  $8 + 1.7$ . So I am multiplying this by  $\pi$  by 180 to convert it to per radian okay.

So that turns out to be 4.66 or 4.7, 4.66. So now you have  $C_L \alpha_{2D}$ . So by substituting in that particular equation, what I can achieve is 4.66 or 4.7 upon  $1 + 4.7$  or 5.86 times 2.86. So what I have is 2.9 approximately 2.92, right. So  $C_L \alpha_{3D}$  is 2.92. So now I know what is 3D  $C_L \alpha$ , so I need to know what is  $C_L$  naught.

Or either I can use this formulation or what I can say is this is equivalent to  $C_L \alpha_{3D}$  times  $\alpha$  minus  $\alpha$  at which  $C_L$  is equal to zero. Am I correct? Because for aerofoil and wing we know that  $\alpha$  at which  $C_L$  zero is same. But let us have this approach to find out what is  $C_L$  naught, right. So I will try to use this approach. So to find out what is  $C_L$  naught.

So  $C_L$  naught is equal to  $C_L \alpha_{3D}$  times  $\alpha$  minus  $\alpha$  at which  $C_L$  is 0, 2.92 multiplied by 1.7 multiplied by  $\pi$  by 180. So this must be close to zero point, so this is 0.086.  $C_L$  naught is 0.086. I have  $C_L \alpha_{2D}$ , which I have used to find out

$C_L$  at  $\alpha = 3^\circ$ . Using  $C_L$  at  $\alpha = 3^\circ$  I now figured out what is the corresponding  $C_L$  at  $\alpha = 4^\circ$ , right as I know  $C_L$  at which  $C_L$  is zero. It was given is it not?

So I know what is  $C_L$  at  $\alpha = 4^\circ$  here  $0.086$  plus  $C_L$  at  $\alpha = 3^\circ$  is  $2.96$  and we have variable  $\alpha$  here, right. Sorry  $2.92$  multiplied by  $\sin 4^\circ$  which is multiplied by  $\pi$  by  $180$ . So what is the value of  $C_L$  at  $4^\circ$ ?  $C_L$  corresponds to  $C_L$  at  $\alpha = 4^\circ$  is equal to  $0.289$ , okay. So what we got is  $C_L$  at  $4^\circ$  and by substituting that value in the  $C_D$ , right.

So by substituting that what I have is  $C_D$  is equal to  $C_D$  at  $\alpha = 0^\circ$  is  $0.02$  plus  $k$  which is  $1$  upon  $\pi AR$  which will be approximately  $0.12$  times  $C_L$  square plus  $0.289$  square. Now what will be the answer?  $0.03$  is the  $C_D$  at  $\alpha = 4^\circ$ . So I want you to calculate for  $C_D$  at  $\alpha = 6^\circ$ , okay. So please let me know what is  $C_D$  at  $\alpha = 6^\circ$ . And also do not use  $C_L$  at  $\alpha = 0^\circ$  approach.

Use the other one that we have discussed, right. Take it as a homework problem and send it to us, okay. So now it is clear, right. We have the data planform like cross section characteristics, it is aerofoil characteristics and then data related to the planform geometry from which we will be able to estimate what is the drag coefficient and lift coefficient at the required angle of attack, right.

So now having this data of drag and lift coefficient I will be able to find out what is the drag acting at that particular velocity, right. What is the velocity here?  $50$  meters per second. If you substitute that what is the value of drag acting on this UAV at  $\alpha = 4^\circ$ ?

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$\underline{sl} - C_L = 0.97, \lambda = 0.167, C_D = 0.15$   
 at  $\alpha = 4^\circ$  &  $V = 50 \text{ m/s}$   
 $D = \frac{1}{2} \rho V^2 S C_D$   
 $D = 0.5 \times 1.225 \times (50)^2 \times 0.03 \times 0.787$   
 $= 36.15 \text{ N}$   
 $D \approx 3.6 \text{ kg}$   
 $S = 2 \left[ \frac{b}{2} \times \left( \frac{C_L + C_{D0}}{2} \right) \right]$   
 $S = \frac{b}{2} \times C_L (1 + \lambda) = 0.787 \text{ m}^2$   
 $C_{D0} = C_{D1} + k C_L^2$  [from drag polar]  
 $C_L = C_{L0} + C_{L\alpha} \alpha = 0.025 + (2.92) \left( 4 \times \frac{\pi}{180} \right)$   
 $C_{L0} = 0.289$   
 $C_D = 0.02 + 0.12 \times (0.289)^2 = 0.03$

At alpha is equals to 4 degrees and when it is moving at 50 meters per second what I have is, so 0.5 times 1.225 is the density of air kg per meter cube, yeah meter cube. So density of air at sea level and I have 50 square multiplied by C D at alpha 4 degrees which is 0.03 sorry multiplied by the corresponding cross-section planform area which is 0.787. So what is the value Prabit? 36.15.

