Introduction to Finite Volume Methods-I

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Lecture – 05

So, welcome to the lecture of this Finite Volume Method. So, now, we are talking about the governing equations. Now, we will look at the equations in the general form; that means, we want to write the equations in the general conservation form.

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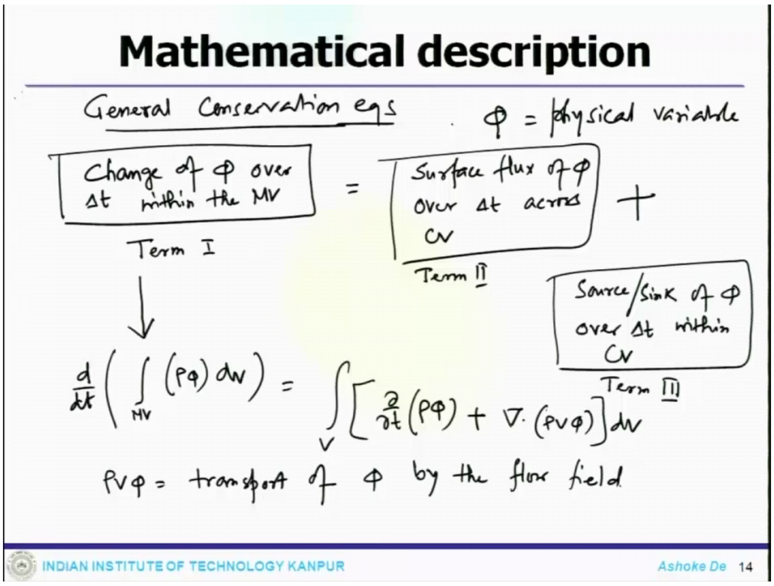

So, which will club everything together ok. So, what we have looked at so far is the mass conservation, momentum conservation, energy conservation and energy in different forms. Now, we will put everything together in one particular form. So, what you can say, let us say the any variable physical variable phi that is what we have been doing it, physical variable phi it could be.

Now, your change of phi over delta t time within the material volume M V ok, this is what is the left hand side of the system equals to surface flux of phi over delta t across control volume. So, that is quantum plus source or sink of phi over delta t within control volume. So, that is essentially my term 1, that is my term 2 and this is term 3. So, if you expand this term 1, so, this is nothing, but d dt of material volume rho phi dV.

So, in the integral, it be del del t of rho phi plus del dot rho V phi dV ok. And, this rho V phi is the transport of phi by the flow field or you can say the transport of the property phi due to underlying velocity field over there, ok.

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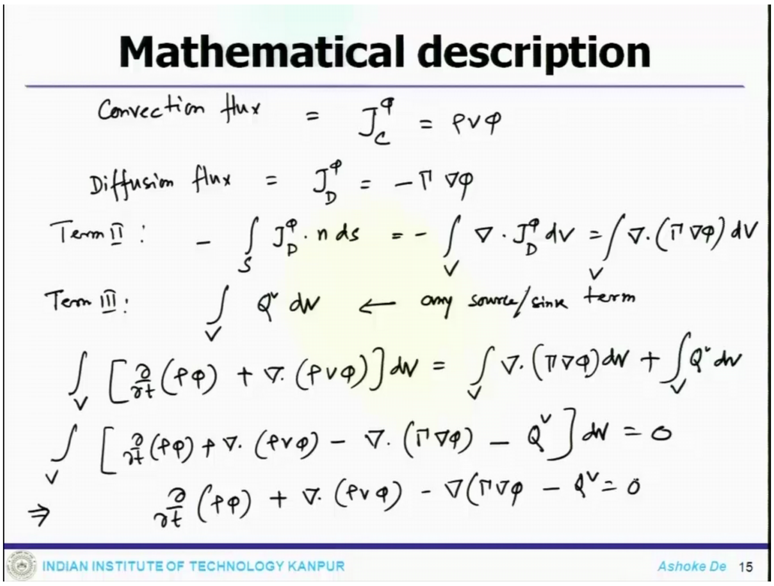

So, essentially this is my convection flux. So, I will designate that 1 convection equals to rho V phi. Now, I have the similarly, the diffusion flux ok. So, that is minus gamma phi delta phi or you can say, simple gamma delta phi. So, my term 2 that becomes minus surface integral J phi dot ds which is delta dot J phi diffusion flux dV and if you write over volume it is delta dot gamma delta phi dV and term 3 which is the source term essentially. So, what the control volume you write Q V dV. So, this could be any source term this could be any source term source or sink term. So, it is in the general notation.

