Introduction to Finite Volume Methods-II
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Lecture – 16
Convection term discretisation-VIII

So welcome back to the lecture series of Finite Volume.

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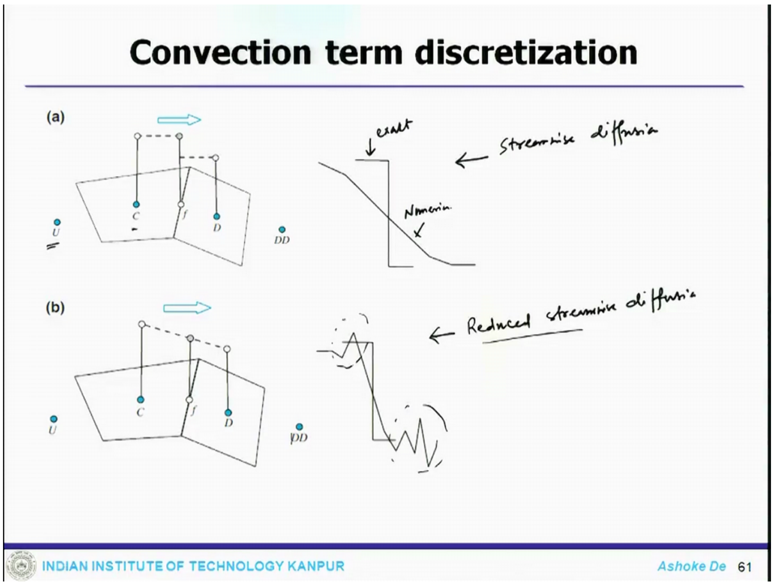

Now, this error when you use for the to reduce the steam wise diffusion or the higher order profile these are also called the dispersion error. Now dispersion error actually shows some this kind of small oscillation in the solution. And this is because of the at the presence of large gradients which render to the solution unbounded. Now if you have this higher order profile then you get this kind of thing.

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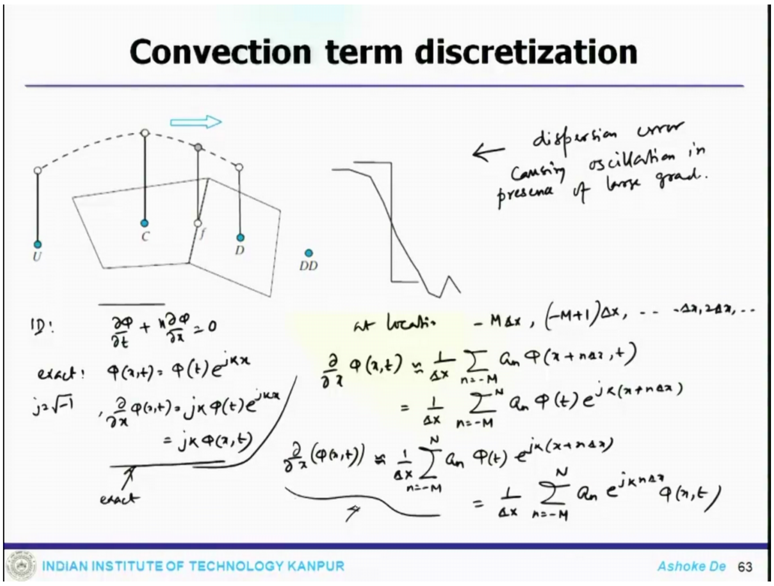

So, now if you move to this, this shows an profile which actually now this is the dispersion error causing oscillation in presence of large gradient so that is what happened. Now one can evaluate this error by some sort of an simplified version of the equation system. Let us say in 1D if you assume this is my unsteady scalar transport convection equation then the exact solution would be phi x, t equals to phi t e to the power jkx where j is minus 1 and the exact value of the gradients become del del x of phi x,t equals to j k phi p e to the power jkx which is jk phi x, t.

Now, with the interpolation profile the numerical approximation of the gradient which is written in terms of phi at locations, let us say minus M delta x minus M plus 1 delta x and so on like delta x, 2 x delta x like that. Then one can write del del x of phi x, t to 1 by delta x summation of n equals to minus M a n phi x plus n delta x, t which is nothing, but 1 by delta x n equals to minus M N a n phi t e to the power j k x plus n delta x.

