Introduction to Finite Volume Methods-I
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Lecture – 03

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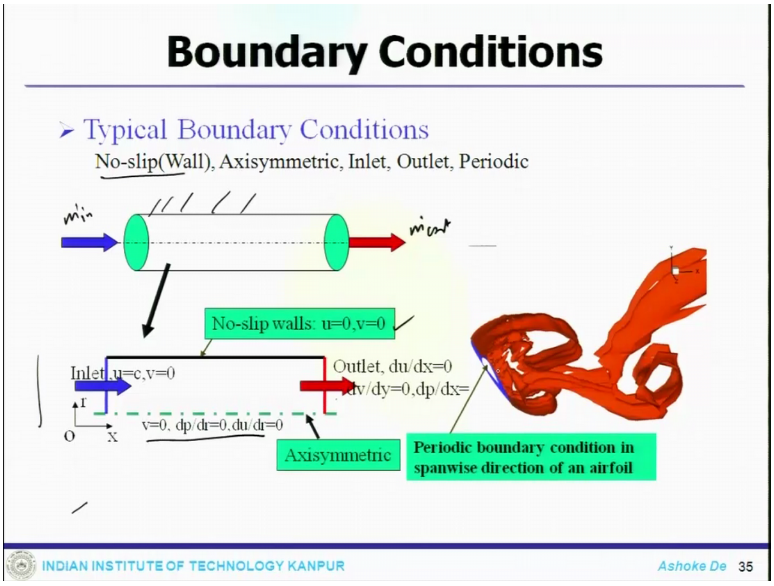

So, welcome to the lecture of this Finite Volume Method. Now, we will look at the boundary conditions. Now, when you look at the boundary conditions, again when you talk about the fluid flow system; Fluid flow system essential you are solving, and a very normal boundary conditions is one of the important boundary condition is the no slip condition; that means, wall you have a no slip. So, this is the channel which we are talking about I am solving the channel when the mass come in and mass goes out. Now, if you want to solve this particular channel through the numerical methods, you need a boundary condition. So, this will be the wall of the channel which will be no slip boundary condition; that means, your flow components are 0.

Then if I solve the axisymmetric problem of the channel; that means, if I cut through the half of the channel and put it here, then this particular boundary the axisymmetric boundary condition would be imposed. So, this is the axisymmetric boundary conditions, and at the outlet there will be outlet boundary conditions, it could be either gradient 0 or some sort of a conductive boundary condition 0 if it is a unsteady problem. Now, when you solve a flow around airfoil kind of structures along the span; you sort of periodic boundary conditions. So, these are the sort of boundary conditions that you use.

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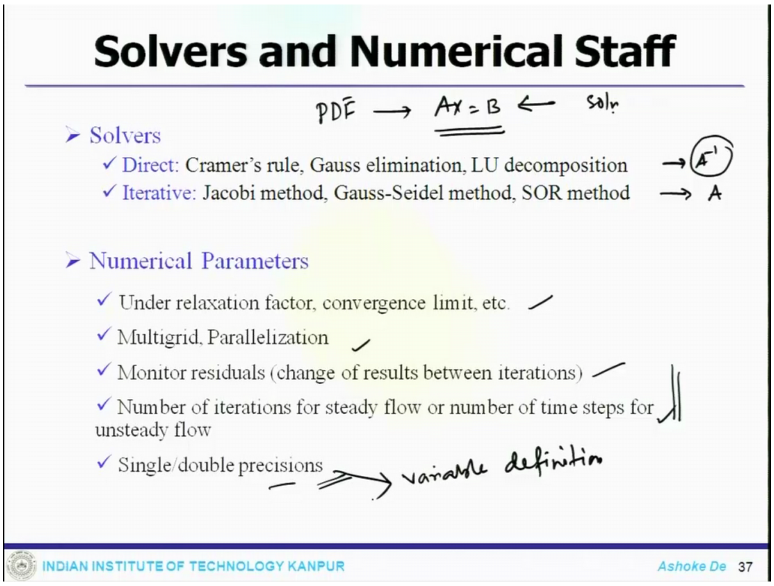

Now, when you look at the numerical things, numerical things mean essentially my PDEs has led to the linear solver of Ax equals to B. So, this is where I have to use different linear solver to get the solution. So now, the solvers could be of the direct solver; that means, I will get it directly A inverse which is always a tedious task, and not a easy task, or it could be iterative solver. So, that means, I will get a solution of the A through iterative process. And the parameter that actually controls the whole business is under relaxation factor, convergence limit.

Then, how do you paralyze the system that means, how I efficiently get a solution of this linear system. Then residual that means, how things are converging with time that means, the error between each of this time iterations or the physical problem the error actually getting reduced. Then, how quickly I am getting iterations over time these are essentially connected with my floating point calculation; that means, you have a powerful computer, you have a efficient numerical technique, you have a programming that can be used to get more and more solutions per second.

Then the precisions that again has to do with the computer architectures. You have a high end architecture, you can get better precisions, you have a low end architectures you can cannot get lower precisions of the solution. And to some extent how you define your variables in the systems so, the data structures and the variable definition. So, this would be the factor how precision I would expect error.

