Introduction to Finite Volume Methods-I
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Lecture – 24
Finite Volume discretization of Diffusion equation-II

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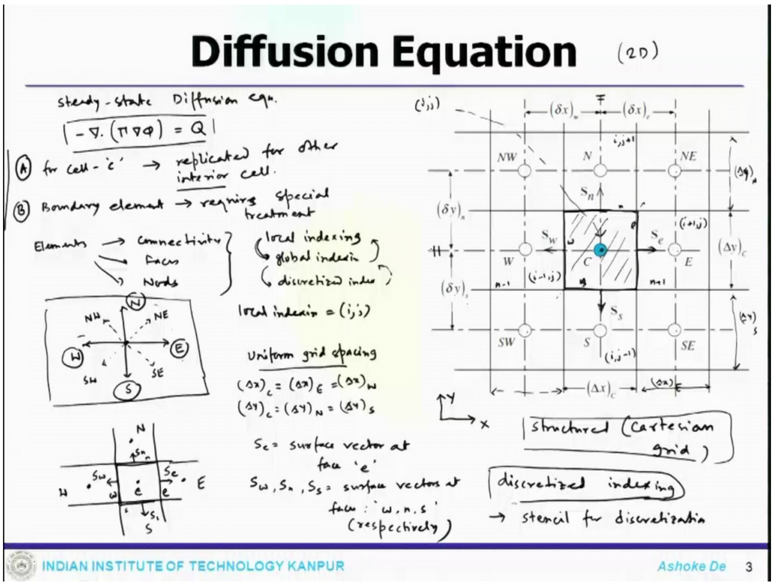

So, welcome to the lectures of this Finite Volume Method.

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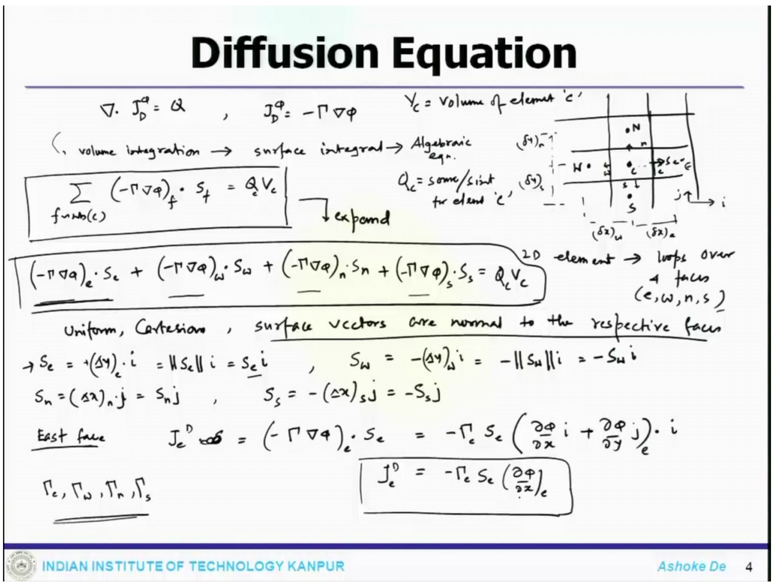

So, once the stencil definition is over we can move to the so, if you keep this picture in mind, this is how we have it this is my cell c this is my e or se this is my north. So, north face this is my west face so, W this is my south face ok. And the distance between this is east, distance between this is delta x e. distance between this is delta x w, distance between this two is delta y south and distance between this is delta y north.

So, this is the small information if you keep in mind, then we can so, our equation is del dot J phi is Q ok, where your diffusion flux is defined as minus gamma delta phi. So, as we have done that you do the integration or volume integration and then convert to surface integral to get back a algebraic equation. So, that we have already done.

So, of you follow the same procedure essentially, doing the integration and all this will turn out to be the summation of f over all the faces minus gamma of delta phi of f dot S f equals to Q V c.

So, we are writing for V c is the volume of element c and Q c is the source or sink for element c. So, that is what it does? You get this algebraic system. So, if you expand this one, now you expand this one, once you expand this one this will now essentially this is written for this element c here and once you expand that if will go over this four surfaces.

So, it is a 2 day element 2 day element so the loop goes over loops over 4 faces. And the faces are east west north south and that is why it is very very important to keep track of this notation east, west, north, south.

So, if I expand it, it will be now written minus gamma del phi at east face dot east plus minus gamma del phi west face dot west vector minus del phi north dot S n plus minus gamma del phi south dot S, s equals to Q c V c. So, that is my expanded equation which is turning out to `be; now this grid is uniform number one Cartesian ok.

So, the surface vector are normal to the respective faces ok, this is very very important. If that does not happen, them we not straight way we have already seen we cannot if they are not collinear or they are some sort of a off the center connections, then we cannot straight calculate the surface vectors.

So, have been said that we can estimate S c, S c would be nothing, but the surface vector which is delta y positive east dot i so, this would be essentially magnitude of S e i. So, one can write S e i, so, the normal vector is here S e which would be these distance into the unit depth.

