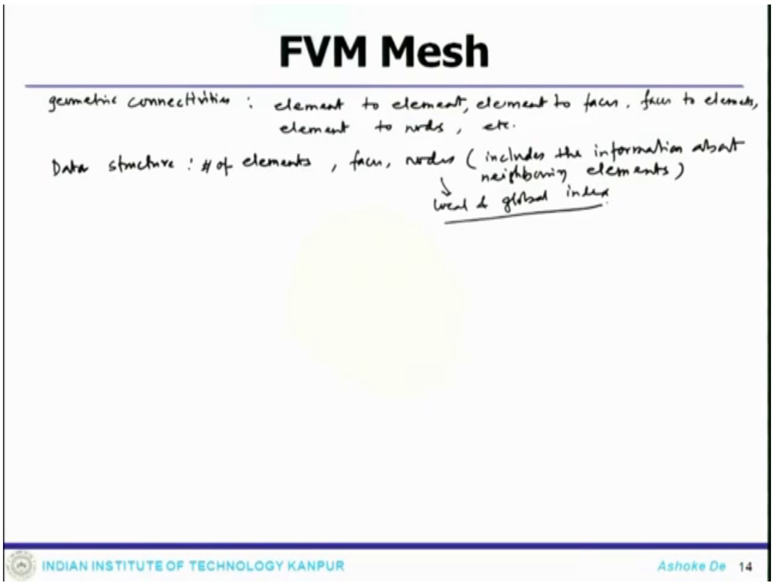
Introduction to Finite Volume Methods-I
Prof. Askoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

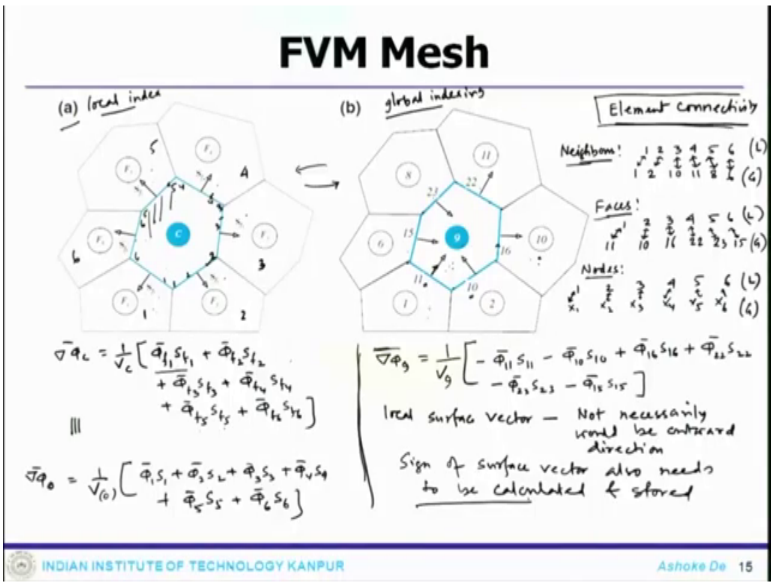
Lecture – 20
Unstructured Mesh System-II

(Refer Slide Time: 00:14)