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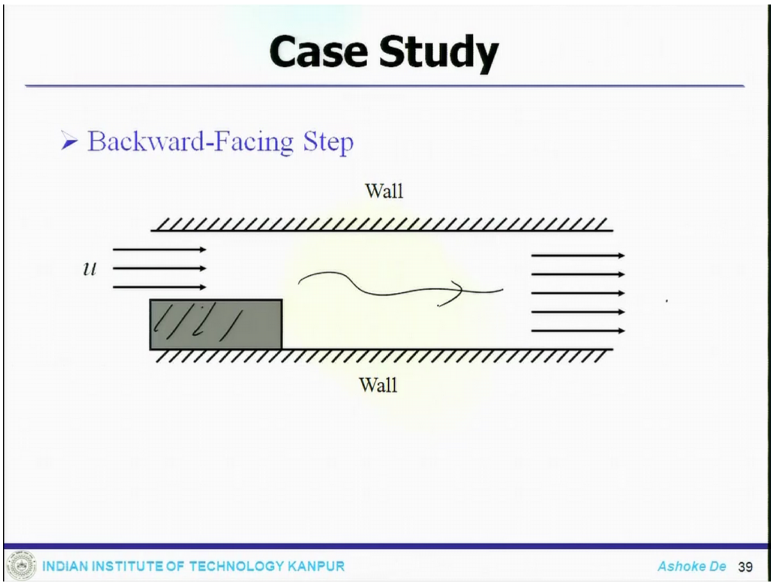

Some particular problem is a simple problem, we have been talking about the flow over a channel, now if you put a block like this then this is also going to act like a no slip problem, flow comes in flow goes out and there are different different flow phenomena, you can expect in this particular process. And using the numerical technique, you can solve this particular problem.

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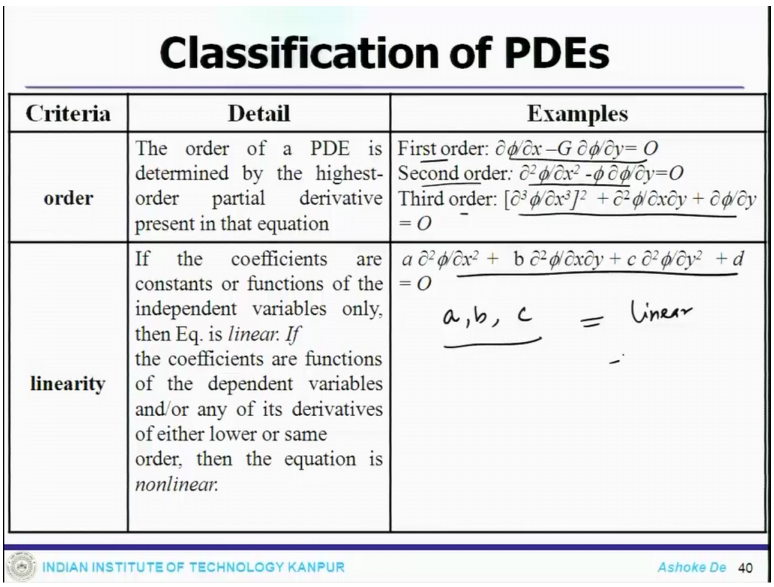

Now, moving at, one thing is there we have been show for talking about the set of partial differential equations. Now, when you talk about the differential equations, these are essentially few important information regarding the partial differential equations are required. One is the order of the equations. The order of the partial differential equation is determined by the highest order. So, if you look at this particular equation which is in a terms of del phi by del x and del phi by del y, the highest order which is present is the first order. So, this is called the first order equation.

Now, if you look at the second equation which is of the second derivative in x, and first derivative in y, these actually give my second order equation. When you look at this one particularly, the highest derivative present is the del 3 phi by del x 3. So, that gives you the third order equation. So, essentially looking at the particular partial differential equation, one can immediately identify what is going to be the order of equation. Whether it is a first order whether it is a second order whether it is a third order the then the linearity.

So, this is generic form of the partial differential equation. When this coefficients this a, b, c this coefficients are constant. Or function of the independent variable then the equation is call the linear equation; means, are not constant or not the function of the independent variable, then they are call the non-linear system. So, any non-linear system is always hard to deal with.

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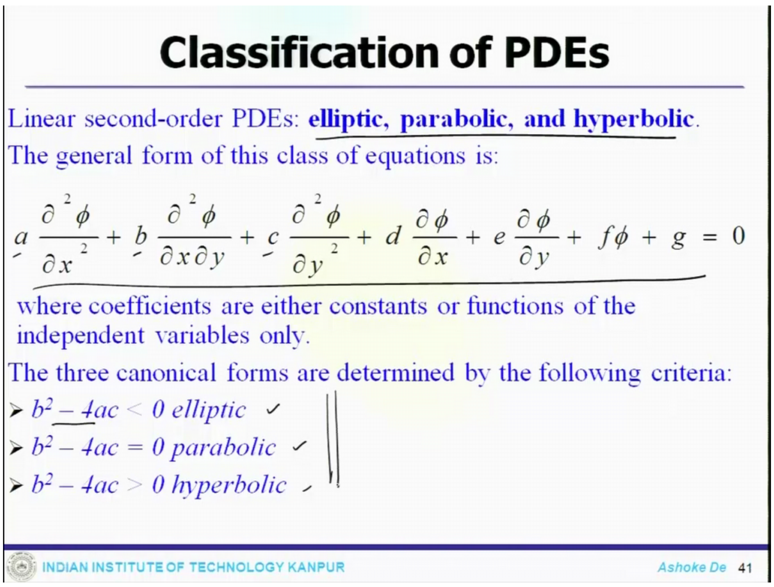

