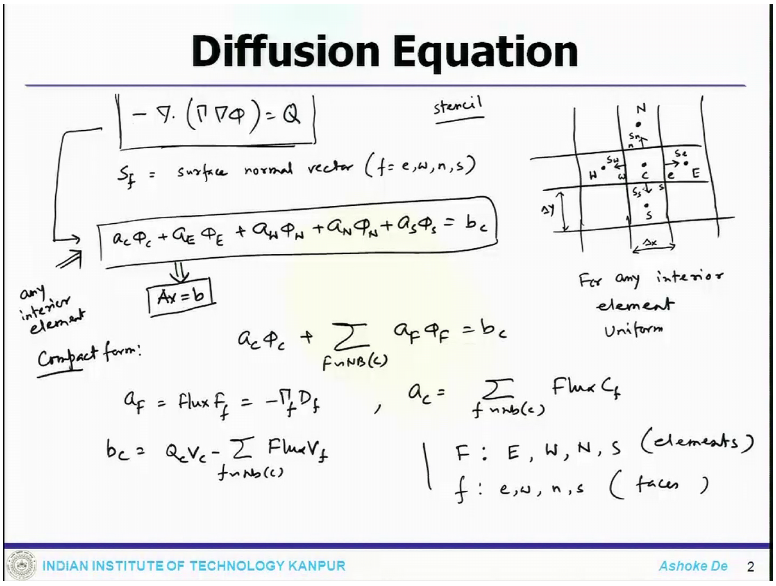
Introduction to Finite Volume Methods-I
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 27
Discretization of Diffusion Equation for Cartesian orthogonal systems-II

(Refer Slide Time: 00:14)



So welcome to the lecture of this Finite Volume Method and we will continue our discussion on Diffusion Equation. What we have done so far is that essentially looking at the 2 dimensional system and just to before begin with we just quickly give you an recap, this is the governing equation that we have looked into and the elemental control volume that we have considered is c and the ahead of it East downstream of it West, then top North and bottom South.

This was the 2 dimensional stencil or the discretized indexing that we have taken for some interior node; this is for any interior element. So, considering an interior element, we have actually derived the discretised system from this system. So, what are the convention?

So, once we finally, use that this is the face E, this is the faced West, this is the face South and North this has the surface vector that are used in the derivation of that. So, this is S e, this is S n, this is S w, this is S s. So, all this S i essentially or S f these are the surface normal vector where f goes from east, west, north, south and also this 2 dimensional grid system is uniform. So, it was a uniform system; so, we had this one delta x, similarly this one also delta y.

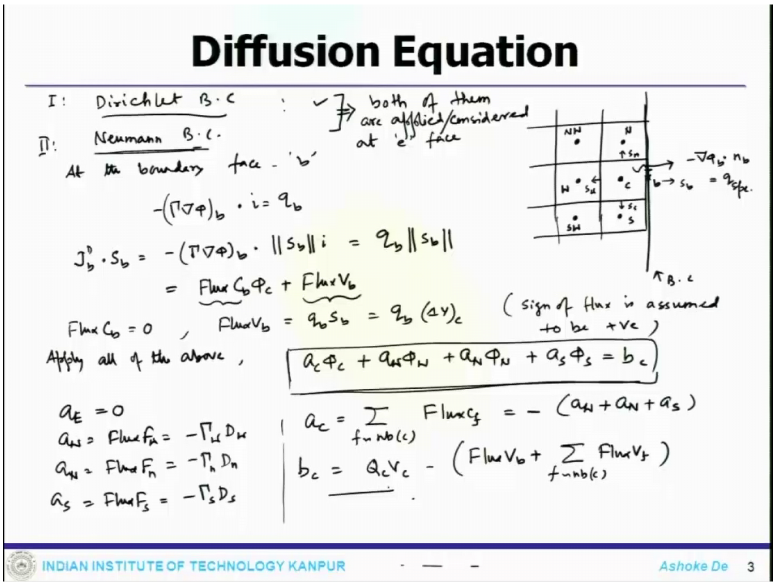
So, once we have done the discretization this governing equation actually gets us this a C phi C plus a E, phi E plus a W phi W plus a N phi N plus a S phi S equals to b c. So, this is the discretized equation that we obtain for any interior element any interior element is governed by this particular set of.