Now, we substitute this of the assume solution becomes del del x of phi x, t equals to or equivalent to 1 by delta x summation of n minus M to N a n phi t e to the power j k x plus n delta x which one can write 1 by delta x summation of n equals to minus M by N a n e to the power j k n delta x n phi x, t. Now this is our exact solution, this is our exact this is our approximated solution.

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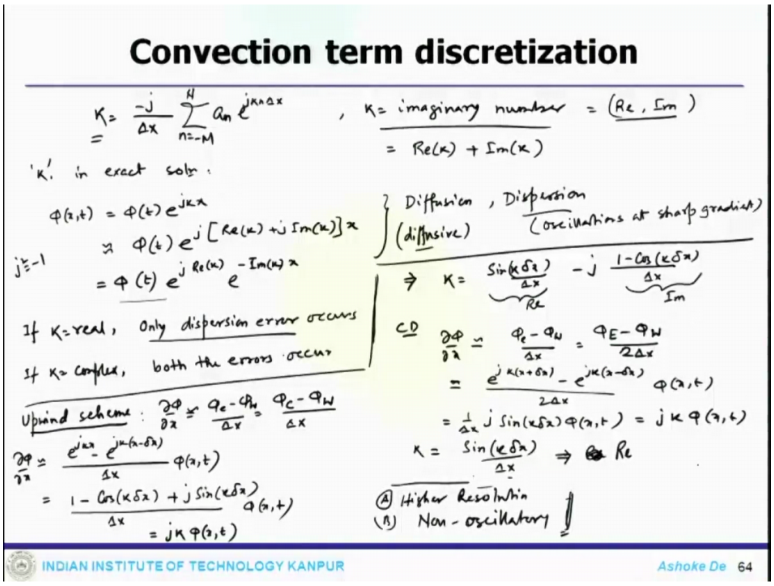

Now if you compare both of them together one can find k equals to minus j delta x summation n is equals to minus M to N a n e to the power j k n delta x. Now, what you can get is that k is essentially and imaginary number. So, k is an imaginary number. So, since k is imaginary it will have two component. One is real plus one is imaginary. So, it will be having two component for this value k. So, now, one can think about writing that real k plus imaginary part of the k.

Now insert this k in the exact solution. So, k in exact solution which you get then phi x, t equals to phi t e to the power jkx equals to phi t e to the power j n Re k plus j imaginary k x which will become phi t e to the power j Re k into e to the power minus imaginary k x. Because this will multiplied with j and j square so j square is minus 1.

So, if you look at the numerical solution it may include both the term. That means, it may include both diffusion and dispersion. So, the numerical solution can have both the term. So, the diffusion which will make the solution diffusive in nature and dispersion which will lead to some sort of an small oscillations in the at sharp gradient. So, either of these cases there are certain issues. Now, if you say k is real; if k is real then this portion of the term goes away.

So, there is no imaginary component. So, the solution would be only this much. So, the error which will actually present there it is only dispersion error. So, if there is no imaginary part if k is real then only dispersion error occurs ok. However, if k is complex that mean it will have both real and imaginary part then this will have both the errors occur or polarise. So, based on this analysis now one can have a value for k for different different scheme. Now k could be different for upwind scheme k could be different from other scheme.

So, now let us say if you look at k for upwind scheme. You consider upwind scheme and try to find out the value for k. So, in upwind scheme what we have del phi by del x what is that del phi e minus phi w y delta x which is phi c minus phi capital W by delta x. So, one can write that is e to the power j kx minus e to the power jkx minus delta x divided by delta x phi x, t which will get you back 1 minus cos k delta x plus j sin k delta x divided by delta x phi x t.

So, now, that this is j k phi x, t. So, which will be so this is equals to j k phi x, t. From here one can think about the k is sin k delta x divided by delta x minus j into 1 minus cos k delta x divided by delta x. So, it is clear that in the upwind this is your real part this is your imaginary part. So, that clearly shows that in upwind scheme you will have both real and imaginary component; that means, both kind of error which is diffusive and dispersive they will arrive.