Now, if you look at the linear second order PDE now, linear second order PDE could be classified in these categories. One is the elliptical, could be parabolic, could be hyperbolic in nature, ok. Now, as I said this is a general form of the equation, where the coefficients are either constant; that means, this a b c these are either constants or function of the independent variables.

Then looking at these term or the discriminate b square minus 4 ac one can say whether the equation is elliptic, then this has to be less than 0. If they are 0, then this is parabolic, if b square minus 4 ac is greater than 0 this is hyperbolic in nature. Now by looking at a PDE, one can immediately calculate these coefficients and look at the nature of the particular differential equation whether it is elliptic in nature, whether it is parabolic in nature, whether it is hyperbolic in nature.

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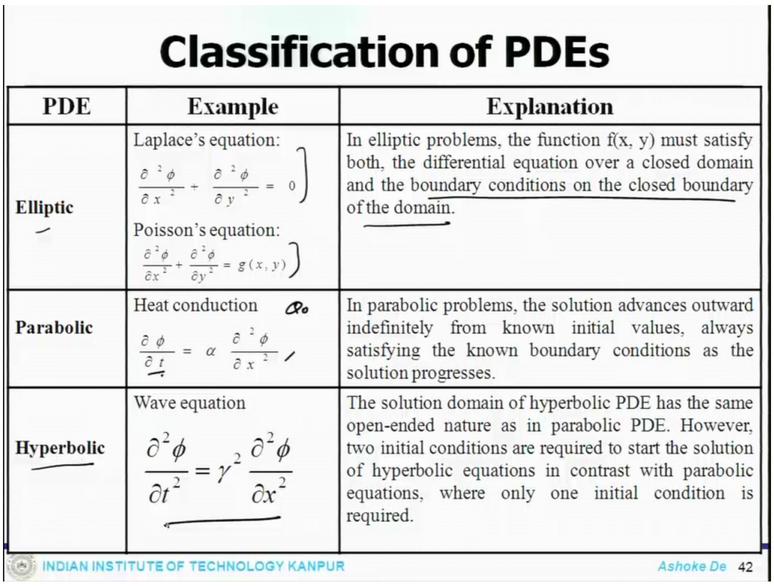

Now, once you distinguish then they come with the certain system. Now, example of the elliptic equation; the elliptic equations like your Laplace equation Poisson equation these are the elliptic questions. Now, elliptic questions are always bounded, and they are sort with the bounded boundary condition in a closed boundary. So, that is one of the important condition for the elliptic equation.

Now, when you look at the unsteady heat conduction equation this is parabolic in nature. Now the parabolic problem the solution always at once out what in final from the known initial value to satisfying a boundary condition as the solution progresses. Because there is a unsteady condition. So, the solution is marching over a time, and this is the solution in marching over time it starts with some initial condition. So, it starts with 5 0 condition, and then it will move along with the time at time progresses and march towards the solution.

Now, if you look at the wave equation, they are hyperbolic in nature. Now the hyperbolic solutions of a PDE open ended little bit. The reason is that solutions are going to go into different directions. Starting from initial conditions it can actually go into different directions. So, these are the nature of different different equations, but when you talk about fluid flow problem what kind of equations they are.

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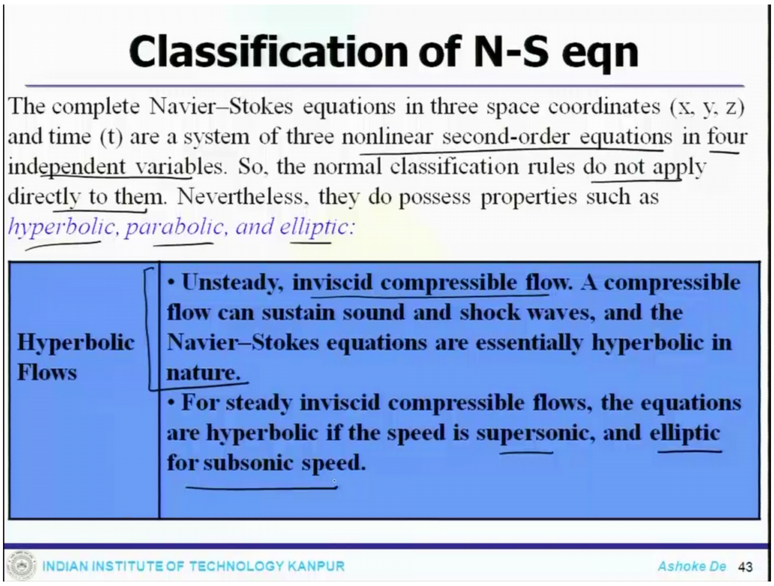

So, actually when you look at the complete Navier-Stokes equations in space or 3 dimensional space and time. This is a non-linear second order equation with 4 independent variables. So, the normal classification rules do not actually apply directly to them. But importantly they do possesses properties of all this hyperbolic parabolic and elliptic systems.

