Introduction to Finite Volume Methods-I

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Lecture - 06

So, welcome to the lecture of this Finite Volume Method. So now, we will move to the second part of it this discretization process.

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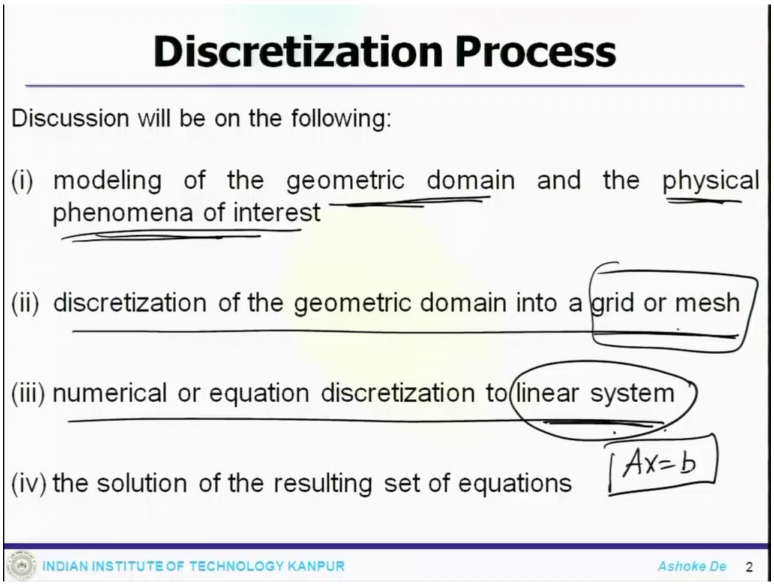

So, once you go there, then we will look at the discretization process. Now what we talk in the discretization process? So, essentially in the discretization process we will be talking on modeling of the geometric domain and the physical problem of interest that essentially again coming back to your calculation of the safety related work, that when you have a problem in hand, how do you convert them to the mathematical problem or the linear system. And finally, get the solution. So, there we will be discussing the geometric domain and physical domain.

So, one is the geometric domain is the physical problem that it will represent and the physical phenomena of interest; that means, it could be flow through channel it could be flow around your automobile vehicle. It could be flow through (Refer Time: 01:24) anything. So, one case it is as a bifurcation of geometric domain and then the physical problem. Then you need to discretize the geometric domain into grid or mesh so, this is one very important point. That means, once I have a geometric domain in hand, so I need to make them infinite number of grid or we sometime call it as mesh so, over which we will have the solution.

Then you actually discretize the numerical equation to get a linear system. So, this is where your physical problem of interest through the mathematical technique getting transfer to a linear system. When is you get transferred to a linear system you get a solution of the linear system.

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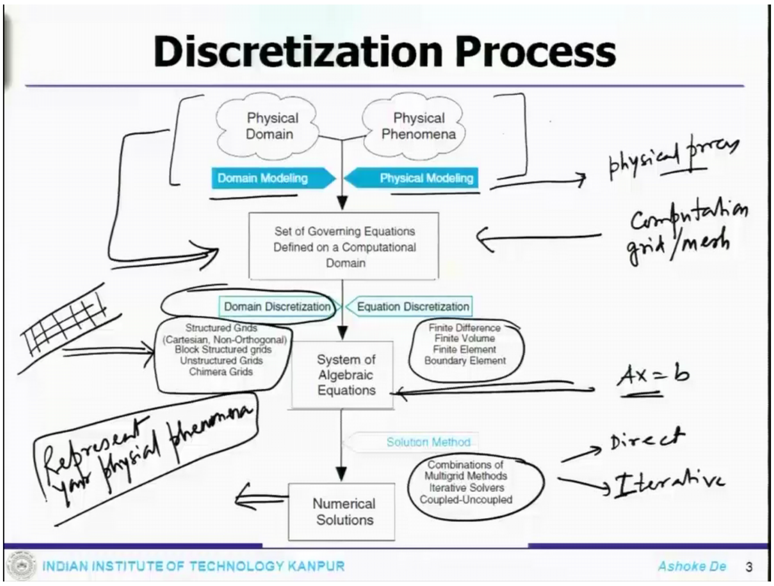

So, essentially this is the solution that you are looking for of a linear system. So, in the discretization process, we will see how a physical problem is transferred to a linear system. Now if you look at the problem, so essentially one hand you have this physical problem, and this is the physical phenomena. So, once you have physical problem in hand, so the physical problem does not come without a physical phenomena.