Now, at the same time if you look at CD. CD scheme the del phi by del x would be phi e minus phi w by delta x which is phi e minus phi w by 2 delta x which one can write e to the power j k x plus delta x minus e to the power jk x minus delta x divided by 2 delta x phi x t. So, essentially this will become 1 by delta x j sin k delta x phi x, t where equals to j k phi x t from this one can get k equals to sin k delta x by delta x which is only real component ah.

So, only real component of the system. So, the real component if it is there then; that means, the CD shows only dispersion error. So, that is what is important that in one case there will be both error present and the other case. So, this dispersion error actually causes some sort of an oscillation an understood and overstood of the results at the location where you have sharp gradient.

Now, once we have some sort of an or having an idea about this kind of different errors numerical dispersion. So, it is quite desirable to develop some sort of an scheme or convective dissertation scheme which are non oscillatory and higher order accuracy. So, we need two important properties number A higher resolution and second the non oscillation; non oscillatory.

So, that will actually lead to a some different class of resolution scheme. And that will have discussion in the follow up lectures. So, where you can see how we can devise higher order resolution scheme, but at the same time preserving the property of the non oscillation. So, that whenever you get an sharp gradient the wriggle should not appear in the solution.

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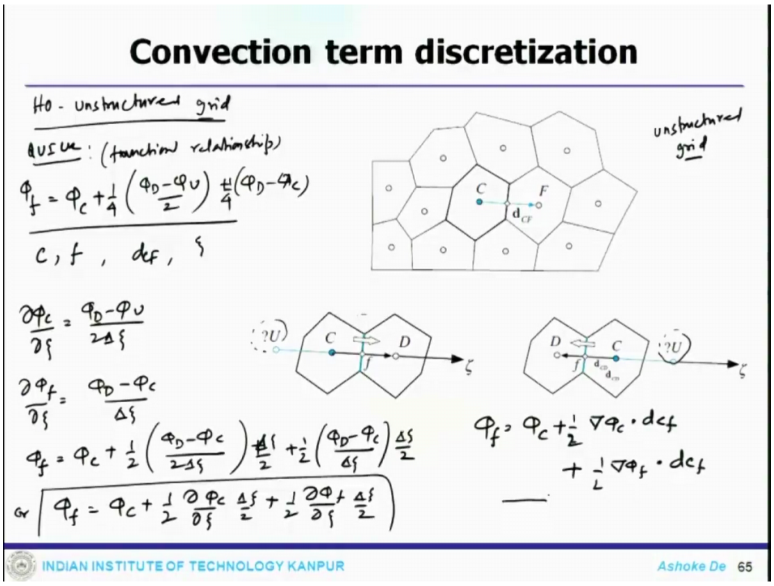

Now, just to conclude this things one more discussion we will need to do is that higher order scheme on unstructured grid. So, how would that look like the higher order scheme and the unstructured grid. So, if you see this particular stencil this is an unstructured grid. So, this is your unstructured grid. And now here in this case you can see this upwind node C and A and they connected with the distance. And higher order higher resolution convection scheme we want to develop given the velocity at the face in this direction and other case in this direction.

So, what is important is that, how to find out the information at the upstream case if the phase velocity in these direction and when the velocity in these direction how to find out this point. So, for structured system this was easy to find out what unstructured system is required some sort of an involvement. So now, how one can do that either let us say whatever profile that we have developed for the structure gives now we can extend and try to use the same thing for here. For functional let us say we start with quick scheme which is having a functional expression the functional relationship. So, the functional relationship provides that phi f is phi C plus 1 by 4 phi D minus phi U by 2 plus 1 by 4 phi D minus phi C this is what we got from the this things. Now you can approximate the gradients at c and f where your direction dcf will be come into the picture or xi direction that we have shown for the curvilinear system.

Now, then what you can get del phi c by del xi equals to phi D minus phi U by 2 delta xi. And del phi f by del xi it would be phi D minus phi C by delta xi. So this is how you can get. Now then phi f can be represented as phi C plus half of phi D minus phi C divided by 2 delta xi plus delta xi by two plus half of phi D minus phi C divided by delta xi multiplied by delta xi by 2 or one can write f equals to phi C plus half del phi c by del xi del xi by 2 plus half del phi f by del by 2. So, one can get in this.

