Introduction to Finite Volume Methods-I
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Lecture - 04

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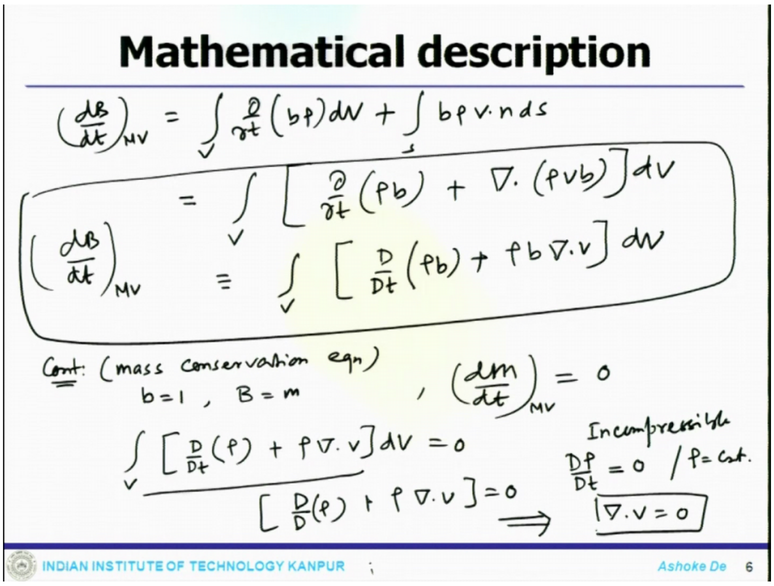

So, welcome to the lecture of this Finite Volume Method.

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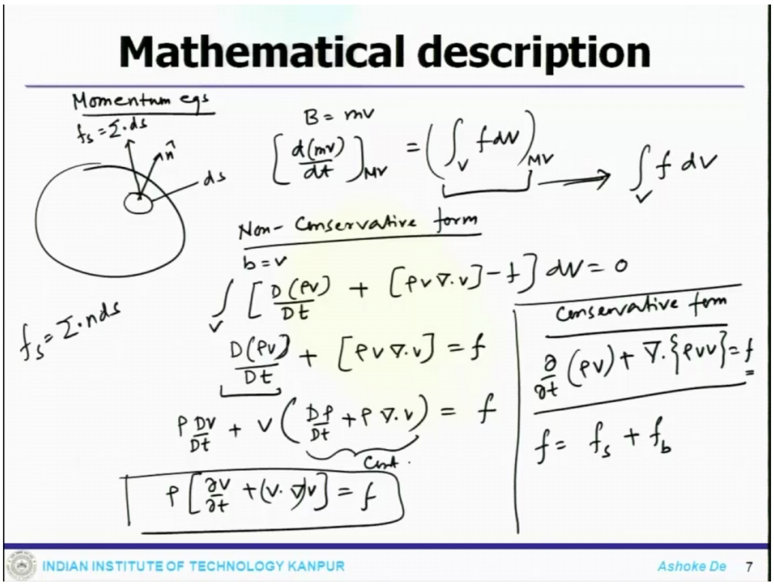

Similarly, if you have a now I we will get the momentum equation, ok. You have a some control volume like this and you have some element. So, this is your ds this is your unit normal, and this is your let us say force vector. So, this kind of element now for continuity equations your system would be now B would be mv. So, then this should be d mv by dt for material volume equals to f dV for the material volume yeah. So, this guy I can write as a integration of f dV.

Now, you can have the equation in two form, one called non conservative form, ok. So, in non-conservative form, if you write the equation, your small b would be v, then this will get you this plus del rho v by dt plus rho v delta dot v minus f dV equals to 0. So, to become this integral true so, this become rho v by Dt plus rho v delta dot v equals to f. If I expand this term the first term if I expand. So, I can get rho DV by Dt plus V d rho by Dt plus rho delta dot v which is f, this essentially my continuity equation.

So, that actually finally, applying the continuity and expressing the material derivative, you can get rho del y by dt plus v dot del v equals to f. So, this form of the equation is called the non-conservative form. Now at the same time, I can have the conservative form. So, to have the conservative form, I can use the other system. So, the conservative form it should be written like del del t of rho v plus delta dot rho v v equals to f. So, this is your non conservative form. Now the important point comes what is this f? F is the so, there could be two component to f, one is the surface force another could be the body force.