So, that means, if I have a flow through a channel the physical domain is the channel and the physical phenomena is the flow which is going through the channel. Now this is physical domain will have a domain modeling, and the physical phenomena will have a physical modeling. So, physical modeling means essentially you are going to have the set of governing equation or the physical equations that will actually represent these physical process, ok. So, this will actually take care of the governing equations. Now that will lead to a set of governing equations defined on the computational domain or computational grid or mesh whatever you call it.

So, essential the physical problem, now getting transferred to a computational system; now you have again to set of system: one hand you need to discretization domain, this is called the domain discretization. That means, if I have a channel like this. So, I have to discretize this one. And the domain discussion will lead to a different kind of grid, and that we have already talked in our previous lectures, that what kind of grid you end up with, it could be structure grid, it could be block structure grid, it could be unstructured grid. And then you have governing equations which actually you discretize through different schemes, ok.

Then you get to the linear system; which is Ax equals to b. And then finally, you have a different solution method. So, solution method could be multiple, because how I can solve this linear system, it could be direct solver or it could be linear solver non-iterative solver so it could be iterative solver. Then once you get the numerical solution that will actually represent your physical phenomena, ok. So, this is how you get the system going.

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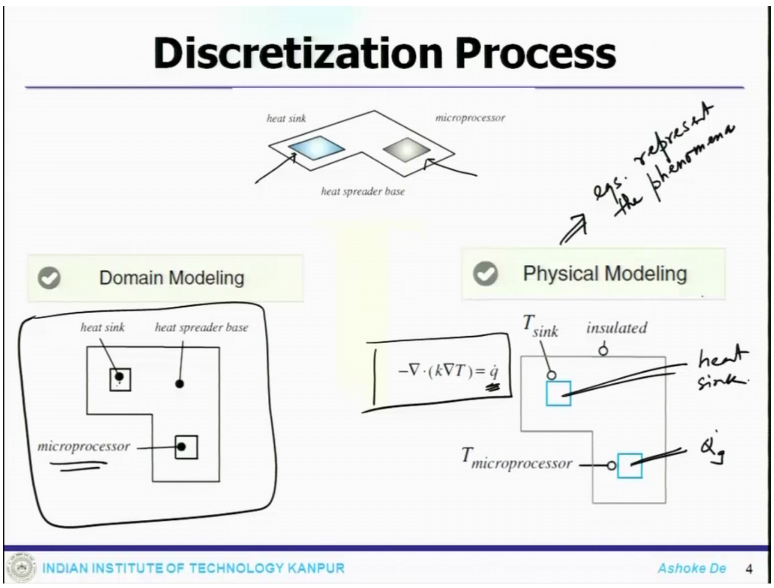

Now, you have an take an example and see how things actually move on. So, here is a base of heat spreader. This portion is the heat sink and this is another patch there. So, it is a essentially a microprocessor. Now if you look at the domain modeling. So, domain modeling if you look at it so this is the how you look the domain modeling. This is my microprocessor, this is my heat sink, and this is the heat spreader base. Essentially the heat spreader base is a kind of an l shaped base where I have a heat sink, where I have a microprocessor.

What does my physical modeling do? Physical modeling essentially talks about my governing equations or the equations that represents the phenomena. So, physical modeling essentially does that; that means, I come up with the equation which can represent this physical process. So, if you look at this heat spreader base, one and I have a sink that means, it is going to be the energy absorber, then I have a microprocessor with might be generating the heat. So, I will be essentially solving a steady state heat conduction equation with a heat generation term. Why there is a heat generation term?

Because the microprocessor sitting in this heat spreader base it is going to be; so this is going to be the heat generator and this is going to be the heat sink.