How? Now when the flow is unsteady in visit compressible flown, a compressible flow can sustain sound and shock waves and the Navier-Stokes equations are essentially hyperbolic in nature. So, particularly under this condition the Navier-Stokes equation or the nature of the Navier-Stokes equation becomes hyperbolic in nature. While, if you look at the steady inviscid compressible flow, the equations are also hyperbolic if the speed is supersonic, but they are going to be elliptic for subsonic speed. So, if you stay even inside the Navier-Stokes equation and if you look at the different different conditions so, that means, the solutions are quite condition depended. So, that is another challenge is there while solving the Navier-Stokes system.

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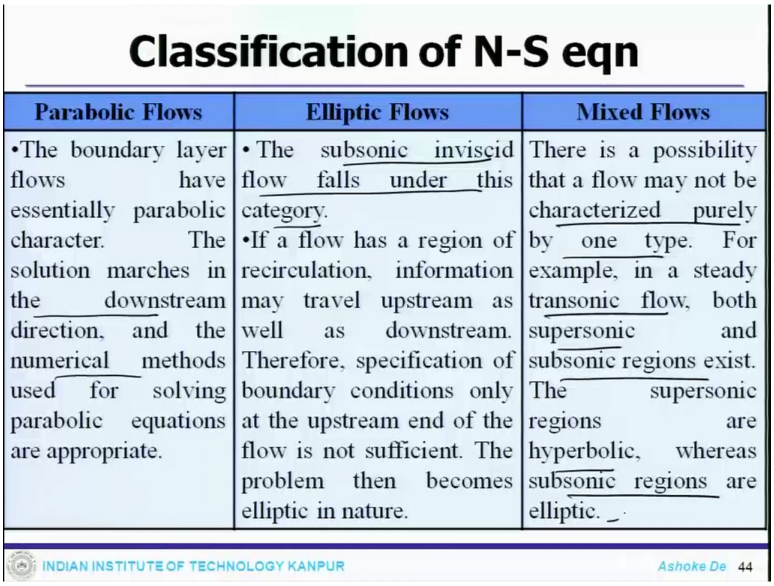

Now, some example of the parabolic elliptic and mix flows; patterning into the application of the Navier-Stokes equation. Now, if you look at the parabolic flow the boundary layer flow essentially parabolic character the reason is that the solution marches downstream direction and the numerical method used for solving parabolic equations are also appropriate.

Now, if you if you look at the elliptical flow the subsonic inviscid flows fall under this category. Now, these are the or if you look at the only diffusion term in the system they will be elliptic in nature. Now the mix flow there is a possibility that flow could be characterized by purely by one type; for example, in a steady transonic flow both supersonic and subsonic regions exist.

So, the supersonic regions are hyperbolic in nature whereas, the subsonic regions are elliptic in nature. So, that means, the application of Navier-Stokes equation is also quiet complicated. And depending on the flow domain flow zone and the conditions or your extra peripheral conditions things could be hyperbolic in nature, things could be elliptic in nature, or it could be mix flow in nature.

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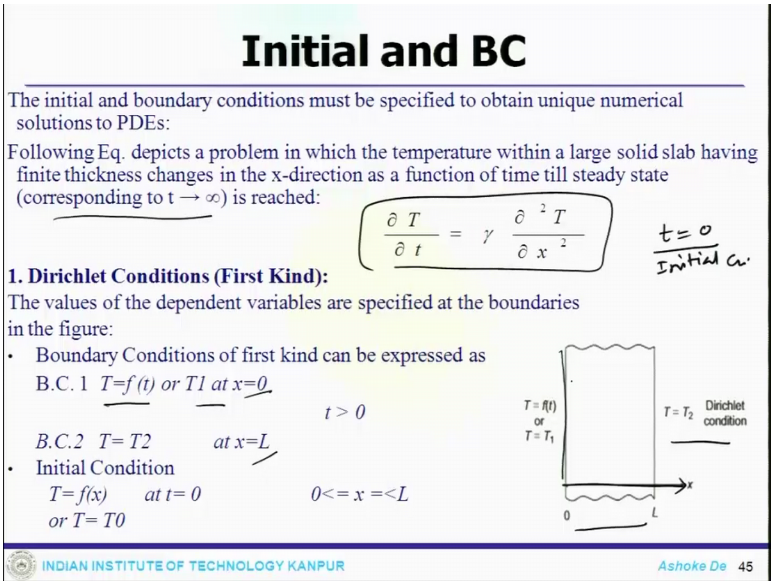

Now, when you talk about this initial condition or initial or boundary conditions, you look at this unsteady heat conduction equation state. So, the equation there must be some conditions at t equals to 0. This is called the initial condition, ok. Now with this initial condition the solution should be obtained. And when the t goes to infinity the solution should is reach to a steady state.