So, essentially this discretized equation is going to get you the A x equals to b. And while doing this all our coefficients where essentially in a compact form. If one has to write in a compact form then it was a C phi C plus summation of F it goes over the faces a F phi F equals to b c. And all these coefficients like a F was flux F f which is minus gamma f D f; a C was summation of small f goes from all the faces equals to flux and b c is Q c V c minus summation of all the small faces flux V f.

And capital F in all these stands for east, west, north, south, this is for elements. So, the element C is sort of surrounded by all these 4 elements; east, west, north, south and small f is essentially the corresponds to the faces. So, the face like this is c; e is a common face between element E and C, north face is a common face between N and C, west face is an common face between C and the west element and the east face is between. So, this is what we obtain and then to get the solution done; we need to apply for the boundary condition.

The first type of boundary condition that we have applied is the Dirichlet boundary condition; so, Dirichlet boundary condition.

(Refer Slide Time: 06:02)



So, this one and then accordingly we have modified the system and the modified equation was obtained. Now the second kind of boundary condition; so, this was also discussed in the earlier lecture; so, the next one. So, what happen when we apply the boundary condition? Essentially this is the governing equation. Now this actually valid for all the interior elements, but when it comes down to the boundary element or the face at the boundary, this particular equation needs to be modified.

For example when we had a boundary at east face; the contribution for this cell will get drops down, at the same time some terms will appear at the source term due to the specified variable. Similarly if the boundary face belongs to north automatically these guy would goes up the north contribution if is at the west; the a W phi W goes up; if it is the south a S phi S goes up. So, that is how for a Dirichlet boundary condition one can arrive the cell. Now will look at the second type of boundary condition is the Neumann boundary condition.

So, for that if you just look at that element then you see how that Neumann boundary condition can be applied. So, this is my face and this would be the boundary face; this is element C, this is South, this is West, this is North, this is North West, this is South West and this face is the and here now this is the point b and that Neumann condition. So, this is the direction of the surface vector this will be the direction of the south face surface vector S W, S N.

Now, the Neumann condition is nothing, but something the gradient is minus del phi b dot n b equals to q specified. So, that is the Neumann boundary condition; so, essentially what does that mean the gradient at that particular face. So, now, if you look at that element C which actually one of its face the east face belongs to the boundary the gradient condition is prescribed. So, the condition which is prescribed at the boundary face that is b is minus gamma del phi b dot i equals to q b.

In other words, one can write the diffusion flux at dot b equals to minus gamma del phi b dot S b i, which will be q b s b which is equivalent to the flux which is defined there C b phi C plus flux V b and these are the coefficients. So, this coefficients one need to find out and if you look at this particular condition they turn out to be the flux C b would be 0 and the flux V b would be q b S b which is essentially q b delta y c.