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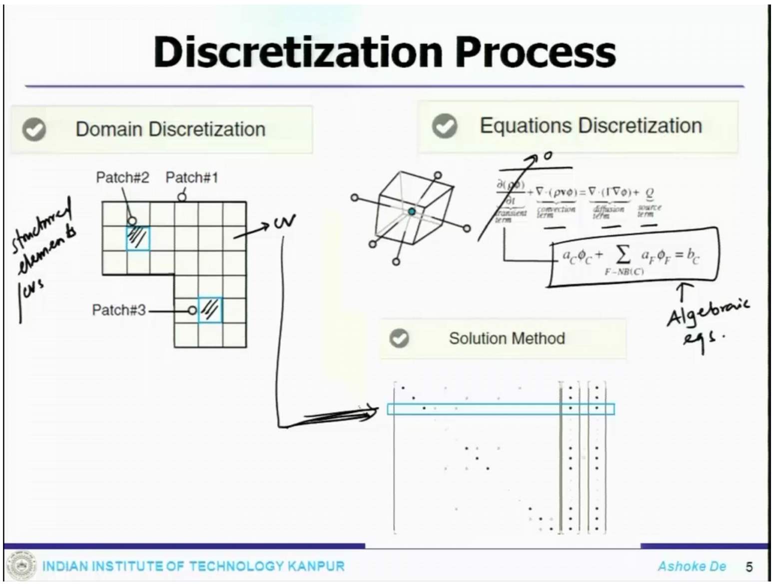

So, both the condition is there so, I have to solve that now any come down to the domain discretization, how do you discretize these things? So, that spreader base is discretize with some sort of a finite elements or the some structured elements or control volumes. So, these are individual control volume. So, you define them in a regular control volume, and this is a patch, this patch is nothing but my heat sink, and this is patch 3 which is the microprocessor this would be heat source, ok.

Now, equation discretizations I have the governing equations with source term. So, I have a unsteady term and convection term, diffusion term, these term, but in these case it is a steady state case. So, that I do not have any contribution from this term. So finally, it comes down to a some sort of this is a algebraic equation, ok. And if you look at that this should be put together in this kind of a matrix form, and this is in a particular one control volume which can represent through these kind of estimate.

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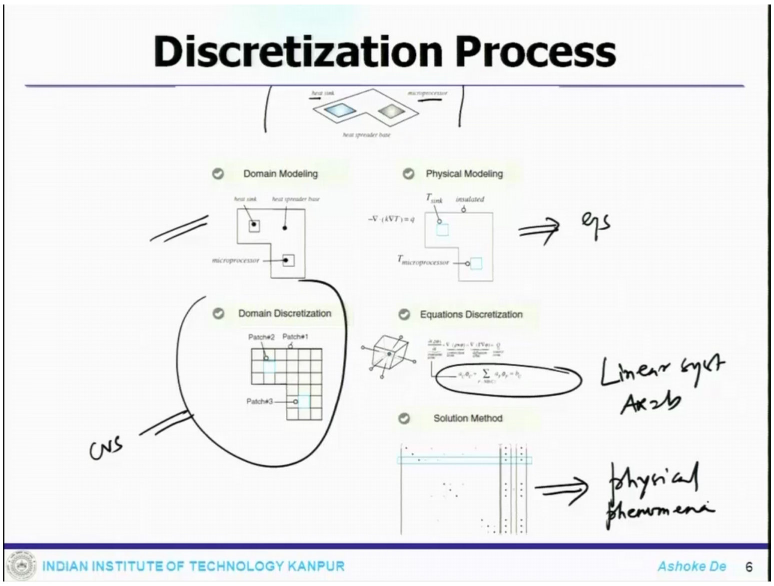

Now, if you look at the complete picture, here this is my problem of interest, where I have a spreader base, I have a microprocessor which is actually heat generator I have a heat sink.

Then domain discretization actually take care of the whole domain. Physical modeling actually gives back to my equation. Finally, domain discretizations takes into account the control volumes, then you come down to a linear system, and you get a solution which will be the physical phenomena. So, that is how the whole process actually works.