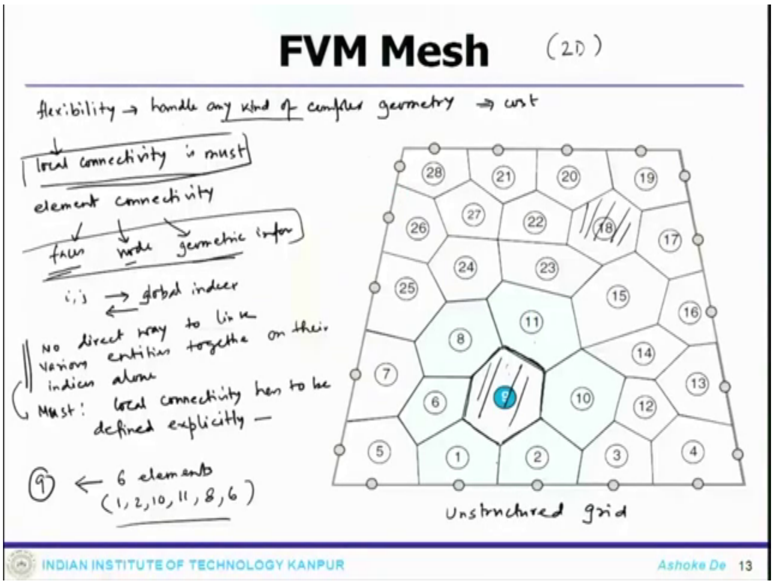
So, welcome to the lecture of this Finite Volume Method. Now, if you try to do the calculation for the gradient.

(Refer Slide Time: 00:26)



Let us say you mark these things properly. So, we have element 9 element 9 is sort of if you look at that and element 9 here this here is the local index so, that means, I have defined 1, 2, 3, 4, 5, 6 like that 1, 2, 3, 4, 5, 6 so, these are my 6 surrounding elements. So, here in this particular figure we stay in the local indexing and if you go to the global indexing so, this is how your global indexing would be associated. This is my element number 9, here this is exactly same what we are interested to calculate that and 9 now 1 to 10, 11, 8, 6 these are there my global indexing.

(Refer Slide Time: 01:52)



So, if you compare the picture here,1 to 10 11 8 6 this is how my global system was there. And I have transform them to a local system and then local to global so, this is how the connectivity is maintained. Now, if you do the calculation of the gradient for this local indexing here. So, this will be 1 by V c and I need to calculate f s 1, s f 1 plus phi f 2 s f 2 plus phi f 3 s f 3, phi f 4 s f 4 plus phi f 5 s f 5, plus phi f 6 s f 6.

So, that takes into account of all these surface normal vector s f 1, s f 2, s f 3 which are connected with our local indexing element that is s f 1, s f 2, s f 3, s f 4, s f 5, s f 6 and these are my local connecting element f 1, f 2, f 3 4 f 5 6 ok. So, one can write other way right this can be equivalently written that 1 by V 0 phi 1 s 1 phi 2 s 2 so, that is meaning is same. So, you write in this fashion or you write in this fashion, they actually point towards the same thing.

So, what typically it says you have this local connectivity, you have this local outward normal vector and you have 6 element which is surrounding to that particular element and their you get all these 1 2 3 4 5 6 normal vectors. So, your gradient calculation would be written like that. So, once you then and if you look at the other part, you can always have these global indexing and you can write this things in a global pattern.

Now, if you just concentrate on this particular element and face the element connectivity information. So, the element connectivity information if you write down then so, the what are the neighbors that is first thing. Neighbors you have 2 indexing the local indexing is 1 2 3 4 5 6 that is my local indexing, now the global indexing that goes back to this is 1 2 3 4 5 6.

So, if you look at global, this would be node number 1 this is 2 this is 10 11 8 6. So, this is my global indexing. So, 1 maps to this, 2 maps to this, 3 maps to 10, 4 maps to 11, 6 maps to 6 or vice versa the global 1 maps to local 1 so, either way is to. So, this is the local to global connectivity for the neighbors so, that takes into account all 6 elements, which are sitting there surrounding that element number 9 so, that is 1.

Now, you can see the faces ok. Similarly, like your neighboring element you would have the face also local connectivity and the global connectivity. So, the local connectivity if you first write down 1 2 3 4 5 6 this is local connectivity now, local face 1 is this one. So, this is mapping with number so, this is my face 1 local mapping or the surface vector. So, this is mapping with the face of 11. So, the global is 11, 2 this is my second surface and the global surface is 10.