Now, what are the surface forces? Surface forces if you look at this system the surface forces would be so; f s is summation dot ds. So, it could be equals to summation dot n ds, ok. And this is nothing but the 9 component tensor.

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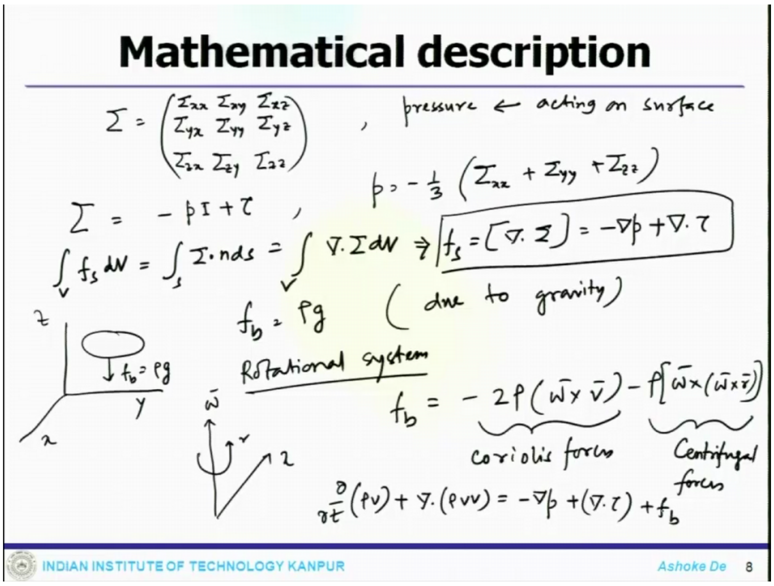

So, this starting with xx, xy, xz, yx, yy, yz, zx, zy, zz, ok now, you have also the pressure which is acting on the surface. So, if you combine those two things together, then this total force can be written as minus p I plus tau, where p is nothing but one third of psi xx, psi yy, psi zz, ok. So, that actually gets back you the; now if you look at that integration f s dV, this will be now surface integral, this dot n ds which is now delta dot dV f s is. So, which is nothing but del p plus del dot tau so, that is my surface force. Now the body force in a system you if you have a, it will be acting like. So, f b is nothing but my direction. So, only the body force is due to gravity ok. So, body force due to gravity is only concerned.

So now this there could be possibility, if you have a let say system rotation rotational system. If you have a rotational system, then you can also have some component. Let us say this is my x, this is my omega and this is goes like that. Then the body force could be due to minus 2 rho omega cross v minus rho omega cross omega cross r. Now this is nothing but my coriolis forces. And this is nothing but my centrifugal forces, ok. But essentially my momentum equation becomes like del del t of rho v plus delta dot rho v v equals to minus delta p plus delta dot tau plus f b. So, that is the equation that becomes. Now, you have a relationship between tau which is the shear stress. So, we are talking about all Newtonian fluid.

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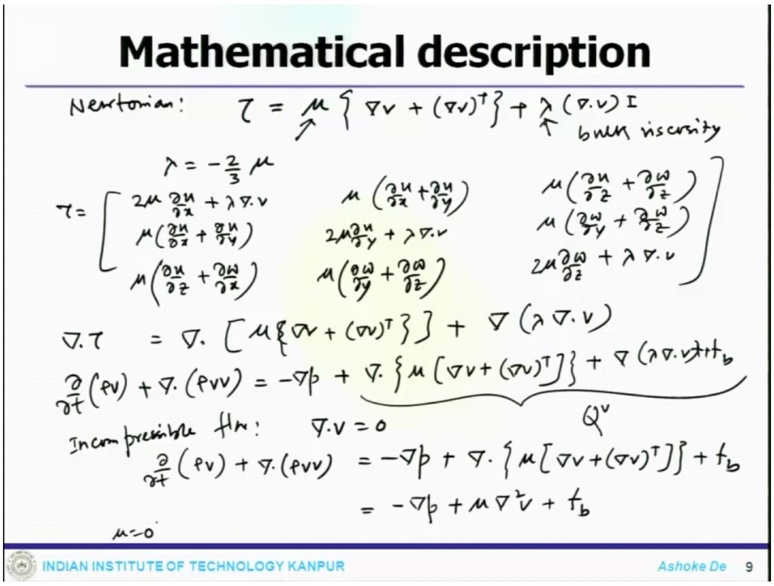

So, the Newtonian fluid the relationship of tau is mu del v plus del v transpose plus lambda delta dot v i, ok. Now where mu is the here mu is the molecular viscosity, and lambda is the bulk viscosity. And the, from the stocks hypothesis we get lambda equals to minus 2 third mu.