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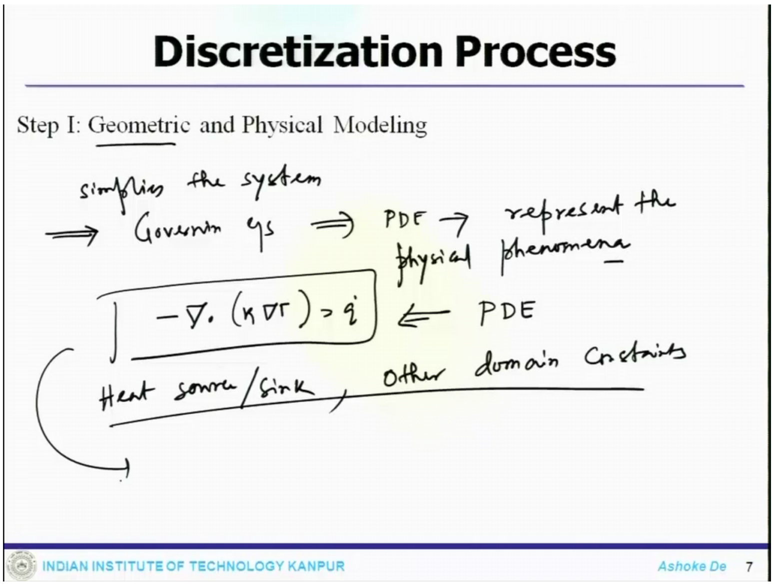

Now if you look at the geometry or physical modeling; so, geometric and physical modeling what we are doing this. This actually want it simplifies the system then it takes care the governing equations; that means, the PDEs which represent the physical phenomena, ok. So, that is what we do.

I mean this case you have a steady state heat conduction equation with a source term this is my PDE. So, first step is the geometric and physical modeling. So, the geometric modeling we identify heat source. We identify heat sink and define the other domain constraints. And the physical modeling will get back to the equations. Now the second step would be the domain discretization.

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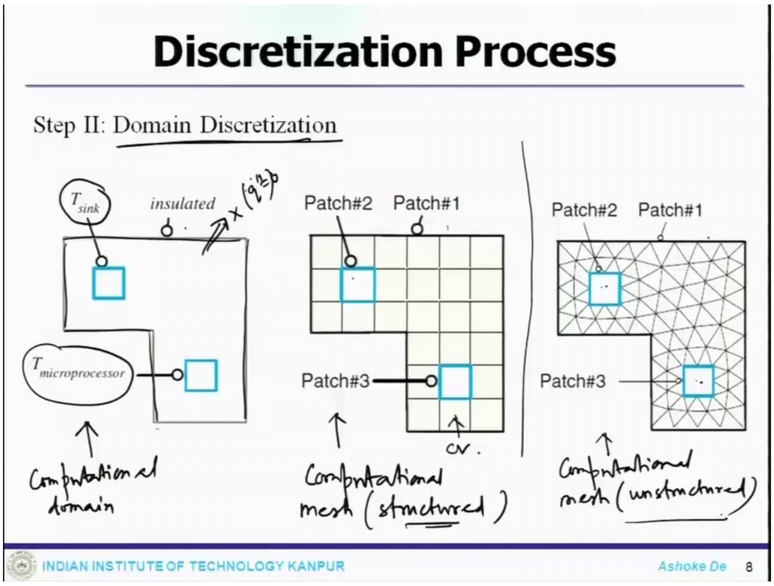

How do we do the domain discretization. Now the second step is very important in the sense how you look at the system or you discretize the domain. So, this is my complete spreader base, this is a L shaped base this.

This portion is the heat sink. So, I have a boundary condition of T sink, this is the heat generator of the microprocessor. So, there is a boundary condition of the microprocessor. Then the outside is insulated; that means, no heat is going out. So, this is stop there is no heat flux, so that is 0. So, this is my so this is the domain that I have to discretize. So, essentially you can think about this picture actually represent your computational domain, ok. This will actually take care of my physical problem in hand. Because this microprocessor will be the heat generated this guy will be the heat sink and then I have insulated base. So, that actually takes care of my computational domain.

Now, the second part is the generation of the computational. So, these portion I have a boundary condition at the heat source portion I have a boundary condition then there is a boundary condition, then if I discretize them with a structure control volume, ok. So, this looks computational mesh. So, this is structured mesh or quadrilateral mesh, or other option this could be option 2. So, this is one option, other option is that still I can maintain this these are the 2 patches one is heat sink and one is the heat sink another is the heat source, this is insulated base and I can generate unstructured grid. So, this is also a one sort of computational mesh, but it is unstructured, ok.

