Introduction to Finite Volume Methods-I
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Lecture – 07

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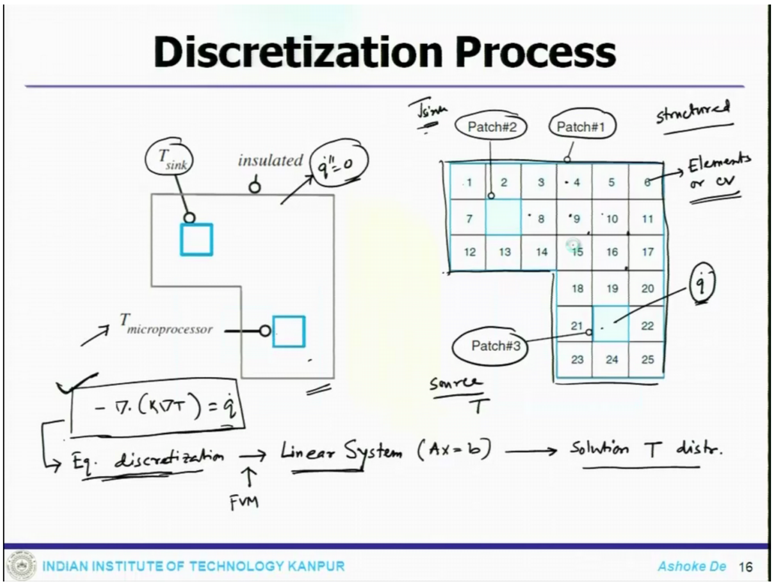

So, welcome to the lecture of this Finite Volume Method. So, where we have stopped actually we will start from there. So, what we are doing is that discretization process of the system. And when we are doing the discretization process there are few steps. And there are steps like 1, 2, 3, 4 and like first we define a domain and then the domain to physical modeling. So, we have already discussed about the domain modeling. We have this is our actual domain, if you recall from our last lecture, actually this is the heat base and you have a microprocessor here; which is essentially behave like an heat source and there is a heat sink here. So, this is particular patch is going to take care of the heat and the outer periphery of the base is insulated.

So, that means, there is no heat flux. So, this is our physical domain, and the physical modeling, that means, for this particular case. The governing equation that we are solving is our steady state heat transfer equation with some heat source. So, this is what we are solving. So, first thing you have a physical problem in hand like this kind of a heat base, where heat source heat sink is there isolated base plate. And the physical equation of the governing equation that actually governs this particular problem is the steady state heat conduction process. And which will get us the temperature distribution essentially we are interested in temperature distribution in this particular heat base; so, which will lead to finally, the design of this particular system.

So, we have done domain modeling, we have done the physical modeling through the equations. Now, we have come to the stage where actually the equation needs to be discretized. So, we are in the process of equation discretizations. And then finally, on we assemble the process, because equation discretization will lead to the linear system. So, this is what we are supposed to do. So, that means, this is where we apply our and this particular step we apply our numerical technique; so, which could be of finite difference, which could be of finite volume, which could be of finite element.

So, for this particular scenario since we are interested in finite volume method; so, we are applying our finite volume discretizations, and from the equation; that means, this is the particular equation that we are interested, because this system is governed by this particular heat transfer equation steady state. Heat transfer equation with the heat source; so, these equations once we do the descriptization, it gets are the linear system; that means, essentially Ax equals to b. Then finally, once we solve this linear system the solution will give me the temperature distribution inside this particular domain.

So, the first step to get the equation discretization that we need to divide this particular domain this is our physical domain. This domain we need to divide into proper volume or the finite control element; to do that. So, this is one of the way one can discretize this is the structured grid. So, from your last lecture if you recall so, this is what one can do. So, essentially the complete domain is divided or sub divided into multiple small finite element. So, each of these calls the finite elements or control volume. So, small small control volume which will finally, lead to this particular heats base.