Now, the boundary conditions; the boundary condition this is a one dimensional plate if you look at it and this is the distance of the plate, and the solution is marching towards that. So, the boundary conditions could be one kind which is called the dirichlet kind. And dirichlet conditions means the particular surface, the boundary values are known which is like this or the other surface also the boundary values are known. That means, at this particular surface where x equals to 0 at this particular surface where x equals to L, my conditions are known. So, that means, while you specify a particular condition, this is known as a dirichlet boundary condition; that means, it is a user defined condition, ok.

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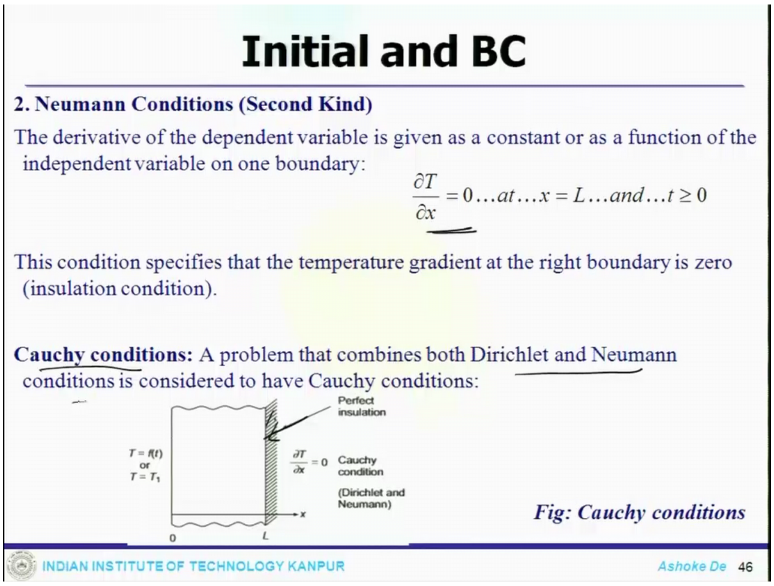

Now, there is a second kind of boundary condition call the Neumann boundary condition. The Neumann boundary condition is the derivative of the dependent variable. So, any particular boundary if you look at these boundary condition, at this particular face the conditions are the gradient exist then it is a Neumann boundary condition. Now, there could be another condition called the Cauchy conditions. So, this combines both dirichlet and Neumann kind of condition, this is a Cauchy condition, ok.

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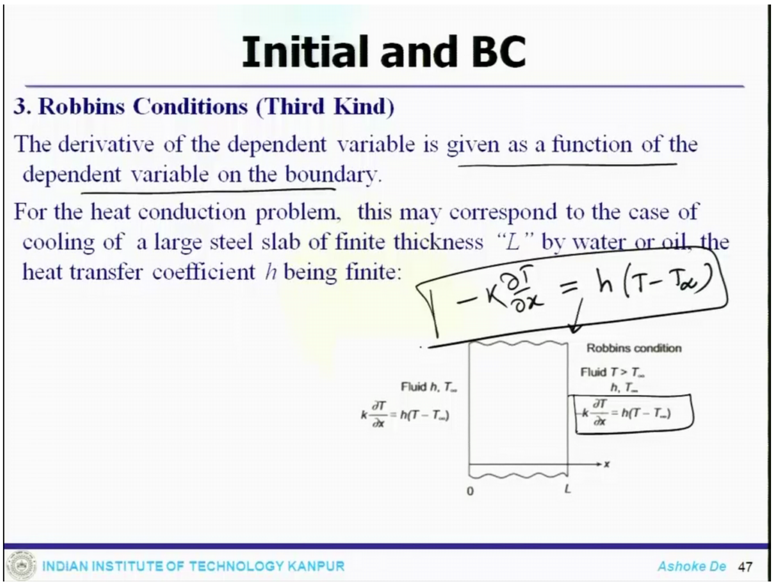

Now, third conditions would be Robbin conditions. Robbins conditions is essentially the derivative of the dependent variable is given as a function of the dependent variable on the boundary. Typically, this is the conditions what is known as Robbin conditions. If you look at the k minus del T del x at this particular phase equals to heat transfer coefficients minus T minus infinity. This is known as the robin condition.