Similarly, this is my face 3, global number of the face is 16; face 4 global number of the face 4 is 22, face 5 global number is 23, face 6 global number is 15 so, that is the global indexing. So, if you see kind of information that a unstructured grid system needs to have that is quite rigorous, because it requires this neighbors and neighboring element, then local indexing of the neighboring element should be connected with the global indexing that is one part of it then the faces; now all the faces in the complete domain are numbered.

Now, for this particular element the local faces where 1 2 3 4 5 6 and they are connected with the global numbers 11 10 16 22 23 15. So, this means the connectivity between local face and the global face also very important and this is how the connectivity is stored and in handling of the data structure one has to do that. So, that takes into account neighboring element neighboring faces. So, what is left here? Now the information of nodes; that means, this particular element does have 6 nodes or 6 vertices at this vertices also belong to this face or global faces and they also belongs to the other neighboring element.

So, the again if you look at it the local nodes must be 1 2 3 4 5 6. So, if you look at that local nodes are 1 2 3 4 5 6 these all local nodes. So, accordingly you can have global nodes and the global node number one can define. So, if you say these are my local node number you start from here 1 2 3 4 5 6 these all local node number and there could be some global node number x 1 x 2 x 3 x 4 x 5 x 6 ok. So, these are my global node number so, that says see it is a and they would be again mapped between local to global numbering.

So, all this information need to be stored in my data structure. So, once we say for a unstructured grid system the element connectivity, the element connectivity must include three things. One is the neighboring element connectivity so, that takes into account the mapping between local indexing to global indexing, faces that takes into account the local face to the global face, nodes local node to global node all these information need to be stored in your data structure before doing any calculations.

Now, if we coming back to the discussion of the gradient calculation, this is what we have done in the local system. So, local system if you do the same calculation of the global one then the global calculation would be 1 by V 9 and then here if you go by one after another phi f 1 S f 1. Phi f 1 is sitting or connected here with this so, this essentially connected with the this things the 16. So, what happens here that this particular term that if you see, this is your local term and this is your global term.

So, this would be the global calculation would be connected with this face of 11 and your normal vector that S f 1 which is actually going inward so, the sign would be negative. So, this would be phi 1 1 and s 1 1 because this is my local phi f 1 S of 1, if I come to the global one my face is 11. So, the flux should be calculated the face 11 and the normal vectors or the surface vector actually moving inward so, the negative sign comes in.

Now, you look at 2 f at 2 S 2, that is connected with the again surface 10 and the vector is pointing towards inward so, this will have again a negative sign. If you go to f 3 S f 3, surface is 16 here the surface vector is going outward. So, it follows the positive sign so, that will be like that. F 4 that means, it is connected with the 22 face number 22 and the surface vector is also going outward.

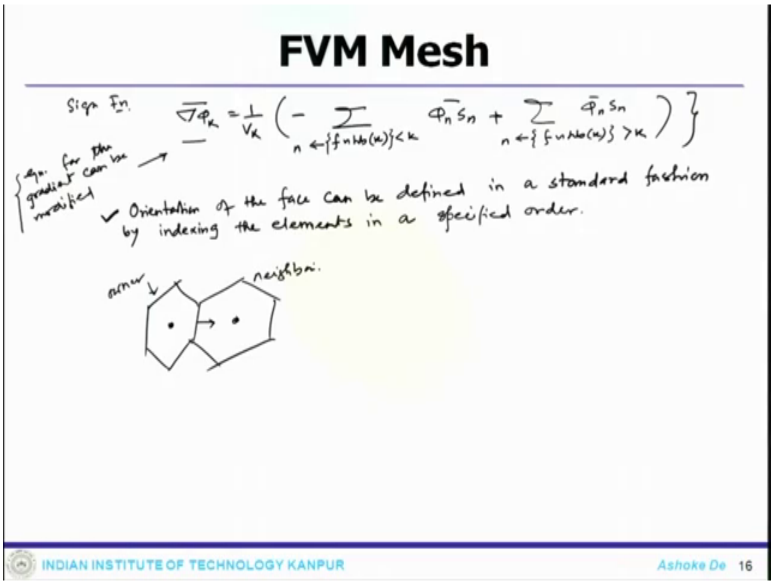
So, this would be phi 22 S 22 phi f 5 and S f 5 this is here if you go there surface is 23 and the vector is coming towards inside, again this will come with negative sign. And the last one is the 6th one; if you go here in the global the face number is 15 and the vector is pointing towards inside so, this will also come with negative sign ok. So, that is how you do the calculation at local indexing you do the calculation and then from local when you go down to global, you require the information of the fluxes.