And the patch the patch 1; that means, along that surface the outer boundary of this heat base. Along this particular boundary, this boundary, this boundary we have a boundary condition, and that boundary condition is essentially the no heat flux boundary condition. As soon as we say it is a insulated system, then we do not have any heat flux boundary condition. And if you look at the boundary condition thing, this heat flux boundary condition is a boundary condition of normal type. And these patches like patch 2 this is the heat sink. So, heat sink means there will be a temperature.

So, it will have a dissolute type boundary condition or constant temperature boundary condition. So, the boundary condition that we have discussed so far, they all at least couple of them you can see the direct application in this particular problem. One is the dissolute kind of boundary condition which is applicable to patch 2 and patch 3. Patch 3 is the heat source so, that will have a high temperature. So, this is the source so, it could have a high temperature or patch 2 which is always the heat sink. So, that means, there will be low temperature. One way are there possible is that since the patch 3 which is the microprocessor.

It could be the heat source, and it could give us the heat generation also. Instead of temperature the source term could be the heat generation in this patch. So, either thing is possible, but essentially that is why the whole system of equation that we are solving here is the steady state heat conduction equation with the heat source. So, this particular patch will lead to the heat source term. And then the equation using our FVM method we will get a linear system. Once we solve the linear system we will get the temperature distribution. So, this is the problem definition that we have been discussing and now how we proceed with that particular problem?

And if you see this particular domain we have assigned 1 to 25, but elements or the small sub elements or control volume, these are all finite control volume. So, what is important here to note that each of these interior points? These are all interior points or points you can say these are the interior vertices, but this control volume or the finite control volume switch at the interior elements. They are connected with neighboring elements. So, what is very important is that keep a track of this neighboring element. So, let us say if you consider this element, then the neighboring element would be 4 number 8, 10, 15. So, this neighboring element will have a impact.

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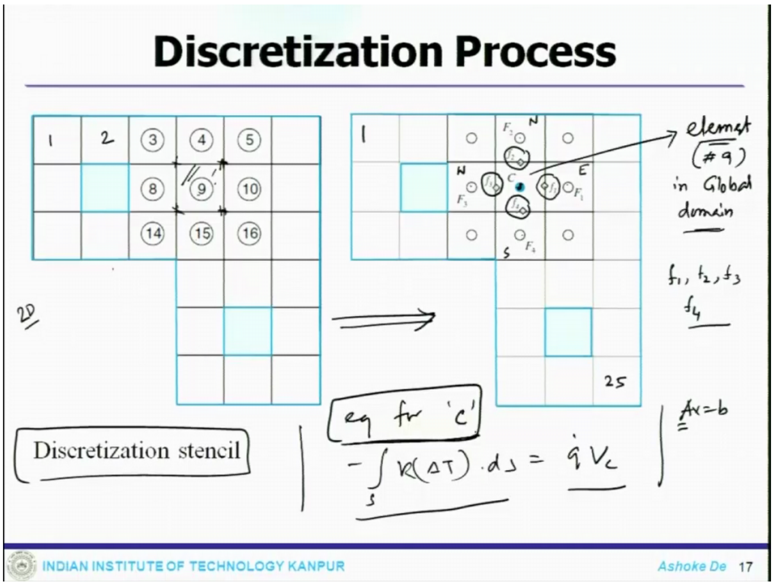

As we move on, if you see that particularly this numbering then this we consider the element number 9. Then element number 9 is surrounded with other 5 neighboring elements; like, like 3, 4, 5, 10, 16, 15, 14, 8. Now, if you just consider only this particular element and I take it here; so, from there if you take that particular element. This is what we have been doing. Let say mark it as a c this particular element number 9 in global domain. So, now if you see the importance of this particular element structure here, that once you mark the complete domain or the computational domain that with a global element number. Now once you try to discretize the equation you come down to the system where it is a local, but the connectivity must be maintained.

And this is called if you consider only one particular element like element number 9, around that this particular called the discretization stencil. Stencil means, the particular finite element for the small element what is surrounded with a different connecting elements. Like, for this particular case the neighboring elements are 3, 4, 5, 10, 16, 15, 14, 8. So, that is how it is surrounded. If you look at the vertex connectivity, 9 and 5 is connected with this particular vertex. 4 and 9 is connected with also this common vertex, and this particular 1, then 8 and 9 having these and these 9 and 15 are having these and this 9 and 10 having this and this.