So 36.15 Newtons, right. So approximately I can say 3.6 kg or 3.7 kg I can say approximately. So this is the drag that is acting. Now if I have to move at 50 meters per second to overcome this drag, I need to give 3.6 kgs of forward force which is thrust in our case is it not? So I want you to find out for what is what happens when yeah what is the drag that you need to overcome when you want to move at 50 meters per second at 6 degrees angle of attack for the same the same aircraft, right. Okay. Fine.

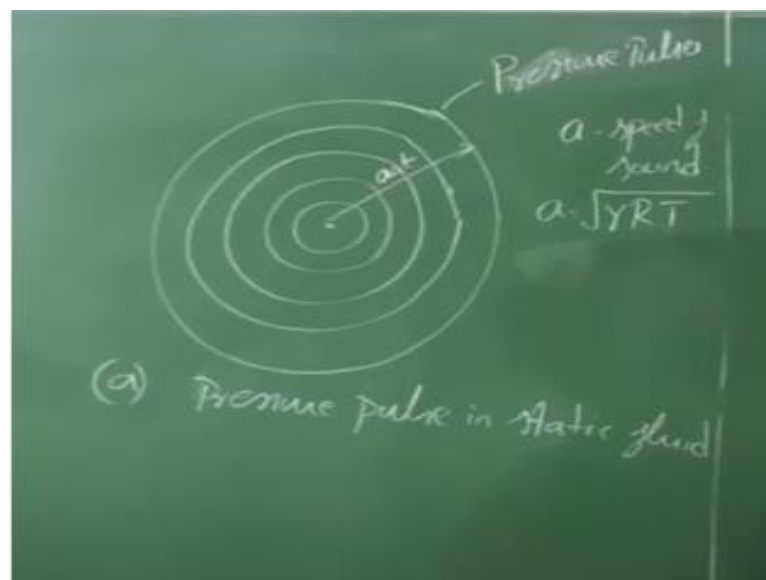
So now let us now get back to that discussion what happens in the supersonic flight or higher subsonic as well as supersonic flight and we are talking about yeah wave drag at higher speeds higher subsonic as well as supersonic speed. So let us look at what how this wave drag is coming into picture as soon as the speed is increasing, right.

So same C L, same let us assume we are flying at the same C L, same UAV and moving from subsonic to supersonic all of a sudden your drag characteristics are changing including the lift characteristics. But let us concentrate on drag

characteristics in the first place, right. So why it is happening? So how do a disturbance in fluid travels, right? That is the first thing we need to understand.

So in fluids any disturbance right will travel with the velocity of sound in the form of pressure pulses, right. For example, consider a pool with a like a pool a pool full of water, swimming pool full of water right and which is calm and still. Now if I hit that pool, if I hit a stone into that pool, so you can see there will be ripples forming around is it not?

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So let us say consider a static fluid say this is where you hit the stone in the pool. So you start seeing the disturbance that we have created traveling in the form of ripples, is it not? Am I correct or not? So similar to that. So the disturbance that we create for example will take will result in the form of pressure pulses, right. For example, what is disturbance here?

Instead of now disturbance in this particular ambient condition, what can be a disturbance? if I just push air like this or if I blow air right, if I disturb the current state of that of this particular ambient conditions, then we can assume that disturbance will create a local pressure difference which in turn travels with the velocity of sound in the form of pressure pulses.

So this particular disturbance that travels is known as pressure pulse pulses, right. These are the pressure pulses that propagates the disturbance from a given at a given

