Introduction to Finite Volume Methods-I
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Lecture – 22
Properties of Unstructured Mesh-II

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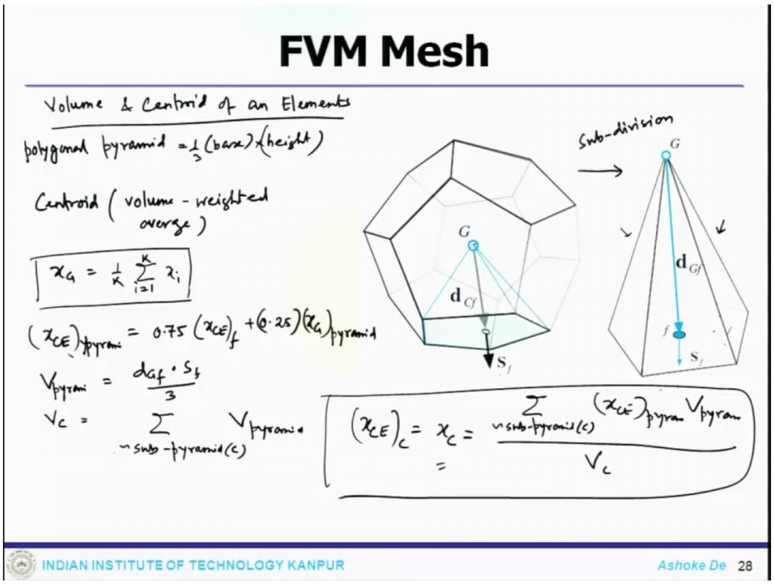

So, welcome to the lecture of this Finite Volume Method. Now, you can find out the volume, so volume and centroid of an element. So, you have in picture here. So, this is a polyhedral. And if you want to find out the volume of this particular polyhedral, typically polygon pyramid this is a polygonal pyramid, for polygonal pyramid, the volume is one-third base into height, this is how you calculate the volume ok.

Now, the base is one of the surface of the element. So, essentially if you look at it this pyramid, the base is essentially the element of one of the or the I mean the base element is one of the surface of that particular pyramid or the pyramidal element. So, now you can actually for centroid calculation, you get the for centroid calculation you get the volume weighted average. Since it is a three-dimensional element, so you get a volume-weighted element. So, my centroid would be 1 by k summation of i 1 to k x i. This is already we have seen. So, exactly the same equation we will use, so for the x C E for the pyramid is 0.75 into x C E f plus 0.25 into x G of the pyramid.

And the volume of the pyramid would be d G f dot S f by 3. So, this is how you get. This is essentially the surface vector from here. This is the distance vector, and you get the volume of the pyramid. So, V c would be summation of sub pyramids of c and V pyramid, so that means this polyhedral is sub divided like a, so this is essentially the sub division. Like our previous case the tetra was divided in the sub triangle. So, similarly this polyhedral is divided into this kind of smaller smaller pyramid. And then once we find out the volume of this pyramid, and sum over those it will get the volume of these particular element.

And then you can get your x C E for this is x c and that would be the summation over this sub pyramid c x C E pyramid into V pyramid divided by V c. So, it is say volume-weighted average that means if I have multiple of this sub pyramids, I get volume of this each pyramid, I get the centroid of this, and then get the volume-weighted average to get the centroid of the polygon. So, once you have a polygon then you can divide into the sub element from the sub element volume and area, you calculate coordinate you calculate, and then you get back the centroid of the polygon. So, this is how you can calculate.