So, they are also connected with these 4 corners. So, it since we are looking at 2 dimensional structure; so, it is only a 4 corner and if you since also you are in 2D domain. The element number 9 if you look at this particular picture, they are connected with 4 faces; the face 1, face 2, face 3 and face 4.

Now, if you same thing if you move to the 3 dimensional domain there will be 6 faces. Because then you will get an hexahedral element. Right now these are quadrilateral element, and quadrilateral element you got only 4 faces. Now this is the element 9. So, I mark it as a c, and now upstream of that there is f 1 north of that f 2 downstream of it f 3 or you can say the west side of you. So, this is how we are calling the convention east, north, west, south.

And this is f 4, and the element c and node 4 or the element 4 f 4 they are sharing the face f 4. Similarly, c and f 1 sharing a face f 1 a and f 2 sharing face f 2. So, and c and f 3 sharing an face 3; so, essentially a particular 2 dimensional element sharing faces like that so, it sharing 4 faces. So, what is the equation that we are solving? Let say we write the equation for the element c so, equation we are solving the integration of these equations with this. So, this is how you discretize over this control volume.

Once we write for one particular element, you can actually continue doing that and get an equation or the complete equation. Because your complete equation would be having Ax equals to B, and these metrics what does it contain. That is what we are trying to achieve this matrix will contain all the coefficients coming from all this element starting from 1 to 25. But if you look at for a particular element, now, in the programming actually you transform the complete thing to a global matrix.

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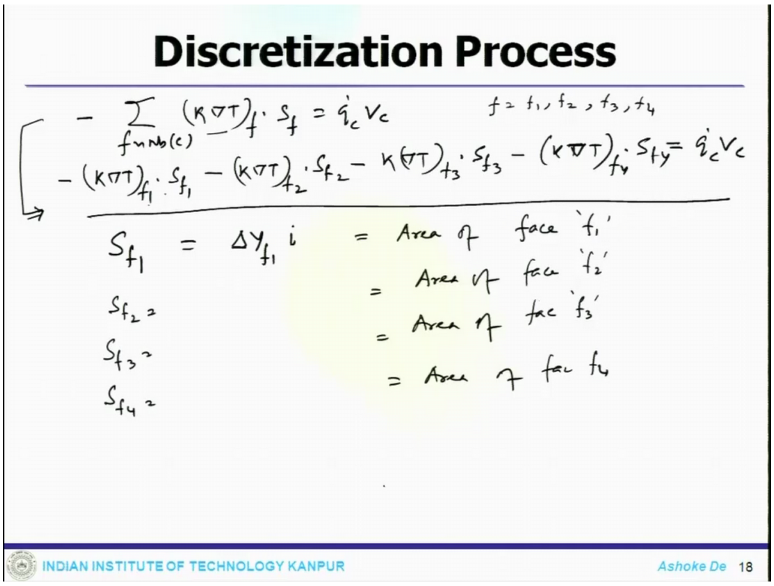

Now, what we have done so far is; this is the equation if you just look at it the equation is in this particular form. So, once you get this particular from, this is the integration over the services so, the integration over surfaces. So, the integration over the surfaces has been done. So, you get a summation of this and f got all 4 surfaces so, f got f 1 f 2, f 3, f 4.

So, all these things are getting over the loop and if you simplify from these step to the step. If you simplify so, if you write down f 1 into S f 1 so; that means, the s is the area of face 1. So, S f 1 is nothing but the area of face f 1. Similarly, if f 2 would be area of face f 2; S f 3 would be area of face f 3. Similarly, this S f 4 that is the area of face f 4, ok or you can say basically this is instead of area you can say that this surface vector rather, you say it is a surface vector so, that is better. So, you call it a surface vector for each of these faces, because f 1, f 2 these are the surface vector.