So, essentially this term now tau, that becomes 2 mu del u del x plus lambda del dot v mu del u del x plus del u del y. Third component is mu del u del z plus del w del z. Second component is mu, again del u del x plus del u del y. This is 2 mu del u del y plus lambda delta dot v. This is del w by del y plus del w by del z. Third component is mu del u by del z plus del w by del x mu del w by del y plus del w by del z. And this is 2 mu del w by del z plus lambda delta dot v so, that is the component. And essentially delta dot tau would be delta dot mu delta v, plus transpose plus delta lambda delta dot v, ok.

Now, if you put everything together in the final equations. So, you get equation of del del t of rho v plus delta dot rho v v equals to minus del p, plus del dot del v plus del v transpose. Plus, delta lambda delta dot v plus f b, yeah, ok. So, this complete term you can say this is q v. And I can simplifying this equation for incompressible flow. Now, incompressible flow I have delta dot v equals to 0, which brings down to this equation as del del t of rho v plus del dot rho v v minus del p plus del dot mu delta v plus the body force, ok.

Now after simplifications one can write in the right hand side is essentially minus del p plus mu del 2 v plus f b. So, this is what your momentum equation for incompressible flow. So, one can make further assumptions. If you have a inviscid flow, then mu could be 0, and then you can get the equation for the so that gets you the momentum equation. Now we get the energy equation.

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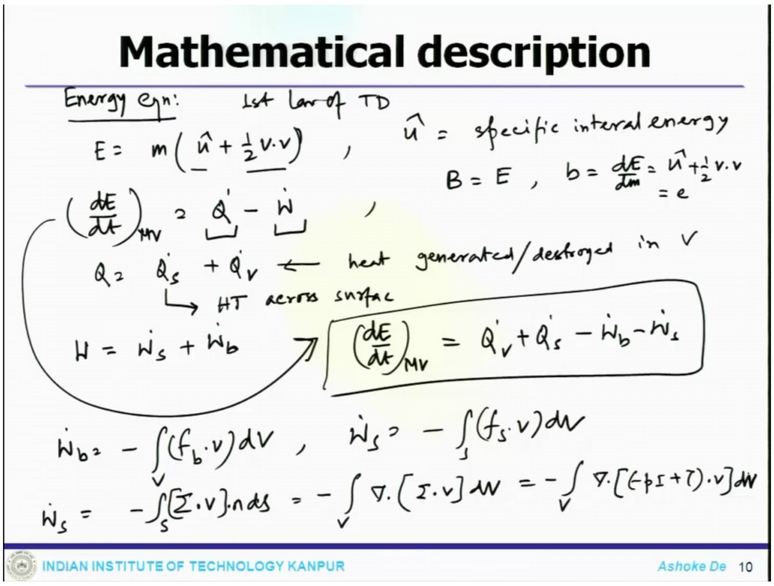

For energy equation this is purely based on first law of thermodynamics, ok. So, the first law of thermodynamics says for the any isolated system. The energy neither can be destroyed nor can be generated. Using that only so, you define the total energy E is the where u is the specific internal energy.

So, the total energy having two component; one is the internal energy another is the kinetic energy. So, from the energy law, I can say that for this material volume, this is heat minus work so, that comes from the system. Now my B here from the RTT, if I write B equals to E and small b is dE by dm which is u plus half v dot v. Which I can say small e, ok. Now this is the heat transfer and this is the work done. So, this is following the general sign convention. Now q will have two component, q is having one surface component plus 1 is the volume component, ok.

So, this component referred to the heat transfer across surface, and this is the heat generated or destroyed in control volume in v. Similarly, w will have two component. One is W dot s plus W dot b so, these are the two components. So, total equation now this equation if I have to write now this is my dE by dt in the material volume is Q v plus Q s minus W b minus W s.

So, that essentially in a nutshell my energy equation now, you can use the definition of the work done, then I can estimate W b would be the work done due to forces and W s would be the work done due to the surface forces. So, my w dot s is nothing but s summation dot v n ds which is over control volume delta dot summation dot v dV. Delta dot minus p I plus tau dot v dV. So, this is my surface force.