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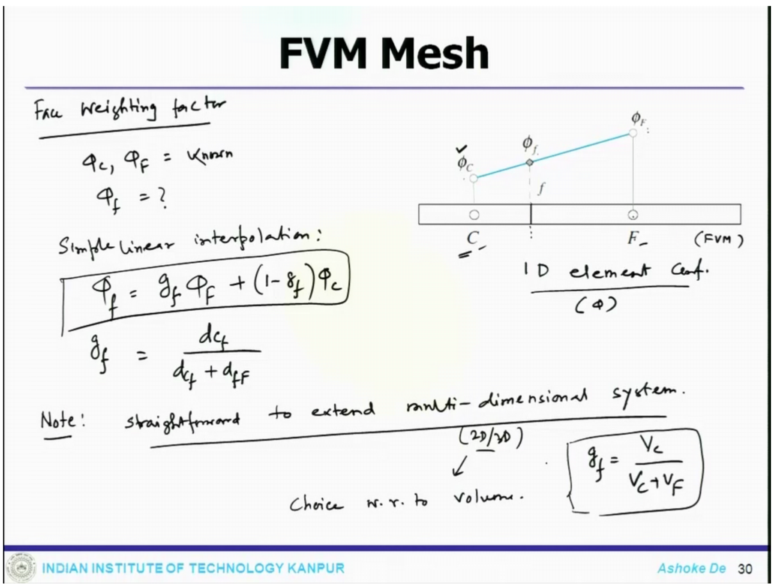

Now, another thing which is very important is that face weighting factor. So, this is what we have used when we have taken the weighted, this is a 1D element configuration. So, 1D element configuration, if you look at it for we just, write it for any arbitrary variable phi, if you write, then the these are shown the element c and element b f. So, these are all in FVM construction. And the face is f, value at this center is phi c, value at the face is phi f, and value at this for this f element is capital F. So, how do you so, you know the value at this center. You know the value at this essentially this is known phi c, and phi F these are essentially known values using that what we try to find out the face value.

So, one way one can do is the simple interpolation, simple linear interpolation. So, if you do that, then your face value phi f the small f, phi f one can write some factor g f into phi f. So, take the contribution from here plus 1 minus g f into phi c. So, that is a simple linear interpolation one can use, where your waiting factor g f would be d c f divided by d c f plus d f F ok. But one can note, which is very important that this is not straight forward to extend multidimensional system, that means this one-dimensional information of the linear interpolation cannot be straight way connected or extended for the 2D or 3D system, because 2D and 3D systems are bit more complicated and involved.

And this case this is much simpler. So, one can do that, but in that case you can have some sort of a choice, because 2D or 3D cases you can have a choice and that choice with respect to volume. So, one can do that like where you can use the g f is a ratio between V c divided by V C plus V F. So, this is one choice or there is one option one can adopt for a multi-dimensional system like 2D or 3D, but again have being said that it is very very important to note here, this is not going to be straight forward, because this may work for certain cases this may not work for certain cases.

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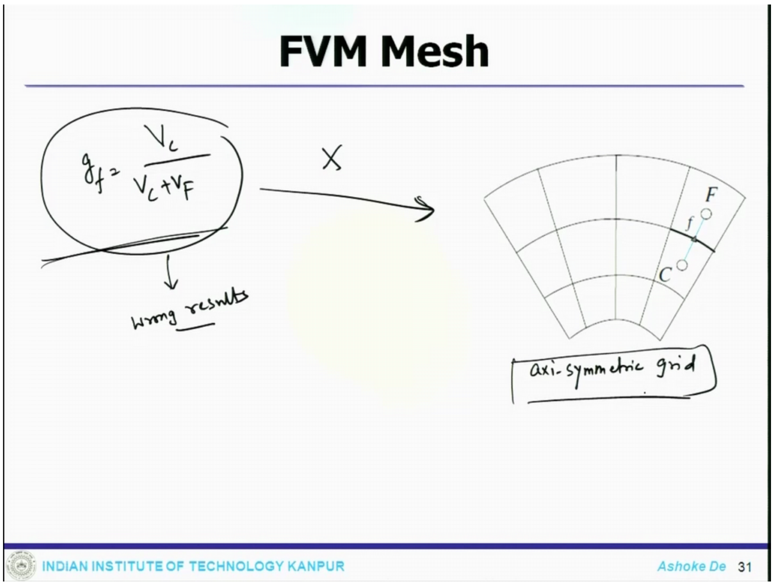

For example, this kind of implications like the definitions of g f, which is V c divided by V C plus V F. This does not work for this kind of case. This is an axi-symmetric grid. When you have axi-symmetric grids, this interpolation does not at all work, this can give you completely wrong results. So, whenever you use this kind of interpolation also one has to note that it is not straight forward or can be extended for any kind of configuration. Configuration to configuration it may change, and configuration to configuration this calculation needs to be handled accordingly. If you have a generic expression for that, then there is a problem for certain cases it would work for certain cases as we have I have shown it may fail.