So, in a domain decomposition step; that means, in step 2, you identify the computational domain, then you generate the grid either structured grid. If it is a geometrically it simpler than you can generate structure grid, if it is geometrically not simpler than you can use a unstructured grid to generate the domain.

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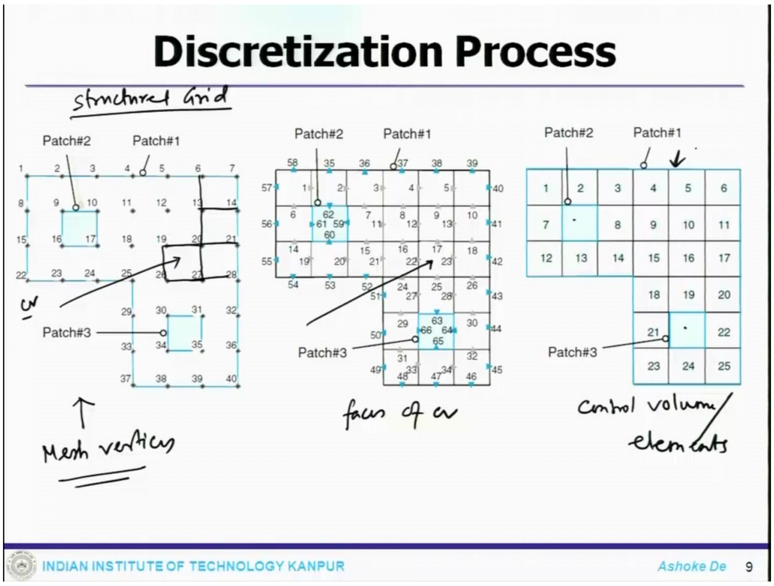

Now, if you identify them with the all sort of points. Now if you consider that structured grid, if you consider that structured grid with the quadrilateral elements, ok. So, this points all nodal points in this particular computational domain they actually represent the mesh vertices, ok.

This is my one individual computational a control volume. So, this is one of my control volume. And the control volume is connected with 4 vertices so these are called mesh vertices. And when this control volume is connected like this that forms a essentially the mesh,. Now second picture if you look at it, then each of these control volume, each of these control volume are having 4 different faces. For example, if you look at this particular control volume. Now this gives you the idea about this faces. So, this are faces of control volume, and what it helps? Individual control volume are connected with 4 faces.

And these faces are going to be very, very important as we see down the line of this lecture, that we need to have the understanding of this particular faces, ok, and finally, if you look at it or if you mark this whole thing 1, 2, 3, 4. These are all essentially control volume or elements; where I am going to have the solution done. And this will be my boundary condition for the sink, this would be the boundary condition for the source, this is the insulated boundary, ok. So now, I can have the mesh details.

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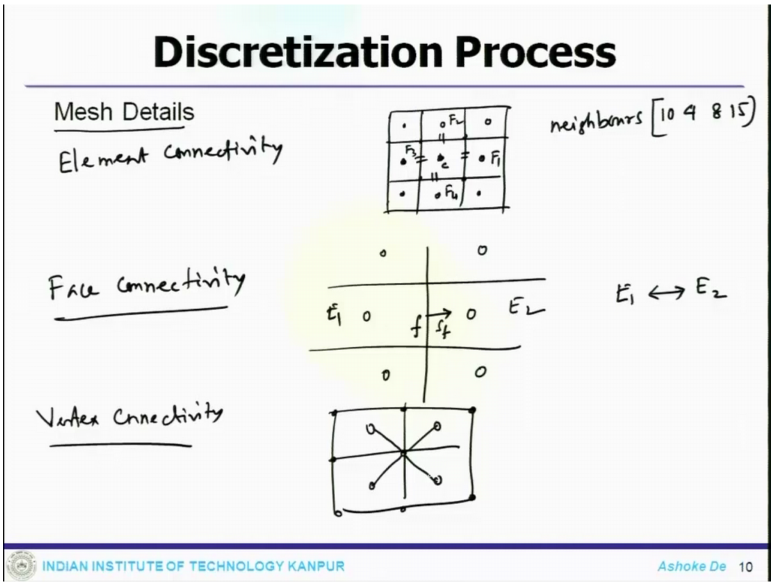

Now, mesh details one could be the important thing is the element connectivity, ok. So, element connectivity if I look at this particular condition. So, this is my domain, and I divide into 9 element. This is c, let us I this is there all, this is F 1, F 2, F 3, F 4, ok. So, this is around c, if you look at the neighbors. So, neighbors are 10, 4, 8, 15 so, based on the numbers ok. So, you can have faces, these are the faces, and these are the vertices. Now, one could be the face connectivity.