So, one thing one I mean important to note here is that the this local surface vector they not necessarily sarily would be outward direction. So, as we have seen this particular example, the local surface vector when we have I mean drawn a local system we have assumed a prop standard sign convention. But, when you do exact calculation I mean essentially for the sake of this example, we have shown some outward some inward. So that means, the local surface vector does not mean that it is always going to be outward direction so, it can be outward.

But, now point here is that if you just look at this element and look at this element number 10. So, 9 is connected with 10 through the face number 16, now when you look at the face number 16 for 9, the surface vector is going outward. But, at the same time when you look at the element number 10 or while doing the calculation for the element number 10, will be looking at these surface it is pointing inward.

So, these gives you an clear idea or reversely you can think about this element number 1 which is here connected through the face 11 with 9 and the local surface vector actually going inward for this face for element number 9. But at the same time when you do the calculation for element number 1 the surface vector would be going outward. So, one case it is negative that is coming inward, the same surface vector would be the positive for the other element which has that common face. So that means, the sign of surface vector also needs to be calculated and stored so, that information is also required and very very important so, how 1 can do that?

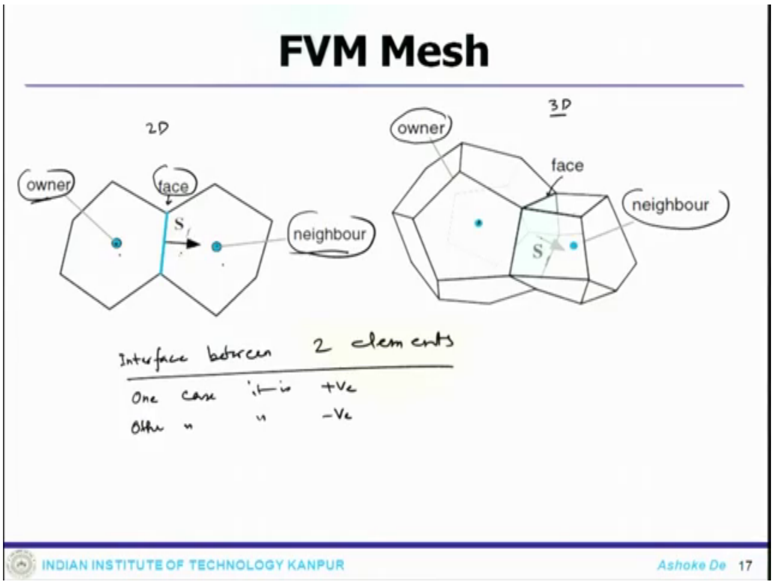
(Refer Slide Time: 18:01)



So, the let us say for sign function, one can use this kind of gradient calculation let us say delta phi k in general divided by 1 by V k minus summation f n b k less than k that goes towards n phi n S n plus summation f n b k greater than k goes to n phi n S n. So, this is how the equation for the gradient can be modified ok. So, this will take care that sign for an individual element when it goes outward or when it comes inward ok.

Now, so, that means, basically the orientation of the face that also can be defined in a standard fashion ok. So, this is very important again, the orientation of the face must be defined in a standard fashion by indexing the elements in a specified order. So, by doing that, you can actually store the orientation of the face and that can be stored by doing this. Now, essentially what that does tell you that if you have 2 particular element then you need to have this orientation one case it can go this other case should be outward so, one case it is owner, other case is the neighbour.