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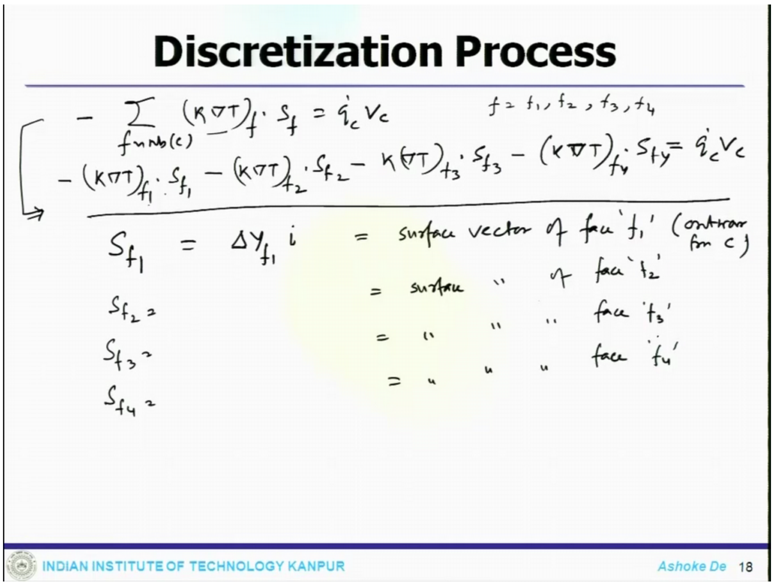

So, it is a surface vector of face f 1, ok. And that goes outward from element c, ok. Similarly, this is surface vector of face f 2. This is surface vector of face f 3. This is surface vector of face f 4.

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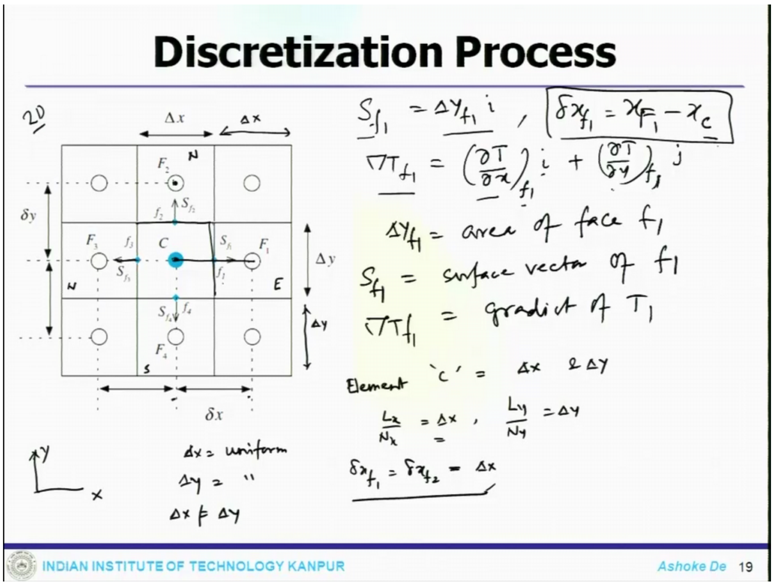

And if you look at that in this particular picture; so, if you look at it. Now this is how it has been done. This is element c it is in the cell centre. So, that blue dot here actually defines your cell centre. And this is your f 1 which is at the east face or east side of the element c, this is your north side of the element c, this is your west side of the element c, this is your south. Particularly for this particular case, we have define this is f 1, this one f 2, this one f 3, this one f 4. And if you look at the vector this connecting c and e that connected with a surface f 1 and S f 1 is the normal vector. Similarly, f 2 which is connected between f 2 and c this one goes to the S f 2 is the normal vector.