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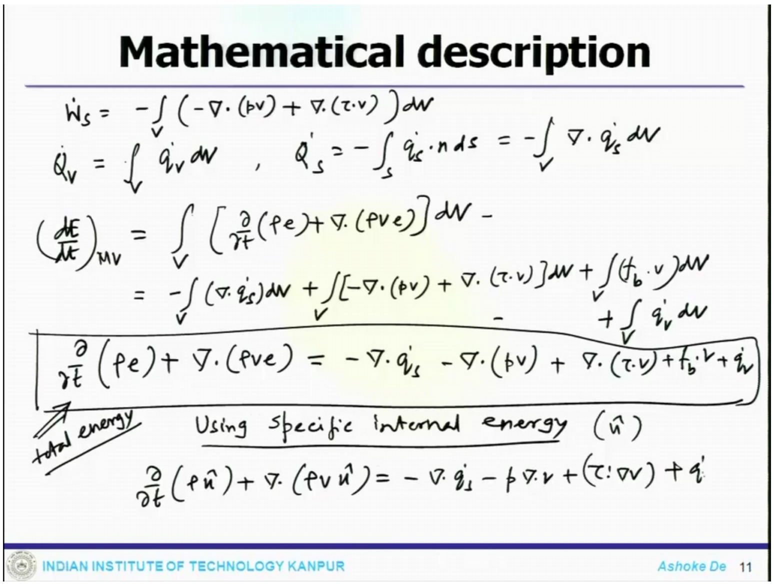

So, if I write it back the work done due to surface force is nothing but my v delta dot p v, plus delta dot tau dot v dV. Now, similarly I will get heat in the control volume. Either it could be heat generated or it could be heat destroyed. So, it can act as a source or sink. So, this could be volume q dot v dV. Similarly, at the surface, it could be surface integral of q dot s dot n ds; which is delta dot q dot s dV. So now, if I put them together in my energy equation, DE t by d m v equals to v del del t of rho e plus delta dot rho v e. This is left hand side; this is equals to minus v delta dot q s dV plus volume delta dot p v plus delta dot tau dot v plus dV. So, these are the all essentially components that you get.

So, if you take everything into the account of the left hand side and the right hand side. Finally, I can write down the equations in this particular form rho e plus delta dot rho v e equals to delta dot q s minus delta dot p v plus delta dot tau dot v f b dot v plus q dot v, ok. So, essentially this is how you get your total energy equation. Now this is written in a total energy form. So, this form is total energy in that constant e is the specific total energy. Now the energy equation could be written in multiple different form.

So, for example, one can write only using the specific internal energy. So, specific internal energy is u bar. So, you can use your momentum equation and the energy equation together and do the algebraic simplifications and finally, you can get the in this format, rho u plus delta dot q dot s minus p delta dot v plus tau double dot delta v q dot v.

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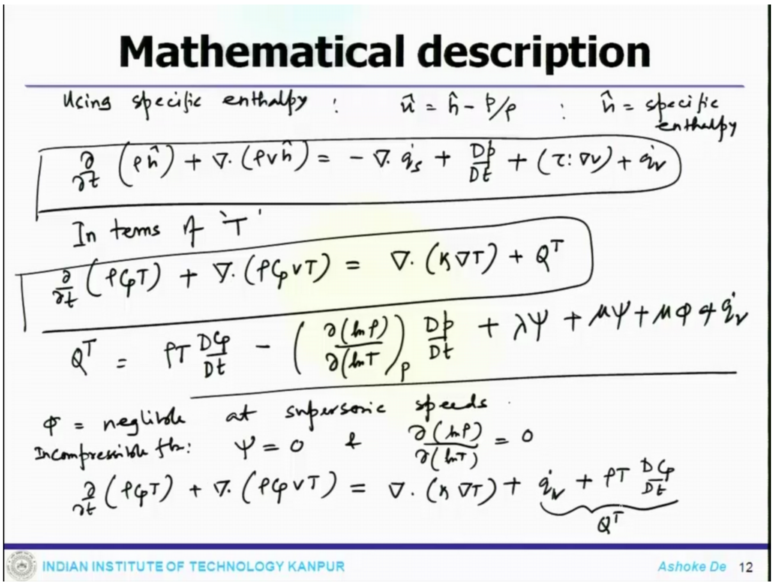

Similarly, if one uses using specific enthalpy; that means, is h minus p by rho. So, this h is the specific enthalpy. So, if you write in that term, then it should be del del t of rho h equals to minus del dot q s plus Dp t plus tau double dot del v plus q dot v. So, that is the form I can have in terms of this. Now the one which are more people are familiar or which is more commonly used in terms of T.