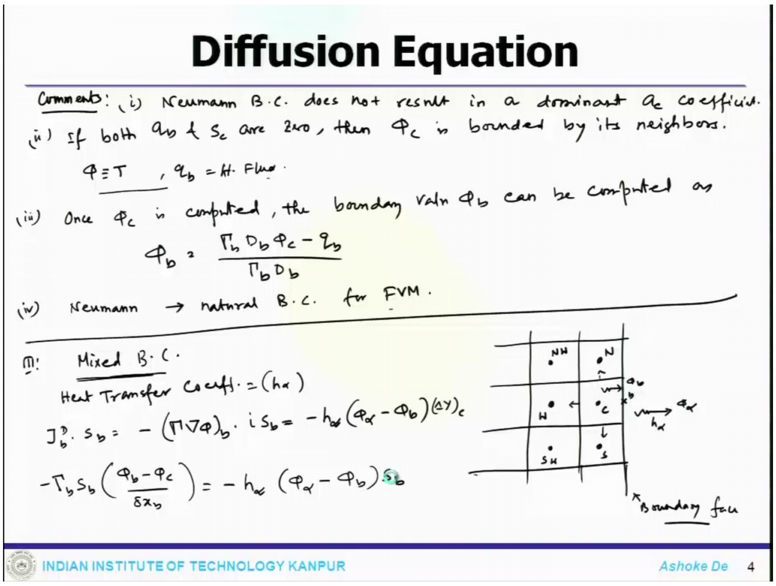
So, that is what you get; now one thing you can note here the sign of the flux component, the sign of flux is assumed to be positive that is why it has been considered pointing outward. Now, this equation of the q b or the information of the q b this could be directly included in the system. And like this could be directly imposed in this particular equation and what we can get. So, once you apply all of the above conditions what we get a C phi C plus a W phi W plus a N phi N plus a S phi S equals to b c.

So, again if you look at this particular equation it looks exactly similar to the condition that we derived for the Dirichlet boundary condition. So, but that does not mean they would be of similar expression the mathematical expression or the discretised system looks similar; the reason being both the boundary condition whether the phi b specified that Dirichlet boundary conditions or the Neumann boundary condition current now currently which we are talking or discussing both of them have been applied at this particular face which is at the east face.

So, the commonality between them is that both of them are applied or considered at east face that is why the representative discretized equation they look similar. But the points which are going to be different is that a is 0 here, a W would be flux F w which is minus gamma w, D w; a N is flux F n which is minus gamma n D n, a S is flux F s which is gamma s D s and we have a C; which is summation of faces and flux c f; essentially that becomes a W plus a N plus a S ok.

And the difference would appear in this particular source term the b c; which is going to be; now Q c b c minus flux V b plus summation of over the faces flux V f. Now this is the difference that would appear for the Dirichlet a Neumann boundary condition. Now, there are few important comments.

(Refer Slide Time: 15:05)



So, or observation for this things; one of the important observation is that this Neumann boundary condition, Neumann boundary condition does not result in a dominant a C coefficient. That is a very important observation, number 2 if both q b and s c are 0 then phi C is bounded by its neighbours; otherwise what can happen? Phi C can exceed or fall below the neighbour value of phi whichever is admissible.

Now for example, and you can get it more clear. For example, if phi let us say represent temperature, phi is equivalent to temperature. If phi is temperature and q b represents the heat flux; so, heat flux applied at the boundary; that means, either the heat is added or heat is taken out from the boundary, then the region close to that boundary it is expected to have higher or lower temperature compared to that interior. So, that is what it says that; whatever information you have accordingly they would be bounded.

Now, third point once phi C is computed; once that is computed the boundary value phi b can be computed; as phi b equals to gamma b D b phi C minus q b divided by gamma b D b. So, from the phi C once that elemental variable is calculated, you can calculate back the specified value. And the last one is that actually this Neumann condition this

Neumann condition can be treated as natural boundary condition for Finite Volume Method for boundary condition for FVM. Because this means whenever we are dealing with some flows or heat transfer related problem; if the flux is specified that is quite handy to implement infinite volume method.