When you look at c and f 3 is the connecting normal vector is S f 3. And they are all pointed outward ok. And how you define them? S f 1 as we have defined so now, this particular element is divided in uniform size. So, c element in the x direction, this is your coordinate direction x and y. So, this is the size of element c or the width of element c this as delta x and delta y. So, this is delta x. And this is delta y. Now the distance between centre of c and f 2, that is also delta y. Because it is assumed that this mark here and mark here they are all at cell centre. So, once you assume their cell centre, and the grid distribution; that means, the delta x is uniform and delta y is also uniform.

But that does not mean delta x has to be delta y. This is not true. It is what is that? Whatever domain length you have that is uniform. Let us say you have a Lx you divide by N number of points, then you get delta x. Similarly, Ly divided by N y number of points you get delta y. So, this is what we mean by delta x uniform. So, this would be also delta x if I consider. So, that is what we meant to say. This is also delta y. So, what does that do? The distance between 2 cell centre is also dell y, distance between 2 cell centre in the downward is also delta y our. So, if I write the surface vector S f 1 this is pointed towards the positive direction of x.

That is why this delta f y f 1 into i. So, it gives you the direction of this is if you it is a 2 dimensional domain, since it is a 2 dimensional domain. The other direction or the spaniel directions you have a unit length. So, if that is the case, this is the length multiply with the unit length is the area and the direction vector. So, it is a area normal vector. So, it gives you delta y f 1. Similarly, delta x f 1 is the distance between this and this so, you get x f 1 minus x c.

So, essentially delta x f 1 for this particular case as we assume their uniform distance; even then if you write delta x f 1 and delta c f 2 they would be actually same, they are all delta x. But for the sake of discretize the equations we continue with this particular format so that you get an idea not necessarily they have to be uniform. If and then if you have a non-uniform delta x you can write this formulation. What it requires is that when I go by this particular direction between c and f 1, I need the distance between this cell centre to this cell centre.

And that is what I mean by delta x f 1, which is more generic in writing, but essentially it boils down to this particular case would be the same. Similarly, delta T F 1 it is a derivative of delta T at face 1 and 2 component and derivative of delta T at face 1 y component with j. So, that is what you write that delta T F 1. So, what even then if you look at it so, if you see the other component, the x c and all these they are the centroid of the c element.

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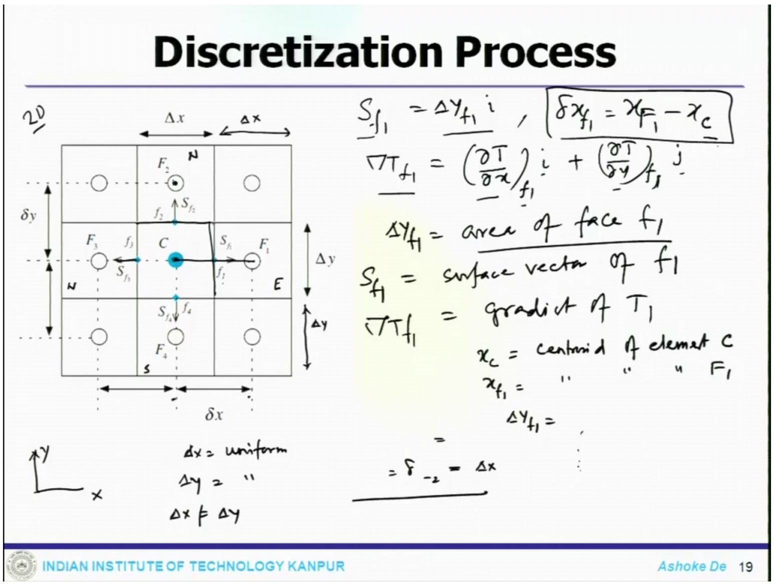

So, you can write down other component and, let say x c; x c is the centroid of element c, right; x f 1 is the centroid of element f 1, ok; delta y f 1 as I have written here area of face f 1. So, similarly you get all other component like f 1, f 2 and f 3 and f 4. So now, if I expand this particular term now, I have delta T f 1 dot S f 1.