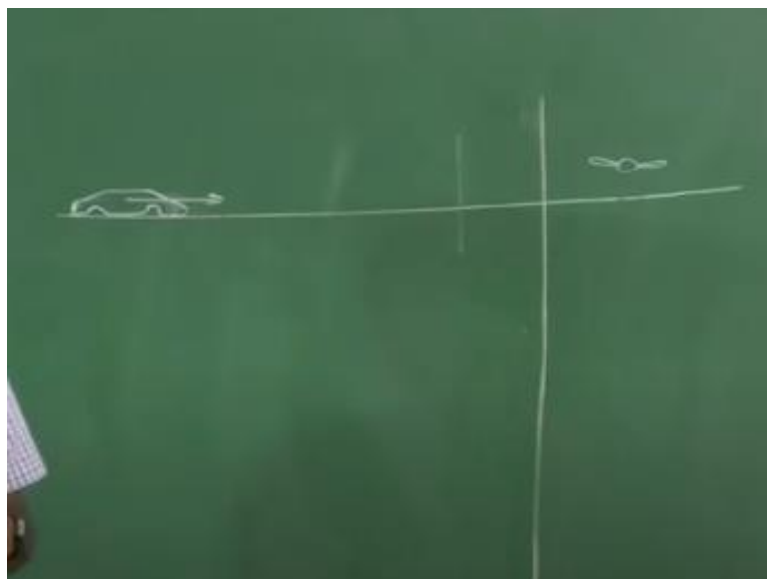
location which we have created at a given location. So this we are talking about a static pressure or pressure pulses in static fluid. And the velocity of this fluid or the velocity of this pressure pulse is equals to velocity of sound in that particular medium.

So let us say if it is in air the velocity of sound in air is about 330 meters per second, right. So now what will be this diameter of this pressure pulse? Let us say it is disturbed at a time like the time difference between the diameter of a like say if it is if you have disturbed at time  $t$  zero and it has lapsed over a time  $\Delta t$ , right. Now over this period say a  $\Delta t$  will be the corresponding diameter of this pulse is it not?

Where  $a$  is the speed of sound right square root of  $\gamma RT$  and speed of sound depends upon the type of medium and the corresponding temperature of that medium, right. And  $\Delta t$  is a time like after which you are interested in figuring out this particular distance that this particular pressure pulse has traveled. So that will be approximately  $a$  times  $\Delta t$ , right.

So when you are static this disturbance travels in all directions in the form of pressure pulse, right. So irrespective of whether you are static or not, this is how it is going to happen, okay. So okay. So let us now consider a general example right. So let us assume we are driving a car or a bike right with a helmet on.

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So when you sit in the car so when you are moving say assume you are on an expressway, so which is about say your vehicle is on the expressway. Okay, so I may

not be able to draw a good vehicle here, ground vehicle definitely. So say okay. So you are moving in this particular direction, let us say there is a bird which is flying here right okay, which is flying far ahead of this car.

So depending upon the intensity of the disturbance this may travel to the corresponding like it may travel certain distance, right. Am I correct or not? But, as soon as if this if the disturbance is in vicinity of this, so this is not affected, the flight of this insect or the bird which is far ahead of the of your car will not be affected much. So when the car comes closer to this bird, so this bird will try so most of you might have observed this right.

So the insect or the butterfly with a good enough reflexes will try to slip over the vehicle right. It will not come and hit your window. It will try to slip over the vehicle is it not? If it has a bit of good reflexes right. So that happens because when you are traveling, the distance that you are creating is also moving ahead, right. It is not that it is just moving back, right.

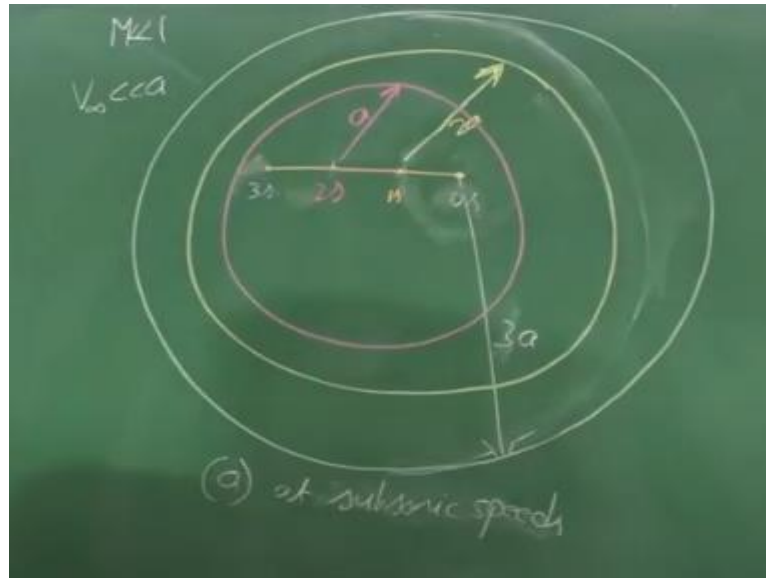
It is also moving ahead and it is moving in all the directions and the flow particles ahead of the vehicle will try to adjust accordingly. So it will try to figure out where the disturbance is minimal right and the fluid particles in the form of the streamlines can bend across this vehicle so that they can smoothly flow over this particular body, okay. Fine?