So, that is what it does similarly you write for S w which is minus delta y w i. So, this would be minus S w i or S w i and why does this minus sign come here? Because if assume this is the local coordinate system, this goes in a positive i and this goes in positive j.

So, this is my positive direction these becomes negative direction ok, S n would be delta x positive n dot i which is essentially n dot j, S n j and similarly S s would be minus delta x s j which S s j.

So, here you get all your surface vector definition. Once you get that surface vector definition you can calculate all this individual surfex surface fluxes the diffusion fluxes. First let us look at the east face so; if you look at the east face so, that is essentially your J e dot S. So, this would be minus gamma del phi e equals to e dot S e so, this is the term essentially.

Now, this term if you expand now this would be gamma e S e del phi by del x i plus del phi by del y j dot product with i because my S e is essentially S e i. So, this is the dot product with i and what you get back here it is a dot product of i. So, you get back with this term only minus gamma e S e del phi by del x e ok.

So, that is my east face diffusion flux, if you look at it now we got a expression will continue this notation for all the faces like gamma e, gamma w gamma n and gamma s just to represent that if there is no non homogeneity, then they would be equal. If there is a if there is a homogeneity they would be equal if they are not homogeneous then they could be different.

And the discretization technique or the numerics essentially, takes into account the non homogeneous contribution of the diffusion term. So, that is what we will retain this suffix of individual faces.

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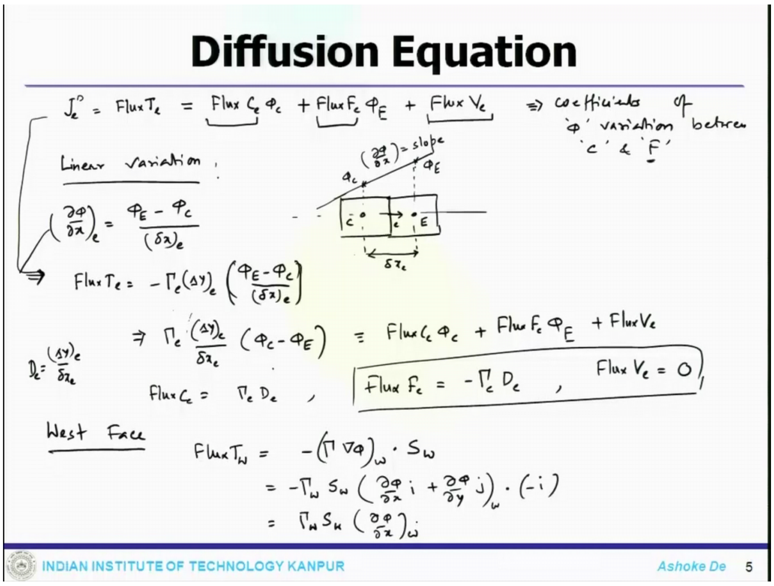

Now once you obtain that so, if you now write the diffusion flux at the east face, which is total flux at east face which is the flux C e phi c plus flux F e phi F plus flux V e.

So, flux C e flux V e and F e these are essentially coefficients of phi variation between C and F; that means, any neighboring element so, these are the coefficients. Now if you assume the linear variation, if you assume the linear variation let us say the phi variation is linear, then you can have the system like this c east.

This is east face, this is the distance delta x e and this is the variation and the linear variation will do the dot. So, this would be phi c and this would be phi E and the slope is del phi by del x. So, this is the slope of that linear variation so, that is a the profile would be the interpolation slope of these two. So, what I can write del phi by del x e with the linear variation phi E minus phi C divided by del x e. Now this expression if you replace in this particular equation. So, combine these two what you get is that total flux is minus gamma e delta y e which we have this is gamma e S e del phi by del x. So, gamma e delta y e phi E minus phi C divided by del x e.

So, if I rewrite that this would be gamma e delta y e by delta x e phi C minus phi E. So, this negative sign is taken into account, which is also equivalent to my expression flux C e phi c flux F e phi F flux, flux V e. So, this would be you can write this one flux E E here.

So, if you take the mapping of the coefficients you get here before that you define these particular system let say D e equals to delta y e by del x e. So, then if you map the coefficients for phi C and phi E you get flux C e equals to gamma e D e.

Similarly flux F e equals to minus gamma e D e and the third one the flux V e is 0. So, this is what you get as the coefficient. So, once you get that then you got your this things, now when you move to the west face we can similarly fine out that flux the total fluxed at the west face is minus gamma del phi w dot S w.

So, which will turn out to be minus gamma w S w del phi by del x i del phi by del y j dot minus i. So, this is as per our definition of S w so, S w is minus i. So, if you do this, this would be gamma w S w by del phi by del x w ok.