So, when you talk about the element connectivity you need to keep track of the elements; that means, one particular element, once you consider you need to see what are the neighbors now face connectivity if I just look at this situation, ok. Here is a element this is a element, these are the element. So, this is E 1 let say, this is E 2, then the neighbor of E 1 is E 2, and the connectivity of this is the face. So, one and it can go this normal of this face goes like this. So, E 1 and E 2, they are connected to each other their neighbor and the F is the corresponding face. And another way you can have the vertex connectivity, ok.

So, the vertex connectivity means, if I have let us say this 4 elements system, ok. And these are there. Now vertex connectivity would be if I connect this with these points. So, this is called the vertex connectivity of the system.

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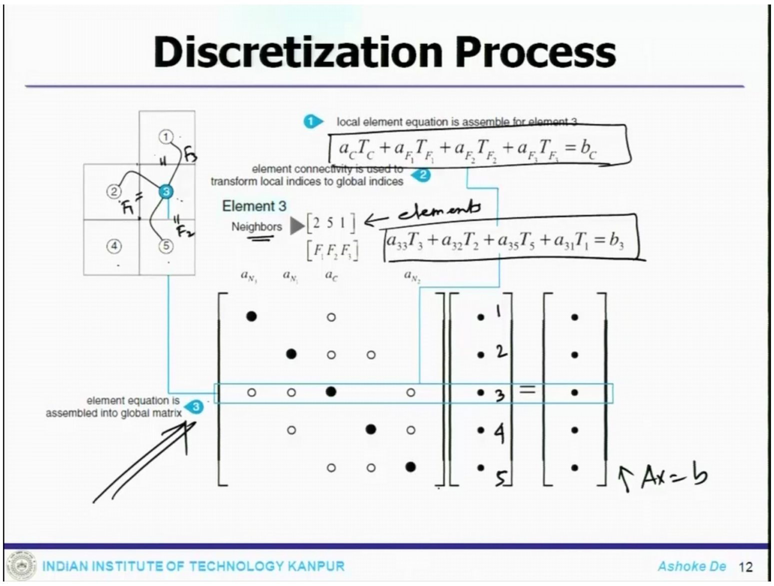

And once you take them into the complete domain so, this is my if you look at this particular some example of 1, 2, 3, 4, 5, now I am interested in the element 3. Now element 3 the equation if I discretize. So, do not worry about this discretization or the discretize equation. We will see how we get back this equation. If you look at element 3 what are the neighbors? So, neighbors are 2, 5, 1.

Because why we are calling into the neighbors 2 5 1, not the 4; because the element 3 and 2 they are having a common face of F this is the common face. Element 3 and one there is a common face, element 5 and 6 there is a common face, but if I look at element 3 and 4, they are connected with one common vertex. But they are connected with common faces. So, neighboring elements these are neighboring elements not neighboring vertex. So, you can say this is F 1, if I go by that this is F 2, this F 3; that means these are the faces. And I can write down my equation for that, ok. And if you look at in the complete system, this probably show the equation is talking about 1, 2, 3, 4, 5. So, all the elements the equations are written, and any somewhere between that one line will represent for the element 3.

So, that means, when you actually discretize the system like this. And you get back to the complete when you actually put them together, you get a system overall linear system like this. And all this in this particular system every individual element they are taken into consideration.