So, if I put 3 together, then my system will become rho phi plus delta dot rho V phi dV equals to delta dot gamma delta phi dV plus the source term V. So, you take everything over one integral and then, you can write certainly rho V phi minus delta dot gamma delta phi minus Q V dV 0. So, which gives me back the system of minus minus Q V equals to 0 ok.

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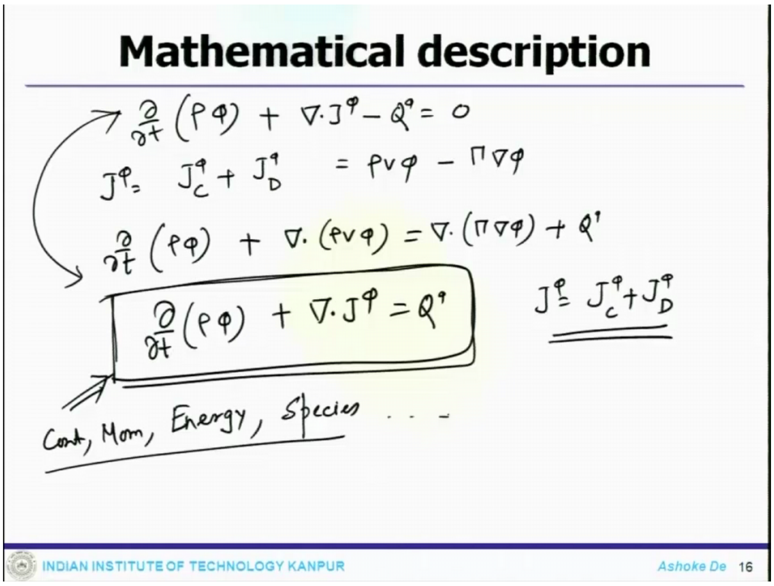

So, if I write them in slightly simplified form, this plus delta dot J phi minus Q phi equals to 0 and J phi is a combination of both, my convection plus diffusion fluxes which is rho V phi minus gamma del phi. So, the equation system will becomes del del t of rho phi plus delta dot rho V phi equals to dot gamma delta phi plus Q phi and in the other way around, if I put thing together. So, these 2 equations are essentially equivalent and I can write in a generic format like del del t of rho phi plus delta dot J phi equals to source term of phi.

And this J phi is the diffusion flux convection plus diffusion flux due to diffusion. So, this will take care everything in one simple form. So, that means, if you look at this particular equation, this can be used for continuity, this can be used for momentum, this can be used for energy, this can be used for species transport equation. So, you take any transport equation, they can be put together in this particular format ok.

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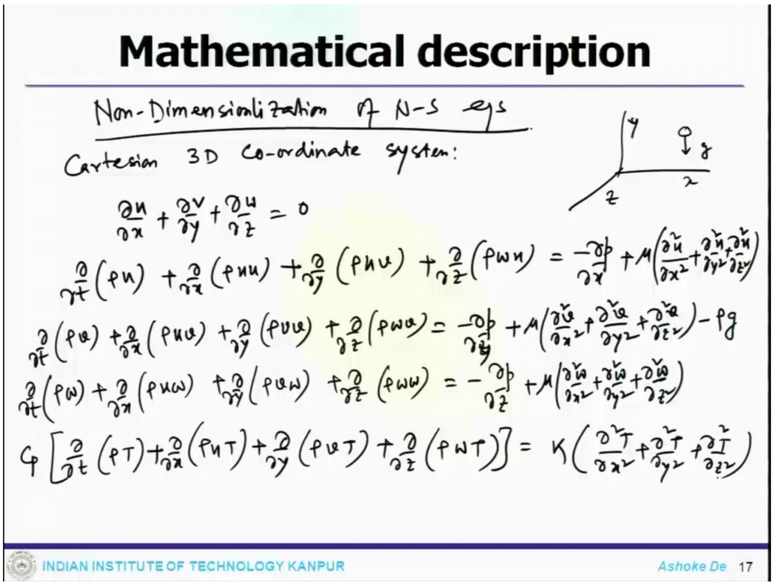