(Refer Slide Time: 21:17)

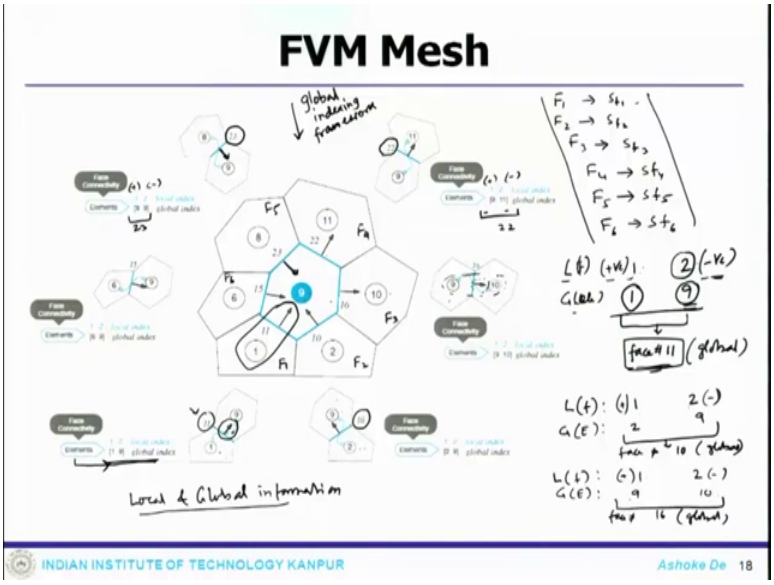


So, this one can see like this, you have a owner element, you have a neighbouring element these are the centre of those 2 elements and this is the surface vector and this is the face. So, the orientation needs to be accounted for or rather stored so, that you can have proper clarity on these elements and the similar thing this is in 2 D.

So, this gives you an idea this is 2 D and this is in 3 D. So, that gives you an idea about owner neighbour and face. So, that you have an idea about the orientation and if you look at the 3 D element, this is the connecting face, this is the centre, this guy is the owner, this guy is the neighbour and this is where the surface vector. So, that means, if you have consider the interface between 2 elements, then one element it is positive so, one case it is positive, other case it is negative.

So, once you look at 2 elements I mean associated with each other, than from the orientation of the surface vector you can get which direction it is pointing whether it is a outward direction or inward direction and then one element if it is pointing outward it would be positive. For example, this case the neighboring element it is taking inward, for this face it would be negative. So, if you go back to this example of 9 and then look at all these information.

(Refer Slide Time: 23:46)



Then you can see each of these element and you can rather explain each of these connecting faces individually. So, let us start with as our convention of the local element this was F 1 locally F 2, F 3, F 4 F 5, F 6 these are all local indexing. So, this theoretically should be face which is connected with the vector. So, F 1 element which will have S f 1, F 2 will have S f 2, F 3 will have S f 3, F 4 will have Sf 4, F 5 will have S f 5 and F 6 will have S f S f 6. So, these are my local element or local indexing of the element that we have used as per our sign convention.

So, now you look at the connections between element number 9, this is now we are in the global indexing framework. So, here the numbers that we get to see these are global. Now, you look at 9 and element number 1 these are the 2 elements they are connected with faces global face number is 11 ok. So, 11 is common for both element 1 and element 9. Now, the local indexing is for this face connectivity elements 1 2 is the local indexing. So, the global indexing would be 1 9 so, these are elements ok.

So, this is how you maintain the system; that means, every for 2 D cases every 2 D system each element with associated with these 2 elements as we said, for when we consider element number 1 the surface vector is going outward for element number 9 the surface vector at surface 11 is going inward. So, when it is going inward it should be with negative sign of that surface vector, now for these surface vector we maintain a global indexing of 1 and 2 which is sort of a universal for any 2 D element associated with one space.

