Introduction to Finite Volume Methods-I
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture-16
Properties of discretized equations

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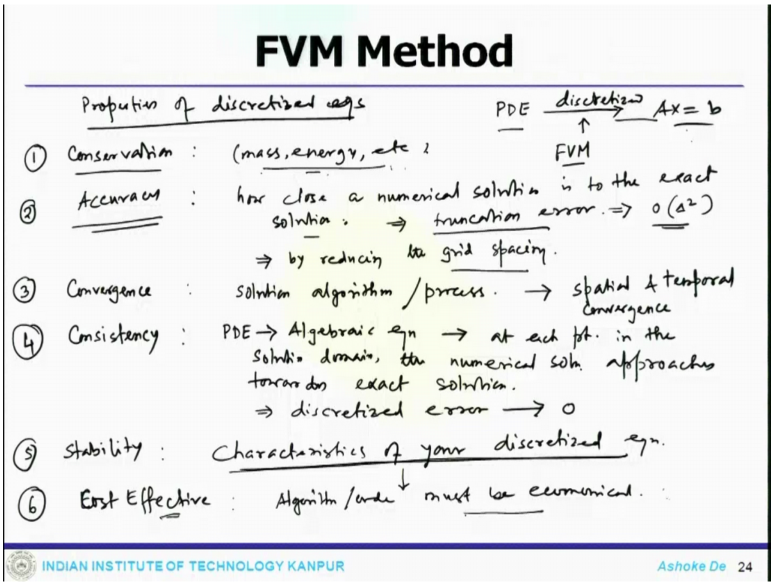

So, welcome to the lecture of this Finite Volume Method. Now when you talk about this properties one of the important another property is transportiveness.

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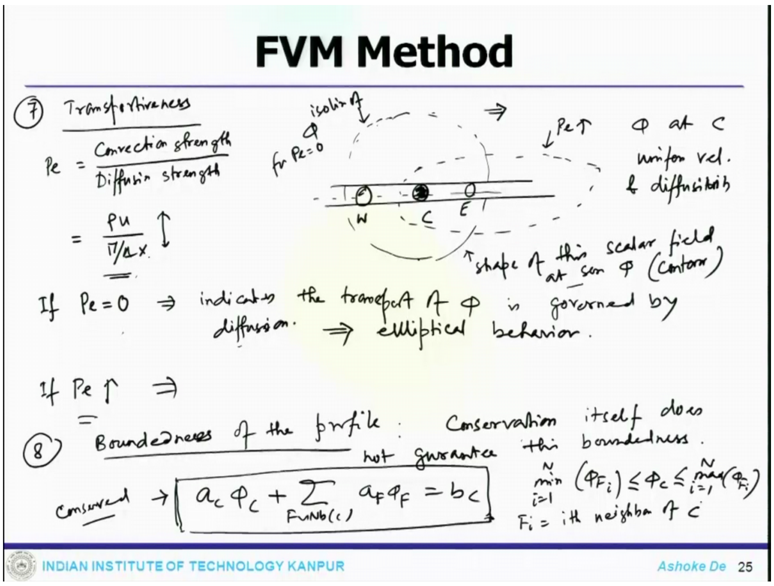

So, this is associated with that you can see an image how that let us say fluid particle, which moves along this particular line it is at location C and it move along this circle some point of time, it come W some point of time it comes to east. So, this has to do with your. So, this is a fluid particle how it moves along that path and that is sort of defined with the number called the pecletpeclet number which is the convection strength to the diffusion strength.

So, essentially rho u divided by gamma by delta x. So, this is very important number that let us say you have a constant phi phi at C in the flow field with uniform velocity and diffusivity ok. So, you have a fluid particle which has some uniform velocity and diffusivity within the flow field; so, the shape of this scalar field. So, essentially this is the shape of this scalar field, the shape of this scalar field actually at some phi will be influenced by this two ratio ok.