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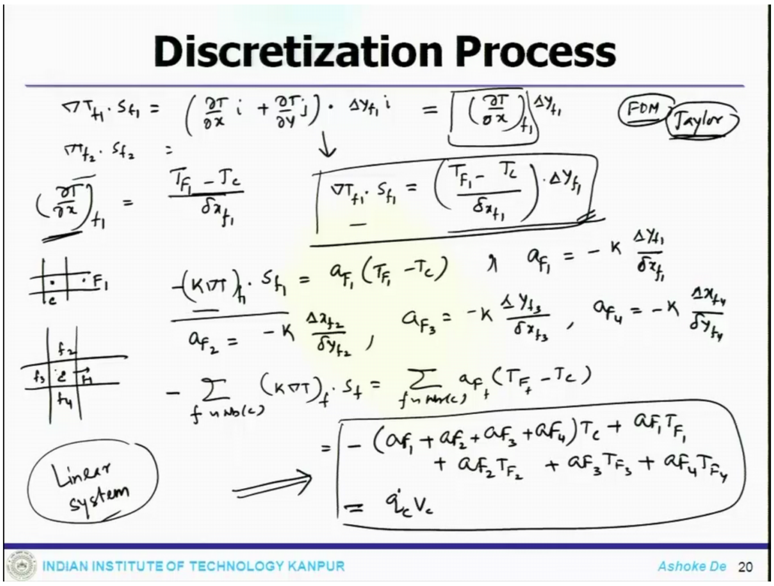

That is what we have got here. So, if you look at this particular equation this is finally, we have got, and it has all the component of all the faces. So, if you look at these components. So, essentially this is delta T by del x by del T i plus del T by del y j dot here delta y f 1 i so, which will get you back. So now, it is a dot product which will get you back del T by del x at face f 1 onto y f 1, ok. So, similarly you can get, now the important thing is that you can get the other like del T F 2 into a S f 2. So, you can get all these other information. Now the point comes if you move ahead along this direction. So, so far we know how to get these dot products.

So, if I get these dot products and put those equations back in the algebraic equations, where all the integration is done over the faces I can get all the component. Now, still one component which is missing that needs to be evaluated is this particular component. And which is true for all other faces like f 1, f 2, f 3, f 4. Now we need to find out how do we do that. So, essentially it is a first derivative of temperature at face f 1, ok. So, here when you come down even. So, what we are trying to do through the finite volume discretizations? Anyway we will have more detail discussions in all these, but that is get you the idea what we get to the linear system or how we get to the linear system.

Now, if you look at this particular derivative now we have to approximate. And this is where the derivative approximation comes into the picture. And towards the end of this lecture, we will also discuss how we find them. And the derivative calculation is exactly similar how you do in the finite difference method so, that using the Taylor series expansion. So, the detail discussion we will follow up later, but now for the time being you say that this is done with a simple arithmetic that the difference between the 2 cell centre. This is my cell centre, this is element c, this is element f 1.

So, the temperature difference between these 2 by the delta x f 1, ok so, this particular equation now boils down to delta T F 1 dot, S f 1 becomes now T F 1 minus T c delta f 1, delta y f 1. Now, if you look at these particular expression here is interesting. You started with a equation partial differential equation. You integrated over the finite volume. And then finally, what you got at least one component what you see is only the information of some algebraic expression or the values which you require is some sort of a floating point data.

Or the double precision data or the so, these all these informations are only scalar quantity. There are no vector involved no integration involved. So, it essentially brings down to the algebraic system with all this component. Now if you put them together so, if I have to look at this term k delta T at f 1 dot S f 1. This will become a F 1 T F 1 minus T c, what is a F 1? So, this is my term a F 1 is where a F 1 would be nothing but minus kappa delta y f 1 by delta x f 1, ok.

So, this is for face 1. Similarly, now particularly this element if you see, this is your element c. So, this side this is your f 1, f 2, f 3, f 3, f 4. Similarly, you can find all other component like this. So, if I have to put them together, I will put the coefficients only, minus kappa delta x f 2 by delta y f 2 a F 3 would be minus kappa delta y f 3 by delta x f 3. Then we have a F 4 which is minus kappa delta x f 4 divided by delta y f 4, ok. Now when you substitute everything in that particular algebraic equations, the equation was like this summation over faces; that means, around that element c equation was k delta T F dot S f equals to summation of now, f around these faces a F into T F minus T c.