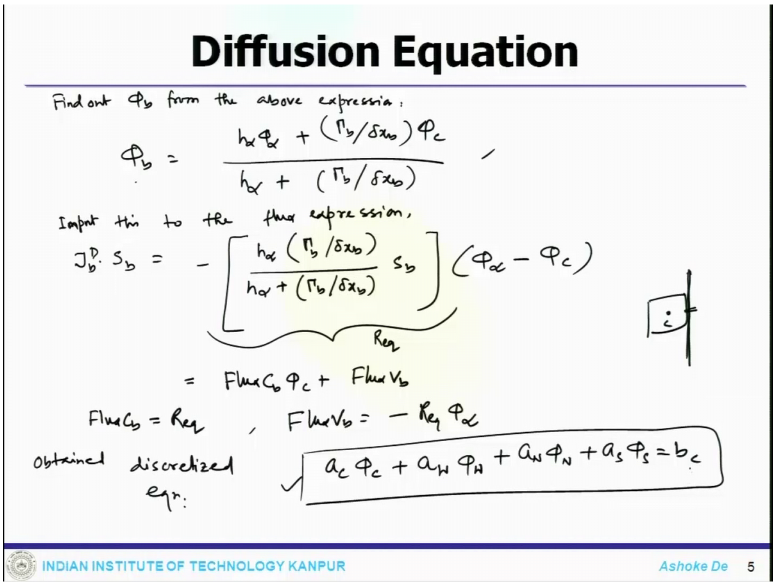
So whether the in terms of discretization of that particular face element or other way round; so, these are certain observation that is there. Now the third kind of boundary condition is the mixed boundary condition. So, mixed boundary condition which will mean if we again draw this; elements and apply the boundary condition. So, this is my element C, this would be north, this is west, this is south and the boundary condition which is applied here this is point b.

So here the boundary condition which is applied at phi infinity h infinity and phi b; so, both of them are specified. So, these are the surface normal vector north west south west and this is the face of the boundary; so, this is the boundary face. Now again the condition here is specified to the mixed condition. So, you have a some sort of a heat transfer coefficient or convective heat transfer coefficient is specified which is based on h infinity.

Now the flux which one can write for that is minus gamma del phi b dot i S b; which is going to be minus h infinity into phi infinity minus phi b into delta Y c ok. So, here you got the both the information in the mixed boundary condition, which is another way people call it a robin boundary condition or mixed boundary condition where you have both the Neumann kind of boundary condition given a flux and also the specified value is also provided. So, both of them are acted simultaneously at the same face.

So, if you rewrite this expression one can write gamma b S b; phi b minus phi C by delta x b which is minus h infinity; phi infinity minus phi b S b; so, that is what it gets you back.

(Refer Slide Time: 22:30)



Now once you get back these you can find out phi b from the above expression which is going to be as h infinity, phi infinity plus gamma b divided by del x b into phi C, h infinity plus gamma b divided by del x b.

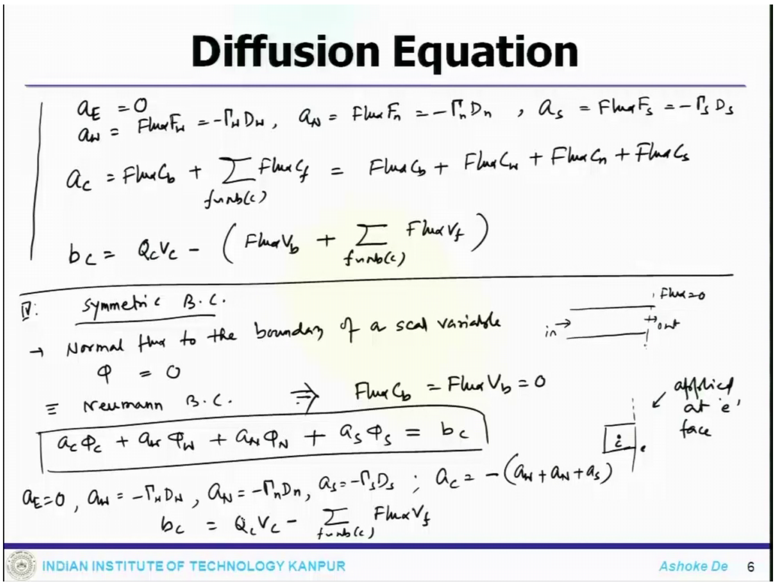
Now, this one you apply it back in this particular flux information. So, the information of phi b; once you input this to the flux expression; one can get J b dot S b equals to minus h infinity; gamma b by del x b divided by h infinity plus gamma b by del x b; S b into phi infinity minus phi C. And this particular term one can think about as an resistance or some sort of a equivalent resistance and you represent this one has Req.