Now, it comes to the normalization of the Navier-Stokes equations or the non dimensionalization of the of Navier-Stokes equation. So, if you just consider a Cartesian coordinate system, Cartesian 3D coordinate system and if you consider that as a sign convention x y z, then you have a continuity equation ok. Then, I have a momentum equation, I mean basically x momentum equation del u u plus del y plus del del z rho w u equals to minus del p del x plus mu del 2 u by del x 2 plus del 2 u by del y 2 del 2 u by del z 2.

Similarly, my y momentum equation would be del rho v plus del del x of rho u v del del y of rho v v minus del p del z plus mu del 2 v del x 2 and if you have a source term. So, you just, this one you put x, this one y, this one z. So, the source term acted in this direction gravity. So, you take rho g in the y momentum equation and del del t of rho w plus rho u w rho v w z plus and the temperature equations if you write C p del del t rho T plus del del x of rho u T rho v T equals to kappa delta T 2 x 2 plus. So, here the temperature equation does not have any source term and if you look at the complete set of equations, they are in a conservative form.

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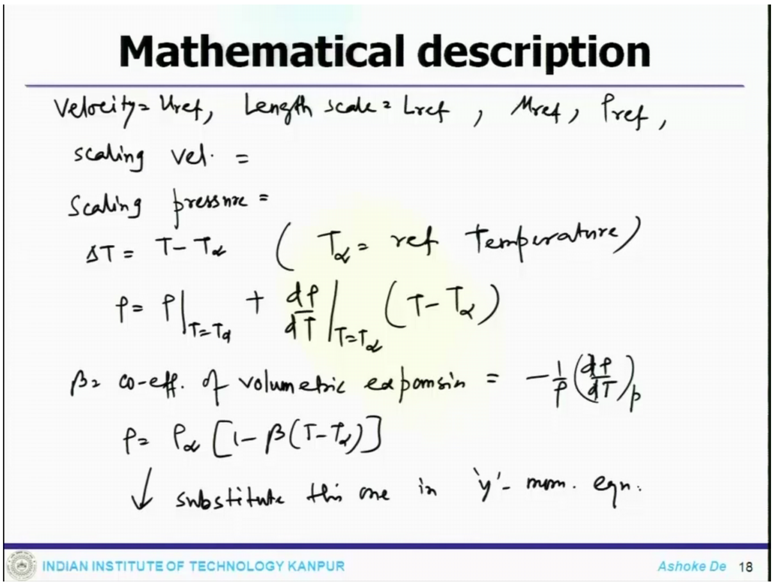

Now, to non-dimensionalize the system, you need the reference in velocity. So, that is, you say that u reference the referencing length scale that is one reference. Then, you have viscosity mu ref density reference. So, all these reference scales you use. Now, the velocity that you need to scale by the so, one is the scaling of the velocity and then, the scaling of the one is scaling of velocity; another is the scaling of pressure. So, once you do the scaling and then, we use them in the equation system ok.

Now, one more important thing that will come is that so, obviously, you will have a time scale and all these things; Now, that since we are using the temperature equation, the delta t with the reference temperature that you take into account. So, T infinity is your reference temperature and the density variation if you take with respect to temperature. So, you take with the Taylor series expansion with the only retaining the first order term and this beta is defined as the coefficient of volumetric expansion. This is coefficient of volumetric expansion.

So, you take the beta is 1 by rho d rho by at constant pressure. So, rho becomes rho infinity 1 minus beta T minus T infinity ok. So, using that, you actually substitute this one, this one in y momentum equation. So, then your scaling parameters so, one is the first thing will be the x.