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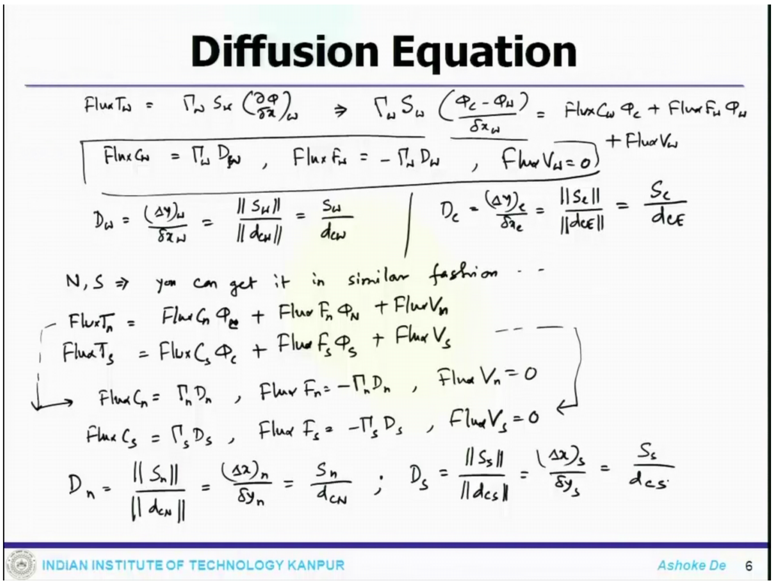

And which can be again written that so, I can write the total flux T w equals to gamma w S w del phi by del x w which is nothing, but gamma w S w phi c minus phi w divided by del x w. And the coefficients would be flux C w phi c flux F w phi w plus flux V w.

If you match the coefficients from these two equation, then you get flux C w equal to gamma w D f by D w flux F w equals to minus gamma w and D w and flux V w equals to 0. So, these are the coefficients that you get and the D w definition is similar to our D e definitions, which would be like where D w is my delta y w by del x w. Which one can think about magnitude of S w divided by d c w which is S w by d c w.

So, that is our definition of so, similarly we could write for S D e that D e we had definition of del y e by del x e which is magnitude of S e divided by d c e, which is S e by d c E.

So, we got the surface of east and west now you have two more surfaces which are left the north and south. So, you can you can get it in similar fashion, we have all the required information which is available. So, if I put this together the flux T n the total flux would be flux C n phi N or flux C n phi c flux F n phi N plus flux V n ok. And the south would be flux T s the total flux would be flux C s phi c flux F s phi s plus flux V s ok.

So, this is the total flux definition and once you do the mapping for each case you get for this case you get where flux C n would be gamma n D n flux F n would be minus gamma n D n and flux V n would be 0. And similarly for this case we get flux C s is gamma s D s flux F s equals to minus gamma s D s and flux V s equals to 0.

So, you got all these coefficients and where your D n would be magnitude of S n divided by d c n, which is nothing, but your delta x n by delta y n which is S n by d c n. Similarly, you get D s definition which would be S s divided by d c s, which is delta x s by delta y s, S s by d c s.

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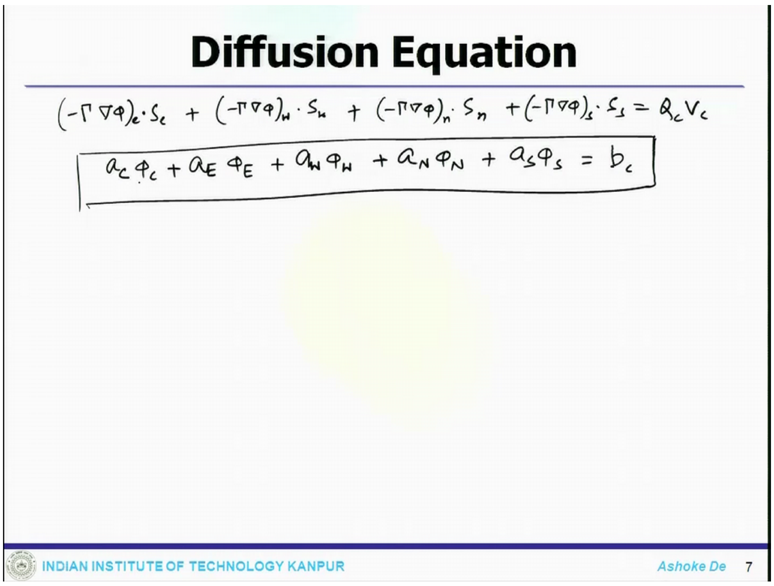

So, you got all this coefficients and now of you put them together in the final equation, equation that you had basically the equation of this that you had minus gamma del phi e dot S e plus minus gamma del phi w dot S w plus minus gamma del phi n dot S n plus minus gamma del phi s dot S s equals to Q c V c.

So, if you put everything together in this particular equation, now whatever we have now obtained if you put them together. So, what you get? You get an equation ac phi c plus a E phi E plus a w phi w plus a N phi N plus a s phi s equals to b c. So, this is the algebraic equation that you get or the linear equation and the coefficients that you can calculate and we will see how to calculate that in the follow up lecture.

Thank you.