So this is true as long as the disturbance is traveling forward. Let us say if you are traveling at a much higher, now as I told you this disturbance travels at the velocity of sound, right. Let us say you have increased the speed to a higher amount right. So this should have a better reflexes is it not compared to that to escape into the flow.

Why because the reaction time it will have is much lesser comparatively, compared to the earlier case. Now assume a case when you are traveling with the velocity of sound right. The moment it feels the disturbance you will hit the bird at the same moment, right. It will hit your car is it not? So both happens simultaneously.

So it will not have the disturbance and that because of the lack of disturbance this may not be able to adjust to the flow, surrounding flow happens that is happening. Similarly the fluid molecules which are ahead of your flight, your vehicle will not be able to adjust if there is no propagation of disturbance ahead of your vehicle, okay. So now consider the case where this particular like the position of this disturbance keep moving, okay.

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So to understand it better let us consider this. So let us say this is your initial point of disturbance and then you are moving in this direction, right. So let us assume this let us analyze this disturbance at say this is a time  $t = 0$  and this location corresponds to time 1 second and this location corresponds to time 2 seconds let us say.

Say 0 say yeah, this is at 0, that is at 1 second and you have traveled another and you are trying to find out the disturbance in say 2 seconds and then at also 3 seconds, okay. So when you are moving at a speeds which are far less than the speed of sound, okay. So if you are, if your Mach number is far less than one which is like velocity of your flight vehicle is far less than velocity of sound.

So in that case, let us say at time  $t = 0$  at 0 seconds, you have created a disturbance. So that disturbance travels in the form of so after 1 second so this may not coincide see, this is do not get confused with that. So at one second or say when you are at one second, right, this disturbance might have traveled a distance  $a$ . Am I correct or not, a times 1,  $1a$ .

So when you are at two seconds, this has traveled a disturbance. So this can be much, so when you are at two seconds, this has traveled a disturbance, I mean this disturbance have traveled a distance  $2a$ , right. So this  $2a$  and this scale are not equal. Please do not get confused. So this disturbance keep, that propagation of this disturbance keep increasing as the time lapses from your right, this thing yeah from the point where you started the disturbance.

So in order to make it better, what I will say is say still this is  $2s$  right. So this is your assume this, the center of the circle is this point at zero seconds, and you are trying to figure out at after say one second, two second, and three second. So this has traveled a distance  $4a$ , right so sorry  $3a$  okay. So when you are at 3. So after that what happens? So after zero seconds, you have already created a disturbance which is two seconds before right?

Am I correct or not, at this particular location. So if you draw another so this will travel when you analyze it at  $0.3$  this will travel a distance of  $2a$ . Am I correct? So when you are here that means you already passed I mean, you already spent three seconds. Within that three seconds the disturbance which you have initiated at a location at this particular location will travel a distance  $2a$  is it not?

So similarly, the one here it will travel a disturbance so sorry, the disturbance will travel a distance  $a$ , which is one second ahead at this particular location, where you are right. Is it not? So that means the disturbance is always ahead of you, am I correct or not? Because, so the radius that you can travel within or the distance say will be  $V$  infinity times  $\Delta t$ .

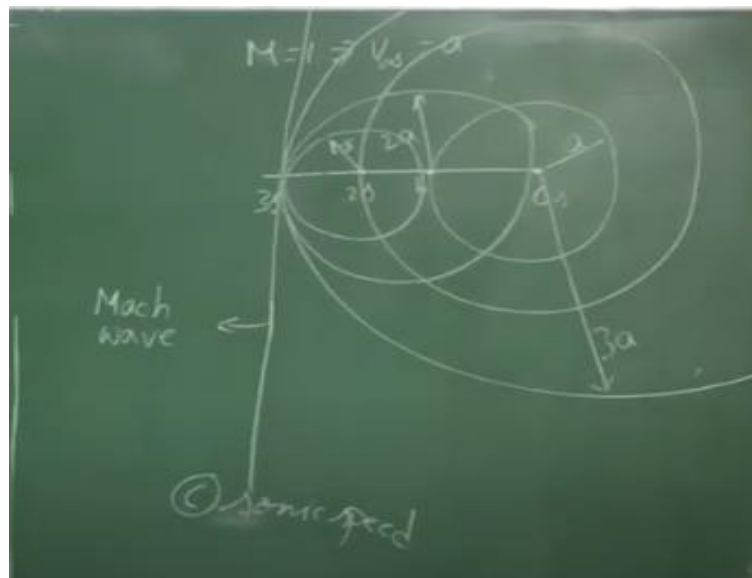
This particular distance will be  $V$  infinity times  $\Delta t$ . So  $\Delta t$  is  $1s$  and  $V$  infinity is far less than that particular  $a$  in the particular region. Am I correct or not? That is what we have it here.  $V$  infinity is far less than  $a$ . So the distance you might have covered is far less than what the disturbance might have already progressed. Am I correct or not?