So, if you write down them together, they will get a F 1 plus a F 2 plus a F 3 plus a F 4 into T c plus a F 1 T F 1 plus a F 2 T F 2 a F 3 T F 3 plus a F 4 T F 4. So, it will and that equals to my q dot c V c. So, that now this particular system is nothing but a linear system. So, if you see how a p d equation through this numerical approximation, you get back to a linear system like this. And this now nothing but a algebraic system, but only thing is that this is for the only one element. What you have to do? You have to integrate over all the elements.

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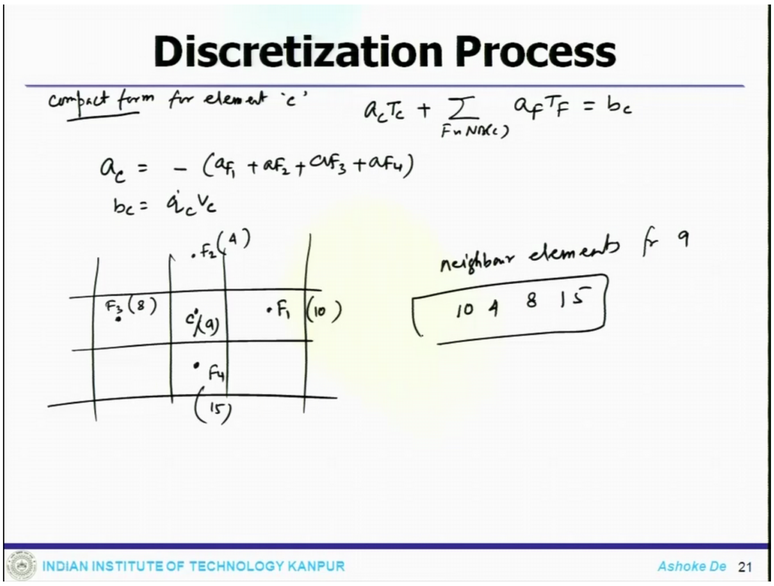

So, if you write in a compact form, if I write more compact form of the same, for element c what I can write is a c T c plus summation of capital N a F T F equals to b c. And in this a c is nothing but minus a F 1 plus a F 2 plus a F 3 plus a F 4, ok. And this is q dot c V c. Now if you see they have been all put together and the elements which are there around that element.

This was my element c or essentially element 9 in a global system. This was my F 1, but in a global system it was 10, this was my F 2 in global system it was 4, this was my F 3 in global system it was 8. And this was my F 4 in global system it was 15. So, neighboring element was for 9 was 10, 4, 8, 15. So, this is where once you bring them together into the global system or the global matrix they will be.

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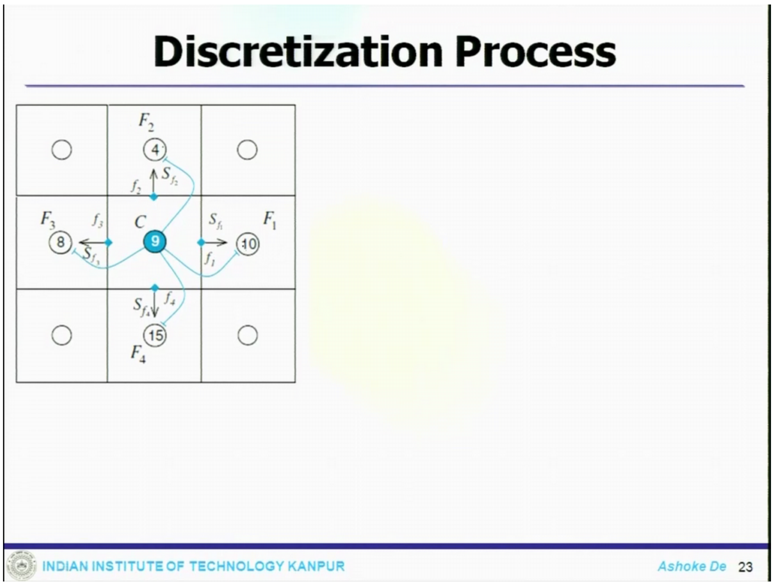

Now, this is what element c now connected with 10 4 in the f 2 8. So, this numbering 4, 8, 15, 10 they are actually the global numbering. And this global numbering once put them together so, you see the matrix like that.