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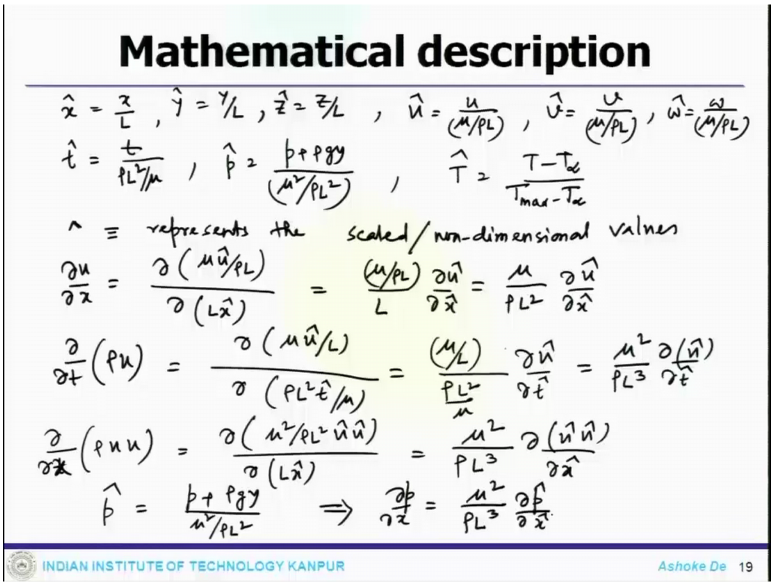

The scaled x should be x by L, y is y by L z is z by L. Then, the velocity scaling u would be u by mu by rho L. Similarly, v is v by mu by rho L then, w is w by mu by rho L, then your time T divided by rho L square by mu pressure is p plus rho g y divided by mu square rho L square and temperature T minus T infinity divided by T max minus T infinity, ok. So, all these tilde represents the scaled or non-dimensional values. Now, if you put them back together, for example, let us see if I take the first derivative or del u by del x which is essentially going to be del del of mu u hat divided by rho L L x hat.

So, if you take that mu by rho L outside and L outside, this will become rho u hat by del x hat. Essentially, this is mu by rho L square del u hat by del x hat. Similarly, I can done the del del t of rho u. So, del del t of rho u would be del of mu u hat L divided by del of rho L square t hat by mu. Mu by L divided by rho L square by mu. So, this will become del u hat by del t hat. So, essentially this is mu square divided by rho L cube del u hat by del t hat ok.

Similarly, you can have a term with del del t of rho u rho u because, this is there in my momentum equation. So, this should be del mu square by rho L square u hat u hat divided by del L x hat. So, it becomes mu square finally, by rho L cube del u hat by del x hat. So, you can do other terms also like for example, if you do the pressure term. So, this will become p plus rho g y divided by mu square by rho L square. So, if you transform them and then take the term of del p by del x, this will become mu square by rho L cube del p hat by del x hat.

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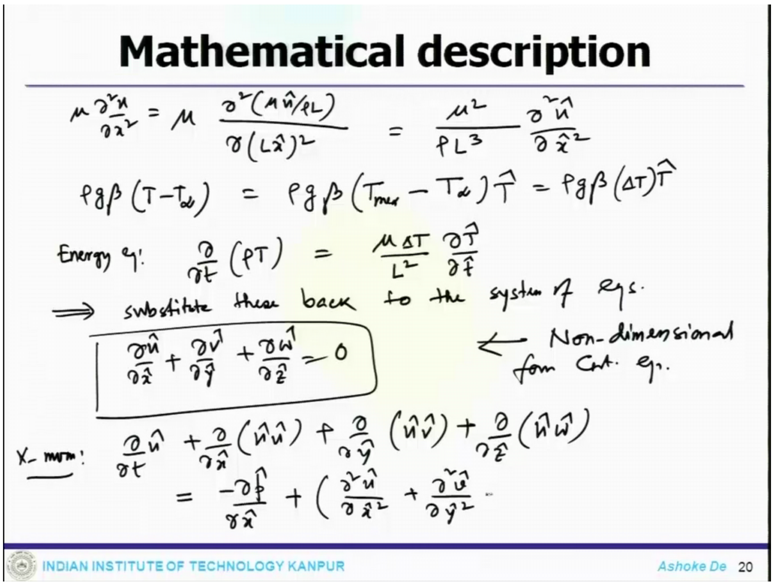