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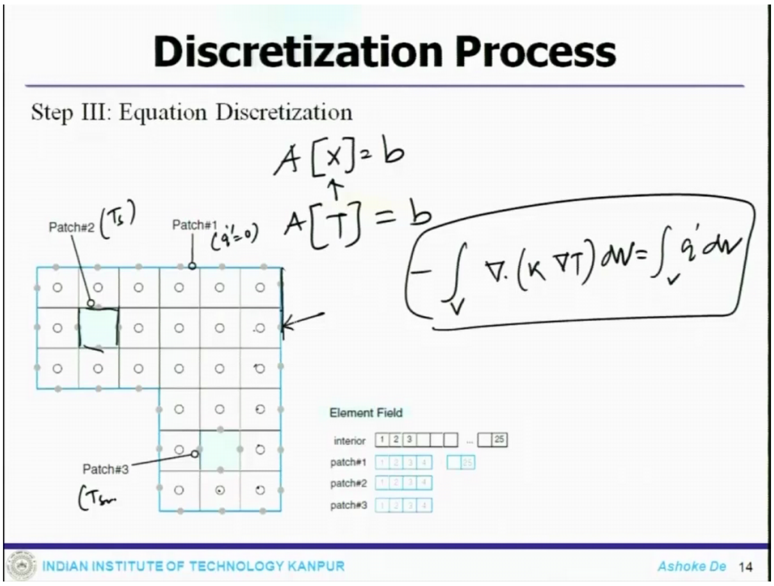

Now, if you go to the equation discretization. So, what is the equation? Basically I need to solve A X equals to b. And here x is nothing but my T. So, I can say A T equals to b. That is what I am trying to solve, ok. And where I am trying to solve? I am trying to solve over this particular computational domain, and these are all my individual element. This patch 2 is the T sink, this is t source and this is q (Refer Time: 20:53) is 0.

Now, here if you look at all these interiors points are 1, 2, 3, 2, 25 these are marked, patch one having some points, patch 2 having certain points patch 3. So, I have to make a demarcation of these faces, because these faces I have boundary condition.

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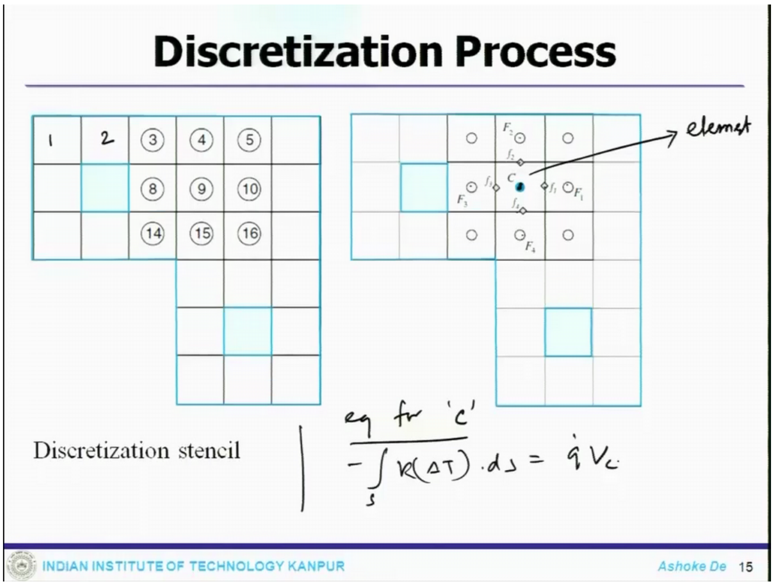

So, once you make a demarcation of these faces, you actually now look at the stencil. So, what equation you are going to solve here for all this is points is the delta dot k delta T dV equals to q dot dV. So, this is my equation, and this equation is valid is inside the domain as well as at these faces, but these faces there will be boundary condition for T. So, I have to apply my boundary condition to have this.

Now, when you look at the discretization stencil; so this is how you number the element. So, basically this could be one, this could be 2, that is how you sequential number the elements. And once you take an individual element, around that you mark the faces. Not only the faces what you have to mark is that the corresponding neighboring element. These elements are very important to keep track of it, because these neighboring elements are going to. So now, if I write down the equation, let us say I will write down the equation for c. There is a element this particular element, if I am I concentrate on that particular element. So, my equation would be minus k surface delta T dot ds equals to q dot V c, ok.