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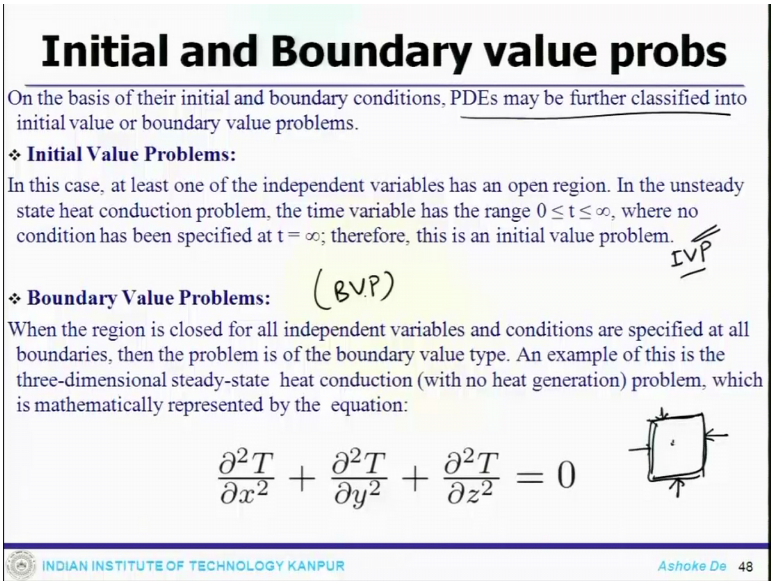

So, now you come down to the initial value problems. So, on the basis of their initial and boundary condition the PDEs can also be classified. One could be initial value problem; that means, when you are solving a time dependent problem or the unsteady heat conduction problem or unsteady any other problem. The initial conditions must be provided so, to get the solution over a period of time. So, that is actually brings that to be initial value problem, so IVP.

Or there could be boundary value problem. The boundary value problems are typically in steady state in nature and elliptic in nature. When the whole boundary is kind of contain in a closed system, and the boundary conditions are provided just like a 3 dimensional or 2 dimensional box or cube like this. So, the solution inside this domain will be bounded by the boundary condition in all these phases. So, that actually get you a boundary value problem BVP. So, the not only the classification of the nature of the partial differential equations like elliptic or parabolic or hyperbolic, they can be also initial value problem and boundary value problem.

Now, if you talk about the Navier-Stokes equation, they do have all sort of conditions. For certain conditions they become hyperbolic in nature certain conditions they become elliptic in nature they are also having this initial value conditions and the boundary conditions.

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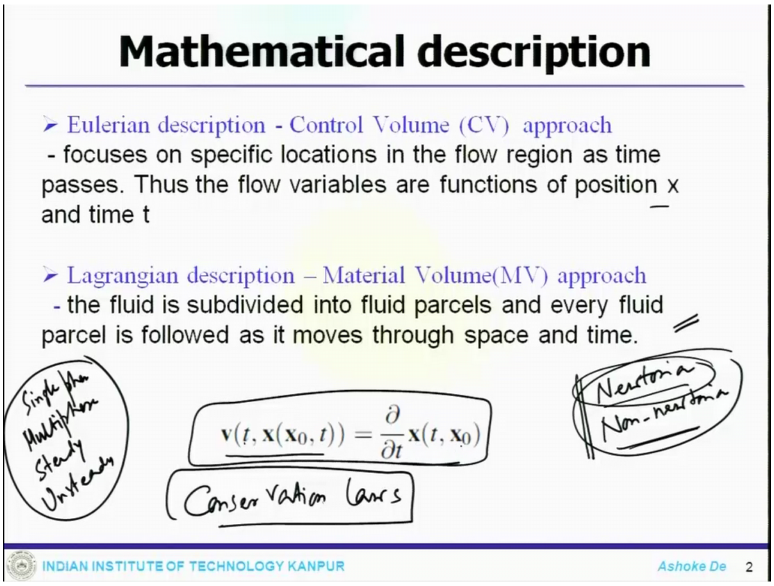

So, get a solution method, you must have this Navier-Stokes equation classified properly or the solution technique that should take care of the things. Now, when you talk about this we will also, now look at the governing questions and how they are derived, because the fluid flow problem are essentially talking about set of partial differential equations. Like, conservation equation, mass conservation equations, momentum conservation equation, energy conservation equations. Now since fluid flow systems they could be classified into two broad categories. One is the Newtonian one is the non-Newtonian.

Now when you talk about the Newtonian fluid flow so; that means, one could be also Newtonian or non-Newtonian. So, this is coming from the kind of correlation they have for a particular conditions for the viscosity. How viscosity is correlated with the stress? So, that will define the Newtonian or non-Newtonian. But no matter what it is or all are governing system will be in the continuum system. So, all continuum mechanics would be valid. And mostly we will be dealing with the Newtonian system, but there are fluid flow systems like flow through the black blazer they are in nature by naturally they are non-Newton in nature.

Where, the viscosity could be a non-linear function of the shear stress, ok. So, this is the classification of based on the viscosity with the shear stress relationship. Now also the fluid flow system could be categorized either 1 dimensional system, 2 dimensional system, multidimensional system, multiphase system. So, it could be single phase system, it could be multiphase system, it could be steady, it could be unsteady. So, all these are different different ways one can classify the system.

Now, essentially when you talk about that, these are leading to the set of conservation loss which are Navier-Stokes system. Now Navier-Stokes system when you talk about, they could be defined or the this fluid mechanical system or the conservation laws; essentially the conservation laws. They are defined in two particular framework. One could be Eulerian description or control volume kind of approach or other could be the Lagrangian approach or the material volume approach. That means, in the Lagrangian approach the whole fluid is sub divided into small small particle, that you track individual particles. So, that is what it call the Lagrangian approach.