Now, diffusion term mu del 2 u by del x 2; so, that would be mu del 2 u hat by rho L hat square. So, finally, it becomes mu square divided by rho L cube del u hat divided by del x square and the final term that is rho g beta T minus T infinity. So, that becomes rho g beta T max minus T infinity T hat. So, this would be rho g beta delta T T infinity ok. And one more time in the energy equation if you look, so, you will have the term del del t of rho T. So, this will become mu delta T divided by L square del T by del t hat.

So, similarly if you put and if you transform and substitute all these back this back to the system of equation so, your continuity equation becomes now only del. So, there is no change with the continuity equation except the hat. So, this is my non dimensional form of continuity equation x momentum equation. So, that becomes del del t of u hat del del x hat u hat u hat plus del del y hat u hat V hat plus del del z hat u hat w hat equals to minus del p hat by del x hat plus del 2 u by del square plus del 2 V by square by del square.

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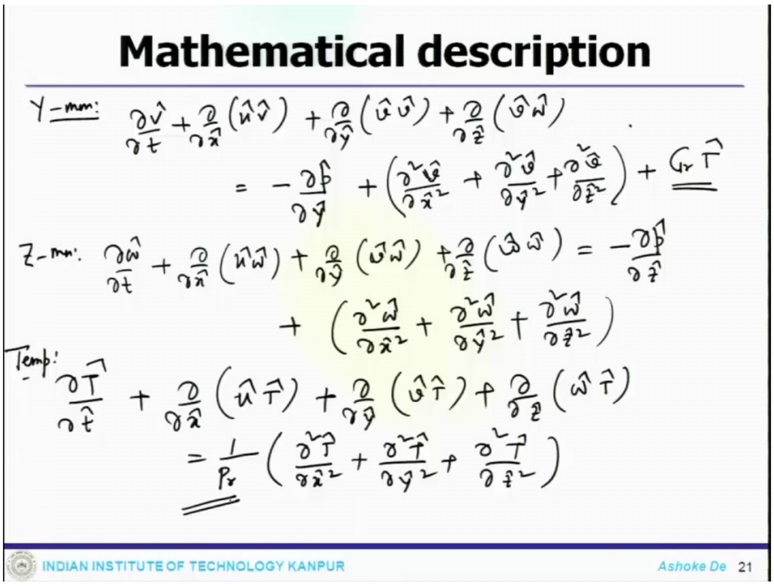

Now, my y momentum equation that becomes del V hat by del t plus del x hat u hat v hat plus del y hat then, del equals to minus del p hat plus del 2 del 2 v hat by del y 2 square plus Grashof number with T hat. So, look at this particular term. So, that is a non-dimensional term or parameters which is coming there and z momentum equation will be again similar. So, there will be no extra term plus the diffusion term.

And the temperature equations that becomes del T hat by del t plus del del y hat v hat T hat plus equals to 1 by Prandtl number into del del T hat square plus del 2 square. So, if you look at it, there is another component or the term which comes in the temperature equation.