So that means the molecules which are far ahead of you, of your location will get the information about your presence, okay. So which have enough time to get adjusted with the shape of your body so that the flow can happen or the fluid elements can smoothly flow over the body, right. The streamlines have enough time to get bent around your object, right?

So they get the information about your presence and the kind of pressure disturbance that you are creating, right. The fluid elements which are far ahead will have that information. So but when you are traveling with a higher velocity, let us assume you are traveling with the velocity of sound. Let us consider a case. So this is like at lower subsonic or at subsonic speeds we can say.

So this is the condition at subsonic speeds okay. So when you move at the velocity of sound, let us say.

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So the distance, what do you mean by that? So let us say you have started at zero seconds know. You created a pressure disturbance at zero seconds. So the distance it will cover within a second will be equal to let us say this is one second. So this distance is equals to the distance that this sound wave propagates is it not or the pressure wave propagates here, pressure pulse propagates.

Am I correct or not? Why because you are also moving at a velocity equals to say when you are moving at Mach 1 which is velocity of your flight vehicle is velocity of

sound, right. So within one second you are here and the disturbance has also travel which was initiated here a distance  $a$ ,  $a$  times  $t$  which is equals to  $V$  times  $\Delta t$ . Here  $\Delta t$  is 1, right.

So the same disturbance when you are at two seconds, right. So what will happen to the same disturbance? You are there. Am I correct or not? So let us say you traveled one more second, you are at three seconds. The disturbance you have is again a concentric circle and you are on the circumference, concentric circle about  $O$  here. Am I correct or not? And this particular at one second location you have also created a disturbance.

So after when you are at three seconds this will say have  $2a$  is it not? Am I correct or not? So the diameter of this will be  $2a$ . Diameter of this bigger circle will be  $3a$ . And at one second it will have a diameter. So this the difference between this is one second, right. So a second before your current location you have created a disturbance and this disturbance also traveled a distance here.

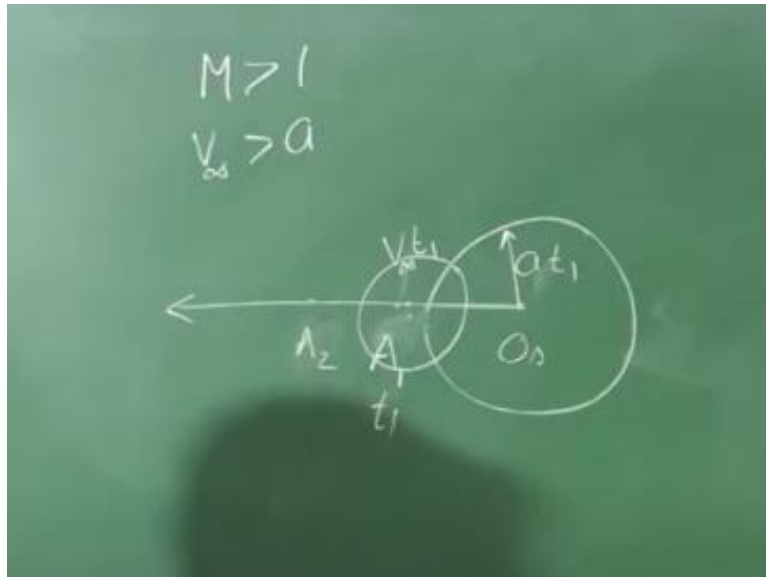
So that means, so at each and every point you are with the disturbance. You are traveling with the disturbance right. So the air molecules. So let us say if you draw a tangent to this particular point. So the air which is ahead of this particular point will not have any information about the object which is approaching those fluid elements, okay.

So that is when we call this as a Mach wave, right. So that Mach wave will create a lot of changes to the free stream, free stream velocity. In terms of pressure, there will be a huge pressure increase as well as temperature and also the density. And downstream of this Mach wave what we have is decreasing velocity, right compared to that of upstream velocity, okay.