So, it is an some kind of an resistance which is applied here which is also it can be written as 2 coefficient; one is flux C b phi C plus flux V b. So, you got the expression for the flux which is applied there and using the information of the given value you find out these coefficients; where flux C b is nothing, but this R equivalence and flux V b is minus r equivalent into phi infinity.

So, once you use this information and again put it in the. So, the obtained discretized equation would have the expression like a C phi C plus a W phi W plus a N phi N a S phi S equals to b c; again if you look at the similarity between your Neumann condition, the expression which is obtained here and the expression which is obtained for the mixed condition; they look similar algebraically or notation wise they look exactly similar.

And the reason is as I have said earlier this is going to be applied at the east face of this cell. This is my C element and all these boundary conditions are applied at the east face if it would have been applied to the other faces like west north or south; those corresponding contribution terms would go away, but the difference is lie in the calculation of these coefficients and also b c.

(Refer Slide Time: 27:01)



So, in the similar fashion for this mixed case we get a E equals to 0, a W would be flux F w which is minus gamma w D w; a N is flux F n which is minus gamma n D n; a S is flux F s which is minus gamma s D s and a C is flux C b plus summation over faces of flux C f, which is going to be flux C b plus flux C w plus flux C n plus flux C s. And my b c is the Q c V c minus flux V b plus summation over faces for flux V f ok.

So, though the discretized algebraic equation looks similar for all sort of conditions like Dirichlet condition, Neumann condition and mix condition, but the differences lie in the expression of this coefficients and the source term. Now have been said that will move to the these are the primary 3 boundary conditions which one can come across while applying the boundary conditions for a physical problem.

Either you have a specified value or you will have an applied flux; let us say for example, if you are dealing with a flow through a channel. This is the inlet this is the outlet at the outlet one can always assume the flux is 0. So, that is a short of a Neumann boundary condition or in some cases where you may have both the Dirichlet and Neumann applied together which correspond to the mixed condition.

Now moving ahead there would be fourth type of boundary condition which can be also used which is a symmetric boundary condition. Now typically along a symmetric boundary the normal flux to the boundary of a scalar variable phi is 0. So, symmetry boundary condition is short of equivalent to Neumann kind of boundary condition; Neumann boundary condition.

So, there is a equivalency between symmetric boundary condition with the Neumann boundary condition because what it says that is the normal to that boundary the flux of the variable is 0. So, if it is equivalent to the Neumann boundary condition so; that means, it will give rise to that flux C b or flux V b they are 0. So, the discretized equation which can be modified for symmetry boundary condition and again can be written as if the symmetry boundary condition again applied at the east face this is my C; it is applied at east face.

So, the discretized equation would look like a C phi C plus a W phi W plus a N phi N plus a S phi S equals to b c; where again you get a E 0 a W is nothing, but my gamma w D w; a north is gamma n D n, a S minus gamma s D s and a C is nothing but minus a W a N plus a S and b c would be my Q c V c minus summation of faces flux V f.

So, we have actually looked at a particular stencil when it exposed to the boundary condition or rather different kind of boundary condition starting from Dirichlet boundary condition, Neumann type boundary condition, mixed boundary condition and symmetry boundary condition; they the discretized equation can be obtained and the corresponding coefficients can also be obtained accordingly.

Now, if in all these cases that we have discussed we have applied the condition at the east face. So, if we apply the condition on the other faces then as I have been saying that the respective coefficients will go away and corresponding contribution or equivalent contribution should appear in the source term. So, that actually talks about the application of these boundary condition. So, will stop here and.

Thank you.