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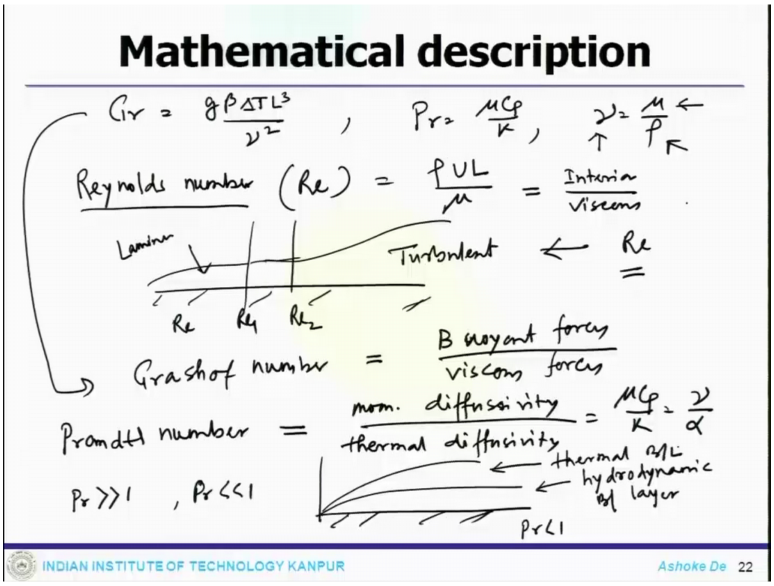

So, essentially, your Grashof number is g beta delta T L cube by nu square and Prandtl number is mu C p by K where nu is your mu by rho. This is kinematic viscosity, this is dynamic viscosity, this is density. So, once you do the non-dimensionalization, then you come across with different non dimensionalizational parameters. So, what are those non dimensionalization parameters? So, one of the important non-dimensionalizational parameter which come across is the first thing is the Reynolds number ok. So, Reynolds number is one parameter which is typically represented as R e which will be rho U L by mu.

So, essentially this is a ratio of 2 forces; one is the inertia forces to the viscous forces. So, that actually dictates if the flow is laminar or turbulent depending on the Reynolds number and where the Reynolds number is going to be dominants. Let us say, for example, if you look at a flow over a flat plate, the flow actually propagates like that and through after some region it goes like that.

So, this particular region up to some Reynolds number, this is a boundary of Reynolds number 1, then this is a Reynolds number 2. So, in this zone, the flow is laminar; then this zone, it is turbulent. So, depending on actually the Reynolds number, one can classify the flow to be laminar or turbulent. Now, the other parameter which is coming is a grashof number. So, Grashof number is nothing, but the ratio of the buoyant forces to the viscous forces. So, this appears due to non dimensionalization in the parameter equation of the temperature equation.

Now, Prandtl number; Prandtl number is another non-dimensional number which is essentially the ratio between momentum diffusivity to thermal diffusivity. So, if you look at the Prandtl number, this will be mu C p by K. Essentially, I can write that nu by alpha ok. Now, depending on the Prandtl number, so, Prandtl number could be greater than 1 or prandtl number could be less than 1.

Now, depending on the Prandtl number, actually it is a ratio between if a same flow if you look at it and the flow evolves over a flat plate, then these are the 2 layers ok. The, if Prandtl number is less than 1; that means, your momentum diffusivity is less than the thermal diffusivity. So, this will be your hydrodynamic dynamic layer or boundary layer and this is your thermal boundary layer while Prandtl number will become greater than 1, this would be completely reverse.

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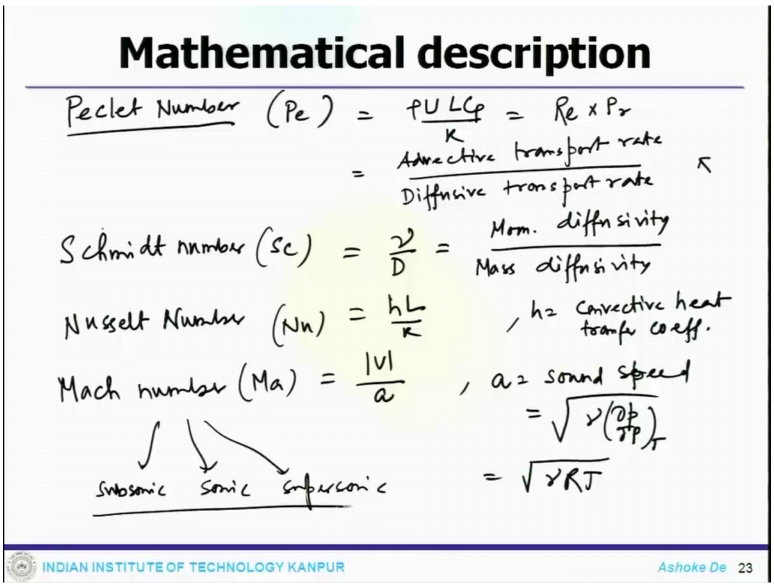