Now let us consider say this is at sonic speeds. So let us say this is  $a$ ,  $b$  and let us say this is at  $c$  is at sonic speed. So let us now consider a case when you are moving faster than the speed of sound there, right. So what happens when you are moving faster than the speed of sound?

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This will be interesting right is it not? So let us consider this is the path that you are moving, right. So say this is at zero seconds. So one second location. So let us not say now this as one second location because you are traveling at a higher speed compared to that of sound. Otherwise, you can still say that one second location. So by the time you reach this the sound will not reach this particular one second.

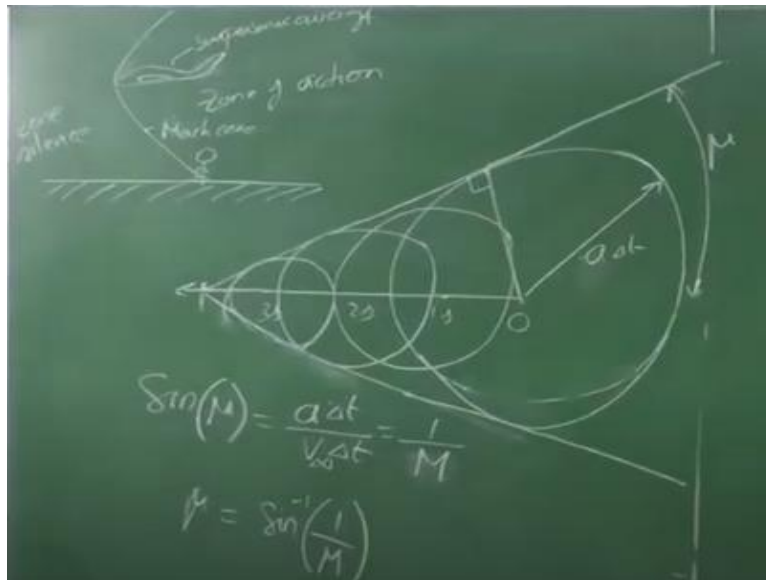
So because your velocity is far higher than this particular yeah sound velocity. So the sound, this one second may not be appropriate here. Let us say  $a t_1$ , right. Let us say our capital A, capital A 1 is the location first location, right. Another location which you have reached within some time say okay. But that sound may not be able to that sometime maybe  $t_1$ , right. Okay?

So the sound may not, the distance propagation will be  $a$  times  $t_1$ . Whereas you have covered a distance  $V$  infinity times  $t_1$ , which is higher than that of  $a$  times  $t_1$ , okay. Why because you are trying you are flying at higher velocity, supersonic velocity where your velocity is higher than this velocity of sound, okay. So  $a$  times not  $a$  times;  $a$  times  $t_1$ , right.

Now, so when you reach point B or A 2 here what happens is. So okay, at A 1 again you have created another disturbance, okay. So when you reach point A 2 this A 1 will travel certain distance, right? That again depends upon the difference between this and the time. So when you are at A 1 or say the by the time the sound reaches this A 1, you are you have already crossed this A 1, right.

And by the time this you the sound reaches A 2 you have already crossed A 2. That means the disturbance is always behind you, right. Similarly, when you that disturbance that you have created at A 1, by the time it reaches A 2 you have already reached ahead of A 2.

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For example, consider this case like let us, so now let us consider the propagation of this disturbance. So let point O at 1s, 2s, 3s, right and you are at some other location here, right. The initial one which you have like that point O that disturbance that you have created so may travel a diameter right which is yeah which is that velocity times velocity of sound times the disturbance here.

And the second one will also be right velocity times the delta t here. And the third one again will also be velocity times delta a delta t, right. So the diameter at each and every point will be the velocity times the corresponding time lapsed, right since you have created the disturbance, okay. Now see because no disturbance will be able to overcome you why because you are traveling at a higher a compared to this.

Now let us say if you draw a tangent to all this know disturbances what you get is a cone, right. So if you draw, so this is not exactly to the scale, that is the reason why we may not be able to get the tangents properly, but say, if you draw a tangent to all this circles, okay. So this particular angle is known as mu, which is a cone angle here right.

So this forms a Mach cone, Mach cone angle okay where  $\sin \mu$  is given by  $\sin \mu$  is what opposite by hypotenuse, right. This is  $a$ , so this is tangent and this is the diameter which is perpendicular to the tangent. What I have is at times  $a$  times  $\Delta t$  upon  $v$  infinity is the velocity at which you are traveling times  $\Delta t$  is what you have. If you have reached this particular location, this is  $V \Delta t$  times  $\Delta t$ .