1 means the connecting global indexing which is here, which is the element 1 2 element 9. So, these global indexing of the element is very important because whenever we have assigned local indexing 1 and 2, once means surface vector would be positive sign 2 means surface vector would be negative sign. So, this is the convention that I mean obviously, it is up to the user how he translate this things in his programming.

But this is typically the convention between 2 element when there is a connectivity and you map the connectivity always you have a local connectivity, this is local and this is global elemental connectivity this is local face connectivity. So, whenever its sees face 1, it will be multiplied with positive sign that means, it immediately points to the direction. So, the vector would be 1, for this particular case if you look at it as soon as we see 1 and the global indexing of the element is 1.

So, one connecting face is 11 so, both the elements has a common face, face number 11, which is a global number. So, when you go in the programming and the look for the data structure, it finds that global indexing as soon as it finds that global indexing of this face it find that a face number 11. So, it will face that information from the data structure face number 11 is connected with element number 1 and element number 9.

And immediately it is also picks up the information of local face numbering that is 1 or 2; for element number 1 it will pick up 1 immediately it understands that the surface vector is in positive side; that means, it would be pointing outward, which is currently shown here. And as soon as that picks element number 9 and the local surface indexing is 2, it will understand this would be minus sign so, that means, it would be inward as it shown here. So, this is exactly what we are talking.

So, that means, every 2 element in 2 dimensional system they are connected with a common face and that face also has a local indexing like this 1 and 2, there will be global indexing to that the face number is a one single face number which is globally indexed that is face number 11. And the global element number which are associated with that global face number is 1 and 9 and for them the local face indexing is 1 and 2.

Similarly, if you look at the other element, it would be much more clear or you can visualize the system. Now, let us move to the next element, element number 9 and between element number 2 so, this is 9 and 2. So, global indexing of the face is 10. So, as soon as it says so, I have a local facing which is again 1 and 2; that means, this is plus this is minus global element number which is 2 9, common face is 10 this is again global and this is face number.

So, whenever it picks the face number 10, the immediately data structure it will find that it is connected with element number 9 and 2 and local face numbering for element number 2 is 1 that mean positive sign. So, this will be pointing outward for element number 9 it will be local face numbering 2 so, this will be pointing inward. So, F 1 and F 2 they are of similar nature. Now you come down to the other case where you see element number 9 and 10 ok so, when you come down there the here 9 and 10 they are connected with a face number 16.

So, your again local face numbering is 1 and 2, which is negative and positive global element numbering is 9 and 10 and common face for the global is 16, this is the face number, this is global. So, as soon as you see that global face number, it will fetch the information from your stored database it is connected with global element number 9 global element number 10. And the local face which is connected with the 1 is that 1, one means positive as its shown here the normal vector or surface vector is pointing outward positive; for element number 10 it is 2 it would be negative; that means, when we considered element number 10 the surface vector going inward so, this is exactly what it does.

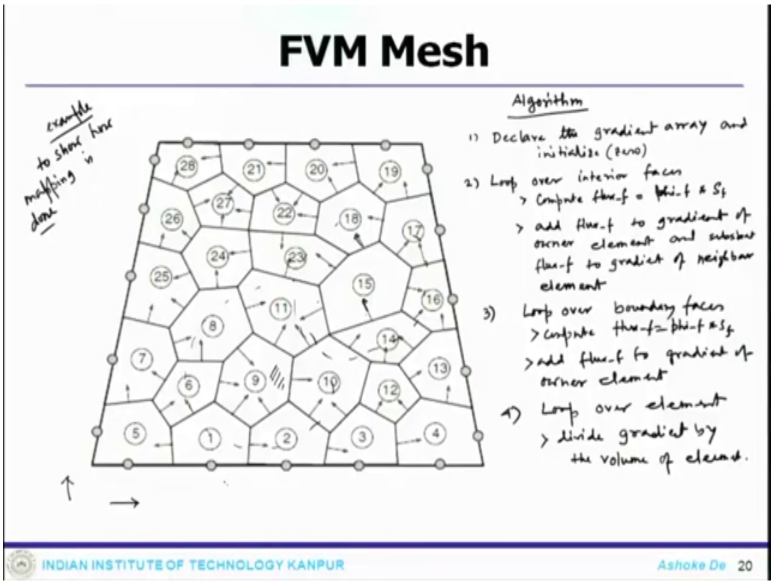
So, similarly, you come down to 9 and 11, the pattern is same. The global face is 22 face connectivity 1 and 2 so, 1 would be positive 2 would be negative global element numbers is 9 and 10 and the common face is 22. So, when you find out the common face 22, it picks the global element number 9, it also picks the global element number 11 and the local face connectivity 1 which is associated with 9; that means, it is pointing outward as it is here and for 11 it is negative.