And when we talk about the Eulerian approach, basically you focuses on a specific location, and in the fluid flow system and as a time goes by you calculate the system. So, which essentially become the function of x and t. Now particularly this system of a particular flow variable of the velocity which is the function of time and x, it could be taken a derivative of the space, this is how they can be correlated.

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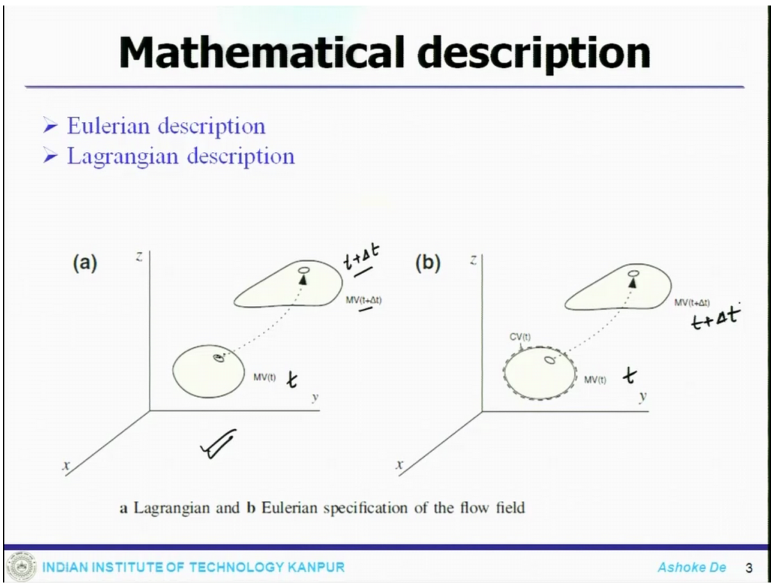

Now, when you talk about this Eulerian and Lagrangian descriptions; this one give you an idea about the Lagrangian description and when this particular control system is divided into subdivided into this small particles. These particles are track with a time. So, this is at the time instant t, and this is at the time instant t plus delta t so, how you track the particle that gives you the framework of the Lagrangian system.

But, rather in the Eulerian system, you have a control volume over the material volume, and then you get the governing from t to t plus delta t and Eulerian system, ok. So, this is where you divide your system and get the solution.

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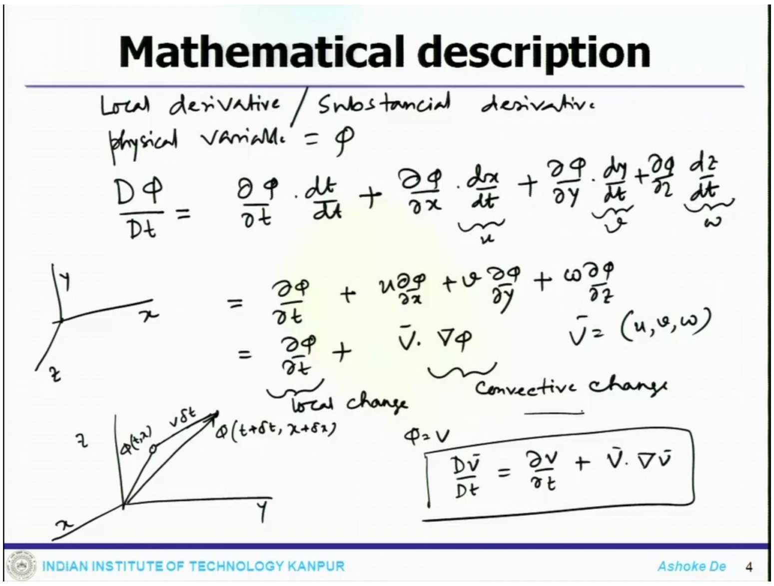

Now, when you get the solution for the different framework essentially, you get the first theme is the local derivative. This is one important parameter that you calculate. Local derivative or you call it a substantial derivative. So, substantial derivative for any variable or physical variable any physical variable fee, that is what you get the substantial derivative.

So, the substantial derivative written in D phi Dt equals to del phi by del t, plus since it is a function of x y z. So, del phi by del x into del x by del t plus del phi by del y into dy by dt plus dz by dt. So, this is nothing but your velocity component in different direction. So, if this is your coordinates system x, y, z then these are the velocity component. So, if I have to write this brings down to del phi by del t u del phi by del x plus v del phi by del y plus w del phi by del z. In other way, I can write del phi by del t plus v dot del phi, where v is a vector, which is nothing but your u, v, w.

So, this is my unsteady term all local change. And this is my convective change, ok. So, essential if I look at this system, let us say I go by x, y, z and this is my particle, which is here, now is at this point it is xt. So, it moves like to v delta t, at this location this is the complete action. So, this is at phi t plus delta t x plus delta x, ok. So, this is how with a time you get this thing. Now if the phi is v then you get dV by dt equals to del d by del t b plus v dot del t, ok. So, that gives you the Eulerian formulation of the substantial derivative of the system which connects the local change and the convective change, ok.

Now, when you say that, we will look at the basic transport equation or the Reynolds transport equation or RTT.