So, in terms of T if you right so, this would be looks in a simplified form as del del t of rho C p T plus delta dot rho c p v t equals to delta dot K delta T plus T. So, this is in terms of temperature, where my Q t is nothing but rho T d C p by Dt minus del del of ln rho by del del of ln t at constant pressure into dp by dt plus, lambda psi plus mu psi plus mu phi plus q dot v. So, these are all different different component of that q t.

Now, one can use this simplification. So, like the phi has very negligible value, it is negligible for large velocity gradients. Negligible at supersonic speeds similarly, for incompressible flow the psi becomes 0. And also this term del del of ln P by del del of ln t this guy becomes 0. So, this equations boils down to a simplified form of del del t of rho c p del t plus delta dot rho C p v T equals to delta dot K del T plus q dot v plus rho T D C p by Dt so, that is nothing but my Q T,.

Say further simplification like if viscosity is neglected, like if you this is neglected, then it can be further simplified to C p del del T of rho T plus delta T dot rho v T equals to del dot kappa delta t plus dp by Dt q dot v.

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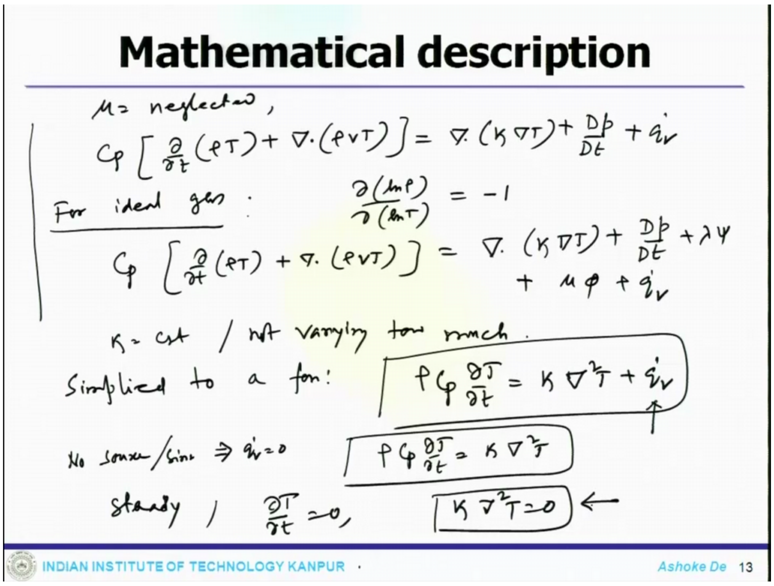

So, there are the different different simplifications, one can make like in a nutshell for all these simplification the equations become essentially, like for if you use an ideal gas. For ideal gas this term rho ln rho by del ln T, this is minus 1. And the energy equation is actually becoming C p del del t of rho T plus delta dot rho v T equals to delta dot kappa delta T plus Dp by Dt lambda psi plus mu phi plus q v.

So, it is basically different different conditions, you can simplify the equation in a different sense. And when you assume this thermal conductivity is suppose is constant or not varying too much, then the equation could be simplified to a form to a form where it is can be written rho C p by del del t by del t equals to kappa delta 2 by del T plus q dot v. So, if you look at it this is one of the very simplified form of the temperature equation which is commonly used in the fluid flow heat transfer problems.

Now, a now this can be also if you look at the terms here this term is the heat source or sink term or heat generation term in the control volume. If no source or sink, then this guy can be 0. Then this equation again becomes a simplified form of the system. Like, del t by del t equals to only kappa del t 2. So, this is the unsteady heat conduction diffusion system. Again if it is steady, then the unsteady term goes off, then it becomes kappa delta 2 t equals to 0. So, this becomes the steady diffusion system.

So, this is how the energy equation could be written in a multiple form. It is starting from total energy, specific internal energy, could be specific enthalpy or in temperature. Now depending on the situation you can simplify the form to a particular system. So, we will stop here, and in next lecture we will take up from here and see how the equations can be looked at in a more generic form.

Thank you.