What you have is  $1$  upon  $M$ , right. So  $\mu$  is equals to  $\sin^{-1} 1$  upon  $M$ , Mach number,  $M$  infinity, right. What we call it as Mach cone, right. So that means all the disturbance that you can see is limited or confined to this particular Mach cone. So if you are looking at the 2d profile of this you can say wedge Mach wedge or when you are looking at the 3D profile, it becomes a Mach cone, okay. Fine?

So all the disturbance is confined within this Mach cone. So whatever like for example, whomsoever stay outside this particular Mach cone will not get to know any disturbance. So to understand that consider an example where a person is standing on the ground, right. So a supersonic aircraft has already passed him okay. And he is not able to hear any sound from it, right until he gets into this Mach cone, okay.

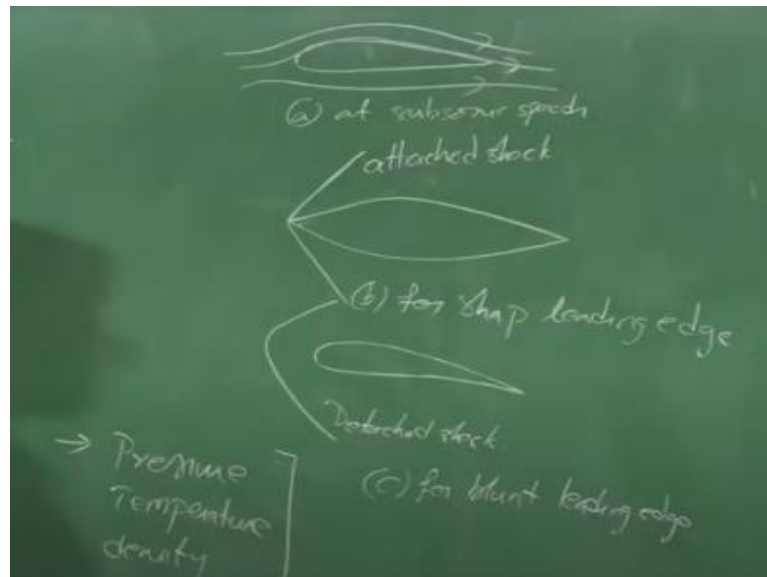
So by the time he hears this he is already, the aircraft has already passed. So what do you call is all this as so this is a supersonic aircraft let us say. This is called zone of action. And this is called zone of silence where there is no disturbance. See the disturbance is not propagating ahead of this okay. So whoever are inside this, this is the Mach cone or we can say Mach cone let us say.

So whomsoever which are inside that Mach cone will be able to feel the disturbance. But all the fluid elements which are ahead of this Mach cone will remain in the silent zone. So due to the lack of this propagation of this disturbance, the fluid elements which are just ahead of the aircraft will hit the aircraft instead of flowing smoothly over the aircraft, right which was the case for subsonic flow.

So the same will not happen here anymore. The fluid element does not have any information about the presence of the object in the fluid or the disturbance that is created by the object in the fluid, right. So instead of passing slowly across the body,

right it will try to hit the body abruptly. So the molecules there will feel a shock know across that like as soon as it encounters this aircraft. So those molecules will encounter this aircraft by means of a shock.

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So in a lower subsonic speeds what happens is a flow will smoothly flow over this object. The fluid elements will smoothly flows over the airfoil at subsonic speeds. At supersonic speeds what happens is if the leading edge of the airfoil is sharp enough then you will have a attached shock to it right. So if the leading edge is sharp enough we have attached shock.

So if the leading edge is blunt we will end up seeing a detached shock for sharp bodies, for sharp leading edge. For blunt bodies this one, for blunt leading edge, okay. So as soon as the fluid particles causes the shock, so there will be huge variation in pressure, increase in pressure, temperature as well as density, okay.

And the velocity downstream the shock as I told you is always less than the velocity of upstream the shock, okay. So this pressure is nothing but like the compression of that mass of the fluid particles which are ahead of the ahead of this flying object, right. So this compressed fluid particles will offer additional resistance to its motion. So that additional resistance to the motion apart from the skin friction as well as pressure drag is the wave drag, okay.

So the wave drag is due to that skin friction, sorry that additional resistance by the fluid particles across the shock for the aircraft to move in that particular fluid, okay. So that is a story about this wave drag, yeah.