So, then your now another non dimensional number is called the Peclet number. So, the Peclet number typically is represented by P e and this is rho U L C p by K. Essentially, it is a product of Reynolds number into Prandtl number. So, it is a ratio of advective transport rate to it is diffusive transport rate. So, it is a ratio between these 2 things.

And this would be the number which will be very important while we will be actually doing the finite volume discretizations and over a cell, we will look at what is the conditions on Peclet number. Now, once you go back, then there will be another number called the Schmidt number. So, that is S c this is again the ratio of the momentum diffusivity to mass diffusivity.

So, this is momentum diffusivity to mass diffusivity. So, this will be another important parameter, when people look at the species mass transfer equation, now in the heat transfer there will be an non dimensional number which you will require is the Nusselt number. So, Nusselt number is estimated as h L by k. So, where h is your convective heat transfer coefficient ok now, one more for the case of Mach number.

So, this essentially is the ratio of your sound speed to the velocity scale and a is your sound speed at a given temperature and you can estimate is a gamma del p by del rho at a typically this is gamma R T. So, these are the number that comes across and this will determine whether the flow is subsonic supersonic sonic or supersonic. So, depending on the Mach number, you can define different resume of the flow field.

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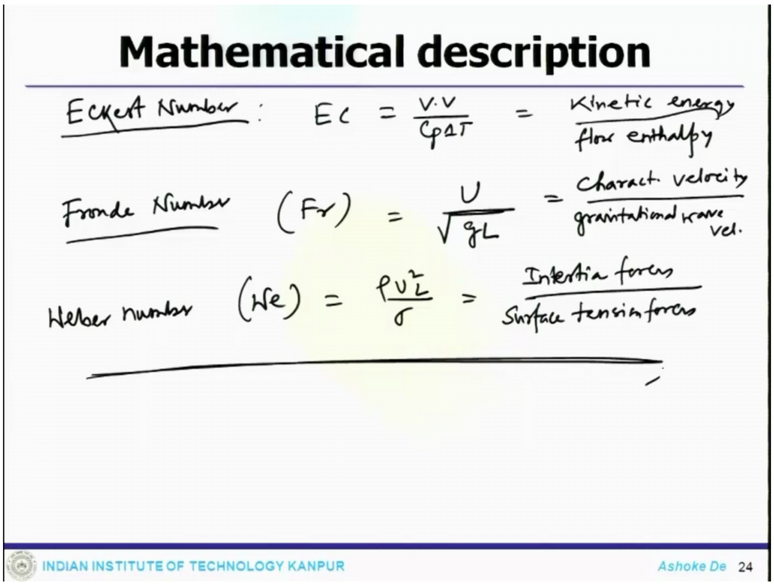

So, this is another number which appear in the heat transfer problem is the Eckert number. So, that actually correlate the kinetic energy to the flow enthalpy. So, it is a essentially the kinetic energy, ratio of the kinetic energy to flow enthalpy. So, this is and when talk about the hydrodynamics, then you get Froude number which is typically represented as F r which is ratio between U by root g L. So, it is a characteristic velocity scale between characteristic velocity to the gravitational wave velocity. It is a ratio between these 2 number.

And when you deal with the multiphase system, then you come across Weber number which is rho U square by L by sigma which is the ratio between inertia forces to surface tension forces. So, these are the non-dimensional number that actually you need to know when you actually non-dimesionalize the system. So, now, once you dimesionalize the system under different different condition, you come across different different number and this is where you need.

So, what we have talked so far is the all the governing equations and their non dimensionalization and once you do the non dimensionalization. So, what kind of number you come across? So, we will stop here today and we will take from here in the follow up lectures.

Thank you.