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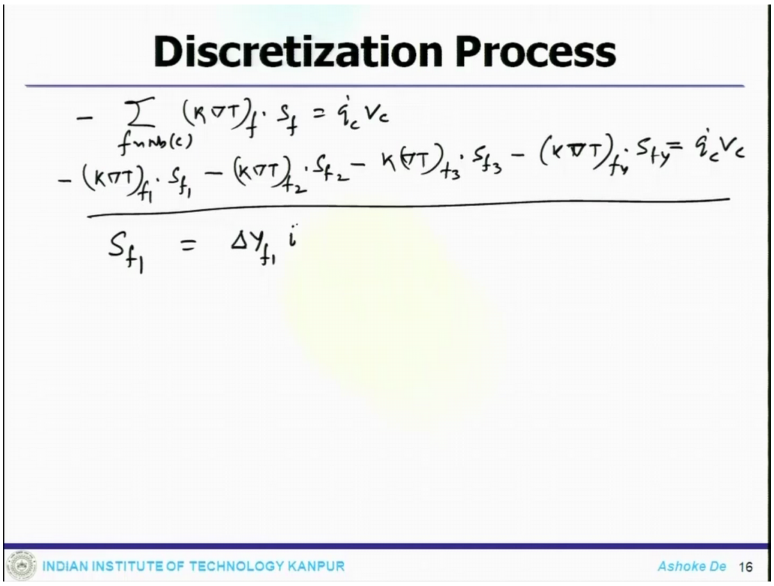

So, if I do that summation so around that c this is k delta T dot s f equals to q dot c V c, ok. And this f goes over that surface; that means, 1, 2, 3, 4 these are the surface. So, the summation would be over those surfaces. So, if I write that k delta T of surface 1 dot s f 1 minus k delta T f 2 dot S f 2 minus k delta T f 3 dot S f 3 minus k delta T f 4 dot S F 4 equals to q dot c V c. So, this is my now you see my steady state heat conduction equation boils down to this mathematical expression.

Now, what is my S f 1 S F 1 would be delta y f 1 i, ok.

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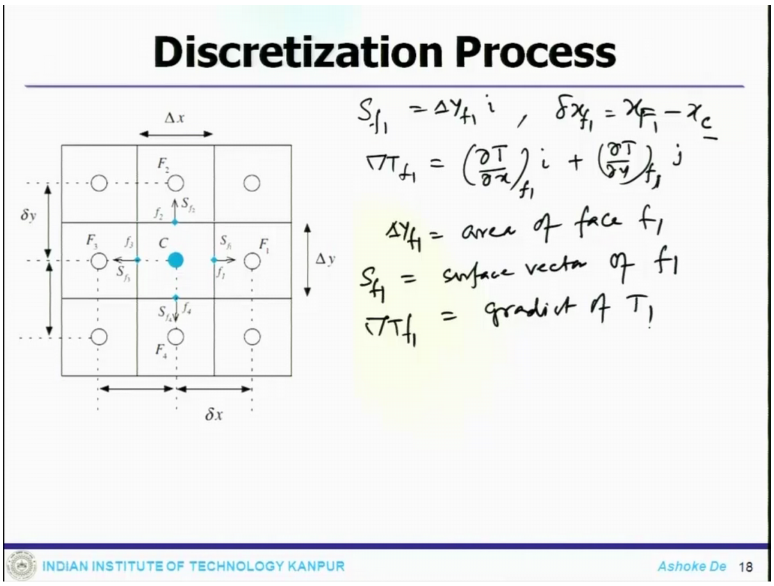

So, if you look at it the particular element c, now I am dividing this in the x direction this is small x the uniform width and this is delta y, then my S f 1 would be delta y f 1 i my delta x f 1 would be x f 1 my minus x c. So, this is the distance between this f 1 is the neighboring cell on the upstream site, and delta T f 1 is nothing but del T by del x f 1 i plus del T by del y f 1 j.

Now what is x c? X c is the co-ordinate of element c except is the x co ordinate of f 1 element then you have delta y f 1 delta y F 1 is the area of face F 1. And S f 1 is the surface vector of f 1. And then delta T f 1 is the gradient of T 1. So, this is how it transform to the system.

Now we will stop here today. And then take it up in the following lecture from here, how we proceed to get that final linear system.