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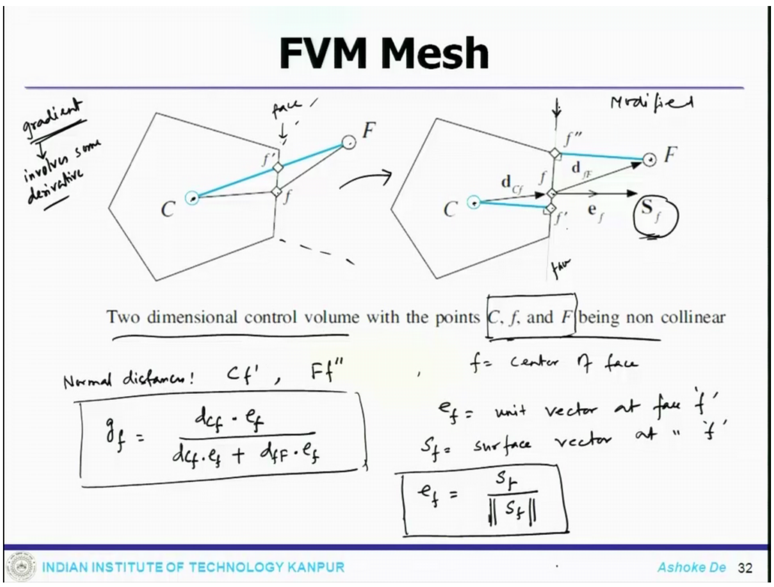

Now, again the simpler interpolation not the linear interpolation may get you in some kind of a problem when you see, this kind of situation. It is a two- dimensional control volume where point C f and F are not collinear that means my cell center for this element sitting there. My cell center for this could be the element like this sitting there; connecting between C and F which will be cutting through that common face. This is my face f prime, and the center of the face is f and there is connection should that means this three points capital C, capital F and small f, they are not collinear.

So, other alternative is that, so once they are not collinear it is very difficult to handle with that kind of linear interpolation, and you may have or find it difficulties. Slightly modified system could be this one from this to this. This is a modified option of that. You have this points F here. So, you find a normal to that f double prime. You have a cell center here. You find a normal to that a prime. You have this face center which is f. From f if you connect to this center of this element neighbor element which is d f F. Once you connect this center with the C d C f and there will be a normal vector which is surface vector e f and in that case the normal distances are there are some normal distances.

So, those normal distances are one is C f prime other case is so this is for element C and then normal layer for neighboring element F and the other normal is F f double prime. So, these are the two normal distances. So, what you have done, since they are not correlated since C, f and capital F they are not collinear instead of connecting them with a one line you get a normal from this point along that face, so along that line. You calculate a normal along that line C f prime. And then here is your surface vector which will be pointing outward. And then you have a distance vector from this small f is the center of the face. You have a distance vector d f F, and from C to small f d C f.

Once you calculate that then you can estimate the weight factor like this g f equals to d c f dot e f ok, so that means these distance to the dot product of that divided by the total dot product that means d c f dot e f plus d f F d f F dot e f ok, so that is actually get you the I mean better interpolation compared to the simple linear interpolation. So, this particular expression takes into account the normal distances and the dot product of the factors. Now, here what is e f, e f is the unit vector at that unit vector at that face at face f ok.

And how that unit vector will be calculated; so, since it is a unit vector this is going along the surface vector s f. So, s f is the surface vector at face f. So, once you have an estimate of that surface vector s f. Then you can estimate the e f equals to s f by mod of s f so that will get you the unit vector along that. So, you have certain situations where things may not be correlated, so that is actually give you an idea about different kind of elements. So, what happens when you move from structured to unstructured it gives rise to different kind of issues.

So, one of that is that handling of different kind of polyhedral then the surface vector and so essentially when you have that you need to get, and proper estimate of this polyhedral, and then the surface vector. And it may possible the connecting neighbors are not being collinear. Then you have to estimate their weighting factor. All these are pointing towards only one thing is the calculation of the gradient, which so this is the calculation of the gradient, which involves some derivatives. So, once you have some derivative calculation then you need to calculate things at the flux, because the gradient calculation is always defined at the face. So, when you come down to face, then flux calculation is must, and there you need to have this normal calculation and all this.