Now you have these vector dcf between c and f. So, you can write the phi f equals to phi c plus half of delta phi c dot dcf plus half of delta phi f dot dcf which is quite suitable use in the context of the unstructured grids which is only required the information related to gradient are the location c and f location. As long as competition of this gradients is second order this can also they way to calculate becomes immaterial and the gives higher flexibility as compared to the original formulation over unstructured grids.

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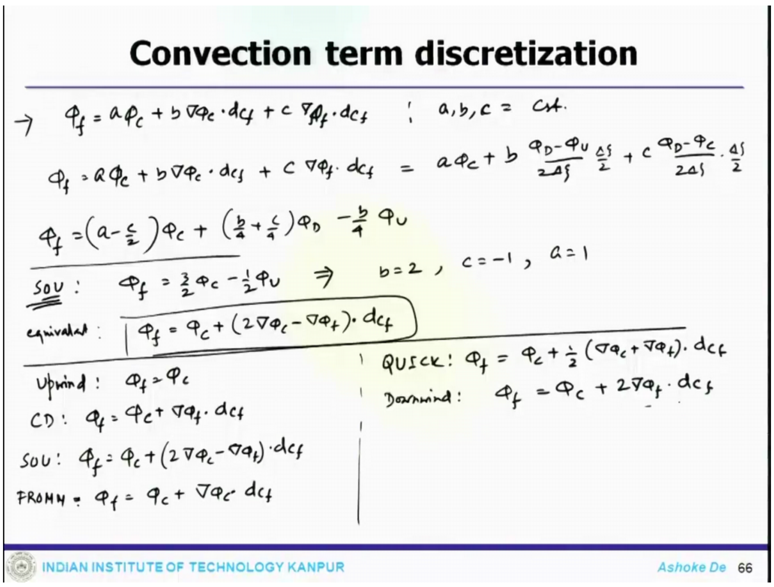

So, now this particular expression suggest that I can write phi f as a phi c plus b delta phi c dot dcf plus c delta phi f dot dcf where a b c these are constant and determined by equating phi f to the profile obtained over structured grids. So, now, the general discretized equation of this can be found once and then used in all subsequent derivatum. So, now what we will do first get this information a phi c plus b delta phi c dot dcf plus c delta phi f dot dcf which is one can write a phi c plus b phi D minus phi U by 2 delta xi delta xi by 2 plus c phi D minus phi C divided by 2 delta xi delta xi by 2.

Now you do some sort of an algebraic manipulations. So, that you obtain phi f equals to a minus c by 2 phi c plus d by 4 plus c by four phi D minus b by 4 phi U. Now you can above expression you can use to calculate abc. For example, second order upwind if you consider. So, one can find the profile in a just assume second order upwind then phi f which written here second order case the phi f is 3 by 2 phi C minus half phi U which will lead to the coefficients b equals to 2, c equals to minus 1, a equals to 1. So, the equivalent form of the sou scheme will be given that phi f equals to phi c plus 2 delta phi c minus delta phi f dot dcf.

So, that is an equivalent system for second order upwind scheme. If you follow the similar procedure one can find for others and I am noting down the final expression for let us say upwind it would be phi f equals to phi c for CD central difference it would be phi f equals to phi c plus delta phi f dot dcf. Second order already we have written, but still you can for the sake of closer you can put thing together phi f is equals to phi c plus 2 delta phi c minus delta phi f dot dcf.

You have from which would be phi f equals to phi c plus delta phi c dot dcf. Now quick you get phi f equals to phi c plus half delta phi c delta phi f dot dcf and downwind scheme will get you phi f equals to phi c plus 2 delta phi f dot dcf. So, we have already seen how to calculate this gradient at c n.