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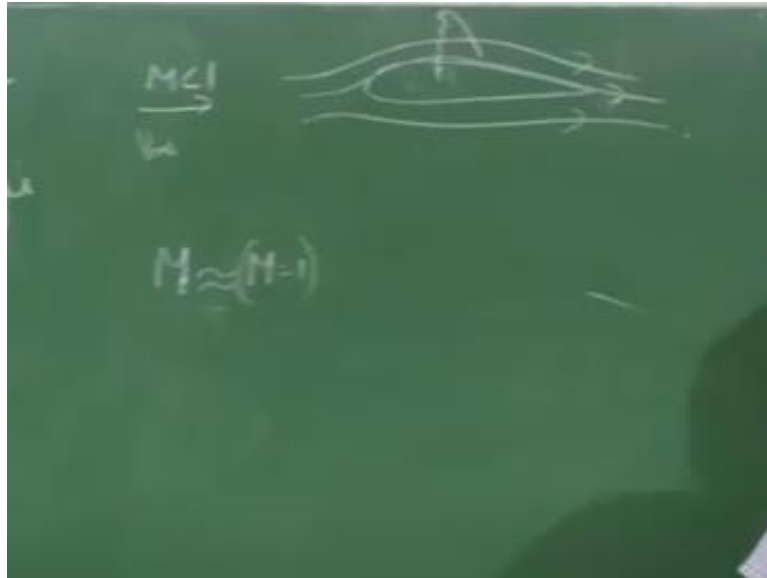


So this profile drag coefficient, we look at the profile drag coefficient. So I am talking about profile drag coefficient here. So if you look at profile drag coefficient, it will almost remain constant with Mach number, right. So until certain yeah until certain Mach number called drag divergence Mach number, right. So beyond which there is an abrupt increase in the drag, okay. Increasing Mach number, so this.

So until certain regime the change in the variation in  $C_D$  or profile drag, what you can say is profile  $C_D$  profile. So the profile drag coefficient will remain constant or can be considered the variation is very insignificant in this till certain Mach number called drag divergence Mach number beyond which there is an abrupt increase in the drag, drag coefficient, profile drag coefficient, right.

So that is the drag the Mach number at which the change or there is an abrupt increase in this drag, profile drag coefficient is called drag divergence Mach number.

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So when you look at subsonic speeds what happens, consider a point P here, right a local point or say the minimum pressure point on the airfoil let us say that. So if this velocity which is less than 1, so on the surface it will increase definitely compared to that of free stream velocity here, is it not? That is what we studied till now.

And then if the velocity is let us say is close to is close to a higher subsonic speeds or close to sonic speed, right. Let us say though it is not close to the sonic speed, the speed stream velocity is not close to the sonic space, but still if it is higher or close enough to the sonic speeds, what happens even when you are flying at a subsonic speed close to sonic, right  $M$  is equals to 1, okay close to.

So what happens is the local flow accelerates and you will see a shockwave on the surface of this wing, right. So the local subsonic speeds at which the airfoil or the aircraft first encounters a Mach wave or a sonic speed is known as critical Mach number. So you are still at a local or subsonic speed, but your wing see a sonic boom on it or a shockwave on it or you can say you see the flow local velocity will reaches Mach number 1, right.

So that particular Mach number at which the you are flying on the flight Mach number of that flight is considered as critical Mach number, right. So critical Mach number and drag divergence Mach number are two important parameters that we need to talk about. So critical Mach number is in general less than drag divergence Mach

number. So though you are flying at, so again you have to delay this critical Mach number as much as possible.

Why because as soon as you see the critical Mach number on the surface, you see on the surface of the wing, you see a higher speeds right. So maybe yeah of course critical Mach number corresponds to sonic speed on the surface of the wing. So that will create a shockwave, which will increase the drag, right. So aim is to reduce that drag.

In the previous problem we saw that when we want to fly at higher velocities, once you solve it for different velocities, you get to know that you need more drag to overcome. You have to you need more force to overcome the drag because the drag increases and the requirement or the input to the system has to increase accordingly. So if you have so the main aim is to decrease this drag.

So there are various way to decrease this drag again right. So we are not going to discuss about that here. But the I want you to understand what is critical Mach number and what is drag divergence Mach number. So critical Mach number is the local subsonic Mach number at which the aircraft first encounters sonic speed on the wing, right on the wings.

And drag divergence Mach number is the Mach number at which the profile drag coefficient start a rapid increase in it, right. Profile drag coefficient will see a rapid increase, right. So that is the drag divergence Mach number. Okay, see you. Thank you.