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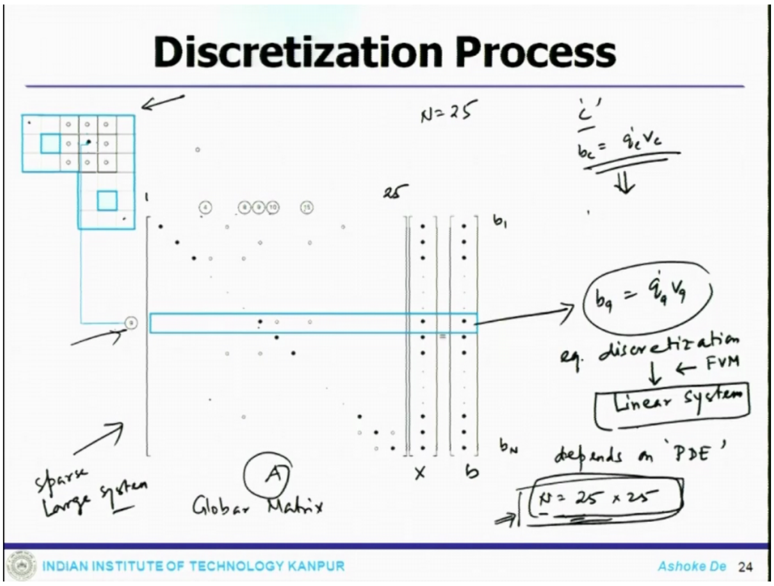

So, what I am looking here? So, this was my heat base or the problem that I was solving or the physical problem that I was solving. And this is the element that we have considered. Now if you look at some row of this particular this is your A. This is your x, this is your v.

And b will contain all these starting from b 1 to b N. For element c, what we have see the b c is essentially q dot v y into v c. So, if you transform them to global system that is that will contain here somewhere, which is nothing but b 9 which is q 9 b 9. Like that I have all the b 1, b 2, b 3, b n. And this is where it goes from 1 to again 25 in this direction N is essentially 25 in this. So, each of these component now see this is 9, and the neighbouring elements are all these 4, 8, 10, 15 only those component will be active for this particular row. What does this system actually tell you?

Once you then you go over a loop starting from on to 25. If you get the system of equation like this, this is the system of equation which you write for the all the elements you assemble them together and you get this global matrix, ok. Now once you get this global matrix, then you need to get the solution. So, the step towards that when you are doing the equation discretization so, your final objective is to get the linear system, ok. And to linear system you apply your numerical technique like ABM, and you get a linear system. Once you get a linear system like this, now the next step or the final getting a solution.

Now, what can happen? Typically, these matrix what you are getting it not necessarily has to be very smooth, not necessarily because once you talk about Navier-Stokes equation they are not going to be smooth, it could be sometimes parse, it could be sometimes essentially banded it depends on the particular pd. So, essentially it will depends on pd that we are solving for. So, in a nutshell you can think about when you solve the realistic problem with the Navier-Stokes equation; this A essentially a sparse large system.

So, here the example that we have taken is a n is 25. Now, when you talk about a big system so, n is 25 by 25. Now when you talk about large scale problem, like problem around automobile problem around the (Refer Time: 30:59) structure, you think about to represent that physical system of what kind of m or the this will be your number of elements, that is required to represent that particular system. So, once N become large, solution of this system becomes challenging. So, we will stop here today, and we will take from here in the follow up lectures.

Thank you.