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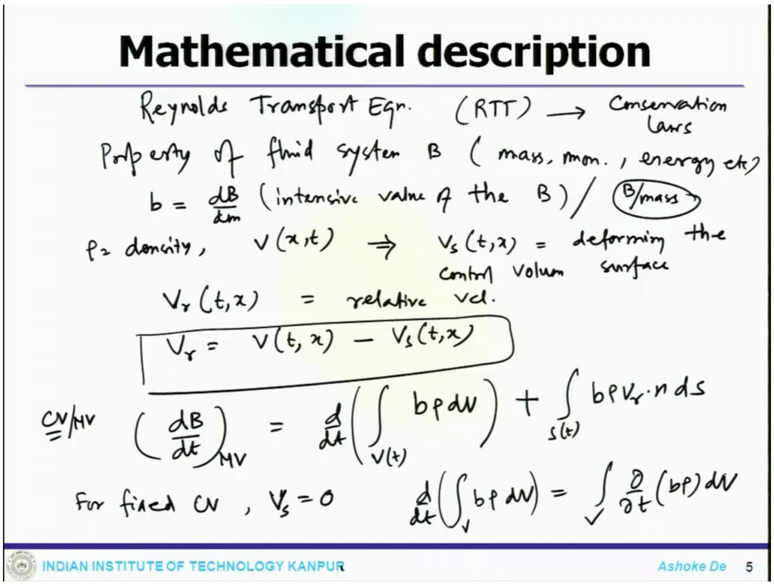

So, from there we will actually get back all other conservation laws. So, it is says that to get this things you say, they you say that any property or the fluid system. So, the property of the fluid system is defined as B, which could be mass, could be momentum, could be energy, etcetera, ok. And b is small b is dB by dm; that is, essentially the intensive value of the B, ok. Or rather you can say B per unit mass. That is what it is the specific volume, ok.

Now you have a control volume, and then you have a material volume. So, the density you say rho is the density. And the velocity scales are b which is a function of space and time. And there could be the two component of the velocity one could be v s which is the velocity which is deforming the control volume surface, ok.

And there could be another component which is called v r. So, essentially this is the relative velocity, ok. So, v r is nothing but my b minus vs tx. So, they are kind of correlated with each other, ok. Now if I write for a control volume the property of this b over the material volume. Then there would be a b rho dV plus surface integral b rho v r dot nds. Now for fixed control volume fixed control volume ds is 0.

So, that brings down to this d dt of b rho db equals to del del t b rho dV. Now this particular equation then simplifies to dB by dt over the material volume equals to volume del rho t b rho dV plus surface b rho v dot n d s.

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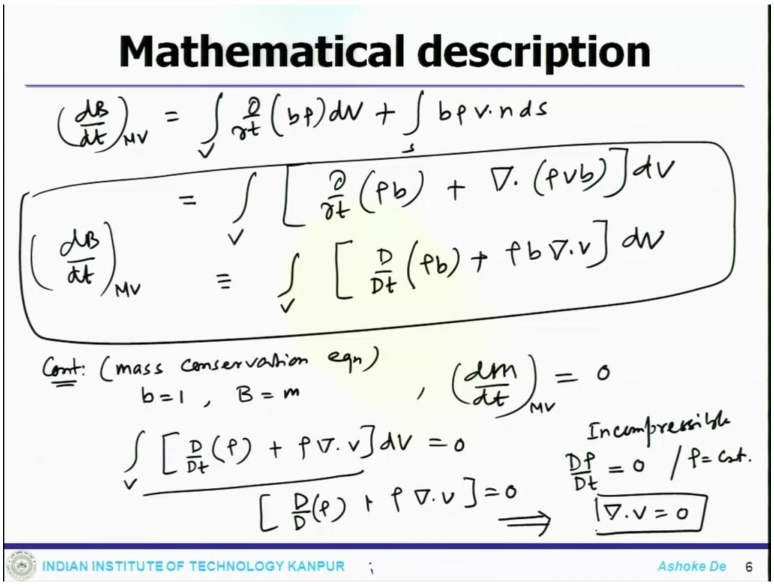

Now, you apply the divergence theorem. So, if you apply the divergence theorem, I can write down this things as a del del t of rho b plus delta dot rho v b or dV ok. So, one can write alternatively same expression like in terms of material derivative D Dt of rho b plus rho b delta dot v.

So, these are the now that gives me a equation for any property for a fixed control volume or a material volume. Here we can actually get the different different governing equations. So, one first thing that we will get is the continuity equation or the mass conservation equation. Now the mass conservation equation, we have this property specific b is 1, and capital B would be m. And to have a mass conservation, I write this equation for a system dm by dt must be 0. So, that get me the system D Dt of rho plus rho del dot v dV equals to 0, ok. So, to have these things true I get plus rho delta dot v equals to 0.

So, when you have a incompressible system, then from here so, you can actually derive that for incompressible system D rho by Dt is 0. So that means, essentially the rho remains constant. So, this get me back a simple equation del dot v equals to 0. This is what you get for the incompressible system.

Thank you.