And then reverse I mean the then you move to the other element 8 and 9, you come here 8 and 9 the global face is 23. Local face connectivity 1 and 2 again positive negative and this is the number. So, based on the global face number 23, it has the information of element number 8 and 9. For 8 local face indexing is 1; that means, it is positive so, if you look at 8 here things are going outward. So, which is positive, for 9 it is 2 negative; that means it is coming inward.

So, similarly it is between 6 and 9. So, that if you see this it gives you an clear picture how you store these local and global information. So, this needs to keep track of not only the element numbers face node local face indexing and so on. So, that now that gives you an idea how involved a unstructured grid system because, this requires lot of data structure issues.

Because, it had I mean you need to define element numbers, you need to define local element number, you need to have a connectivity between local element to a global element, you need to define face number, you need to have local indexing of the face number which sign convention. So, that actually takes into account the orientation, then you need to have global face, you need to have connectivity between local to global face of a particular element and so on.

(Refer Slide Time: 36:11)



So, it so, completely if you see; if you see this whole business this is how it looks like this is really in one way one can say it is really ugly, because this was not the case when you have compared the completely structured system. Structured system was much simpler to look; this is completely different wall game. Here it actually gives you an idea these are random, there is no sequence this is just for an example to show how mapping is done ok.

So, this is the element 9 which we have been discussing and the surrounding element like this, but if you go back to just one other element like let us say 11 or 15, you see this is connected with this surroundings elements and their vector some of them going out some of them coming in. So, that is true for the other things this is how it is actually done.

So, similarly like your structured system, if you put the algorithm in a place. So, how you find and do the calculation? 1st thing is that one you declare the data structure; declare the gradient array and initialize ok. So, you can initialize to 0 so, you do that. So, you declare the gradient array and initialize; number 2 second step you loop over interior faces and there what you need to look for you compute the flux compute flux equals to phi into S f.

So, you go over all the interior faces; that means, you first declare the gradient array then loop over these interior faces so, you go by these faces and then compute the flux. Once you compute the flux then you add these flux to gradient of owner element and subtract flux to gradient of receiving element or neighbor element ok.

So, that means, you have gone over this loops all the faces, you have done the calculation. So, this means you go over all the faces and 3rd step you loop over boundary faces ok. There are again you compute flux equals to S f, again you add flux to gradient of owner element, then 4th step you loop over all the elements and divide gradient by the volume of element ok.

So, the complete idea is that you define the data structure and essentially the data structure for the gradient array, initialize them, go over the faces and then you look for each faces here and compute the fluxes. So, that means, I go to this face, this face, this face, this face and compute this flux. Now, when you complete the flux then you for this you add to either this element is the owner element so, you add to it this is the neighbor elements so, you subtract from it.

So, that is where your local indexing comes into the place. Then you loop over all the element and then finally, you do the same adding and finally, you treat the boundary elements and divide by the volume to get the gradient. This is how you do for the unstructured case and we stop it here and then look at the element in the subsequent lecture.

Thank you.