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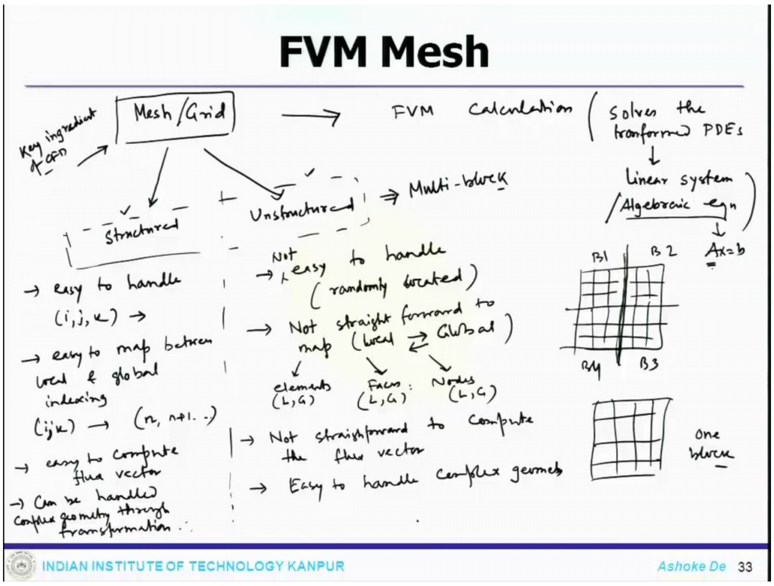

So, if we sum up the whole business so the mesh is or the grid this is a inherent component for any finite volume calculation ok. Calculation means where you are actually solves the transformed PDEs. Transformed to a linear system or algebraic equation ok; so, this is one of the ingredient or this is one of the key in gradient of CFD the mesh. The mesh could be two types it could be structured or it could be unstructured ok.

Then another category which is possible both structured and unstructured can be also can be also multi-block that means I can have for example, if you have a simple rectangular geometry, and you can generate grid in multiple blocks. So, this would be the blocks. This is block 1, this is block 2, this is block 3, this is block 4. So, I can either way have a simple single block system like this. This is a one block system or I can have a multi-block system. So, mesh could be structured or unstructured.

Now, once you have a structured it is easy to handle, because in your i j k direction can be tracked easily. This is not easy to handle, now because the cells are randomly located. So, when you have a structured system, you can easy to map between your local, and global indexing that means you go from i j k to n, n plus 1 and s, on. And this local to global indexing is very very essential as we said your linear system which involves a solution of A x equals to b. And this matrix only can be defined on the global indexing system. And there we have seen how local indexing to global indexing can be done, and this is much easier for the structured system.

And also you keep track of the discretize system there surrounding elements and all those things. So, also to some next easy to compute flux vector ok and at the same time here not straight forward I would not say easy, it is not straight forward to map that means, local to global or vice versa global to local. Here you have to keep track of the elements, keep track of the faces, keep track of the nodes including local global. And once you keep track of that then only you can do the connectivity. Also not straight forward to compute the flux vector, but that does not mean one cannot be, but it is easy to handle complex geometry. Here can be handled complex geometry through transformation.

And as soon as you do that it actually some sort of an equal kind of waiting, it requires like your unstructured system, because as soon as you do the transformation you need to handle lot of the data structure, you need to compute Jacobian, you need to calculate some derivatives cross derivatives and all these things. So, in a global sense, if you look at it, there are some advantage which are associated with structured system some advantage which are associated with unstructured system. And there are certain communalities I mean that of the systems you adopt you need to handle the local and global indexing which contains both your elements mapping, your face mapping, node mapping then accordingly calculation of the flux vector

For the unstructured system the calculation of the flux vector is not that straight forward whether the structured system the calculation of the flux vector is quite straight forward. And have been said that you might have certain kind of geometry like axisymmetric situations or when the neighboring elements or connecting distance between neighboring centers or not collinear some difficulties to calculation of these vectors, but this would be discussed in details when you go down to finite calculations. So, we will stop here.

Thank you.