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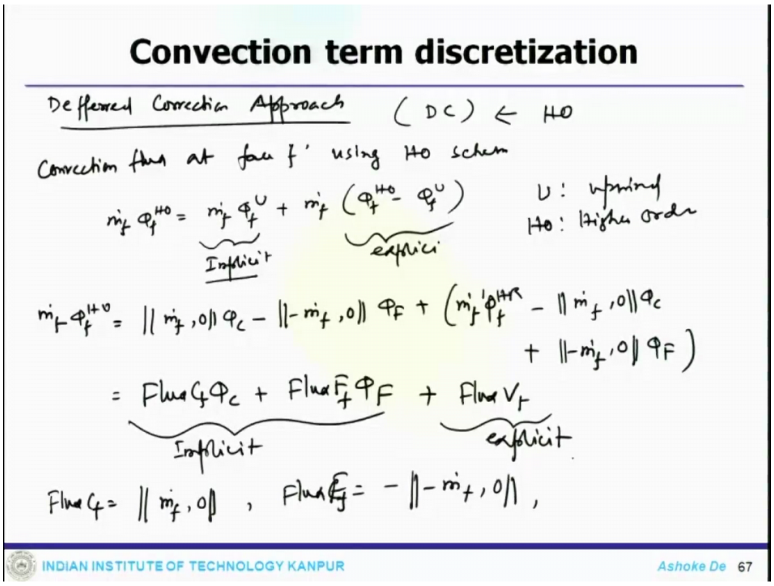

Now the final one which can is that the default correction approach. So, this default correction approach or DC which enable us to use higher order scheme. So, essentially that allows you to higher order scheme and then which can be also written in a very flexible fashion. So, the method which is used on writing the convection flux.

So, if you writing the convection flux at face f for using higher order scheme. Then you can write m dot f phi f equals to m dot f phi f upwind plus m dot f phi f higher order minus phi f. So, this is something comes as implicit this is explicit. So, where U stands for upwind and Ho is higher order scheme. By expressing the flux in fashion so the first terms first and term in the right hand side is implicitly evaluated and the second term on the right hand side is explicitly evaluated.

And while doing that so one can express his m dot f phi f Ho is nothing, but m dot f 0 phi c minus m dot f 0 phi F plus m dot f phi f h are m dot f 0 phi c plus minus m dot f 0 phi F. So, which is equivalent to getting your flux C f phi c plus flux F f phi F flux flux V f which is again implicitly evaluated and this is explicitly evaluated. And what are the terms? The terms which are there we can see the flux C f equals to m dot f 0 and flux C f equals to minus m dot f 0 and flux V f equals to m dot f phi f HR minus flux C f phi c minus flux F f phi F like that.

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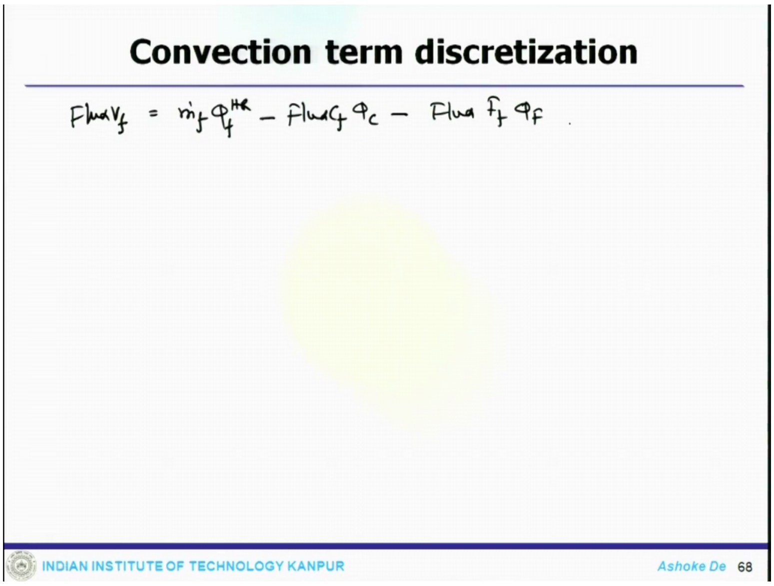

So, now, we can substitute this one and put back and get the discretized equation, but we will stop here today. And final discretized equation we can obtain in the next lecture.

Thank you.