Now, for example, let say if your peclet number. So, this dotted line actually refers to the shape of this scalar field for some value of phi or rather contour, you can think about the contour of phi. Now the fluid particle having some uniform velocity and diffusivity, now if the peclet number is 0 what that indicates it indicates the transport of phi is governed by diffusion so; that means, which will have an elliptical behavior. So, which will be a elliptical behavior.

Now, in that in this case the phi is surrounded by this two two location. So, it will have some elliptical behavior and the isoline this is the isoline of phi for peclet number 0 and they are influenced by the neighbouring elements at e n. Now if you increase the peclet number if you increase the peclet number. So, essentially this is the fluid particle which are having some uniform velocity and diffusivity now the motion of this fluid particle, which will actually be represented with some iso contour of phi now they are governed by this two ratio which is given a number peclet number between the convection and the diffusion.

If peclet number is 0, then primarily it is governed by diffusion. So, the iso contour will move along the circle and the point at c will be influenced by the neighbouring point ahead of it and downstream of it. Now if i increase the peclet number; that means, my convection is going to now slowly dominant over diffusion. So, this circular shape will become elliptical shape. So, this is the case when peclet number increases and the phi will be shifted. So, this is how your solution of the fluid particles they are controlled by the different transportive properties and this will be one of the property which will be very much required when we have the convection diffusion system or the discretized convection diffusion system.

Now, the another important is the boundedness of the profile ok. So, this has something to do with the first property of conservation; so, conservation only itself. So, does not guarantee the boundedness. So, the conservation itself does not guarantee the this boundedness so; that means, when I discretize the partial differential equation and the discretized equations, they need to be bounded ok. If you recall for a element c we have written that a c phi c plus summation of f n b c a F phi F equals to b c. So, these discretized systems also need to be bounded.

If it is not bounded because when you arrive this discretized system from the pdes that through the volume integral and surface integral and we come down to this algebraic system, this is conserved. All your conservative properties in your pdes and the properties in your discretized system they are all conserved, but they do not guarantee the boundedness. So, to have the boundedness you must have a criteria like minimum of n phi F i must be less than phi c maximum i to N phi F i. So, F i represents the ith neighbor of C and N is their number.

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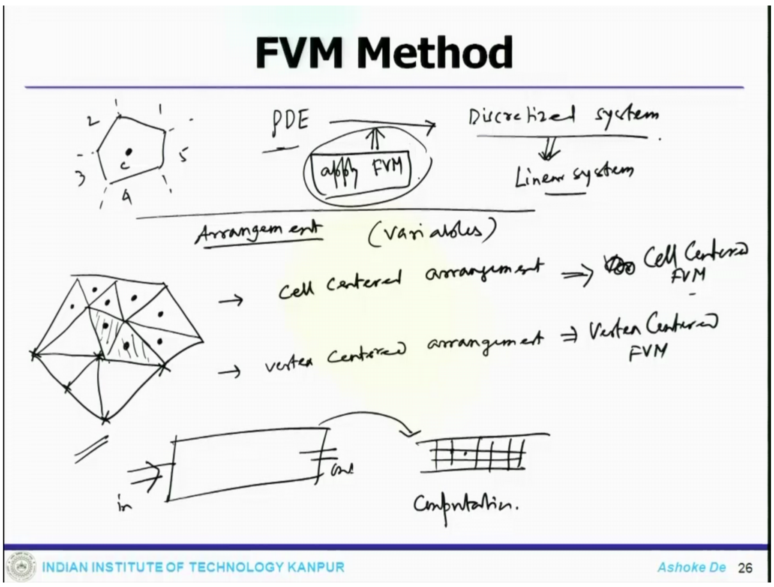

So, essentially if you have a element like this and you have all the neighbor 1 2 3 4 5 So, there would be certain boundedness which are associated with this particular this things. So, these are the important things that is essentially need to be satisfied by your discretized equation. So that means, when I come down from my PDE to a discretized system, I must fulfill those properties. From the discretized system essentially I will get by linear system ok.

So, that is what you get and when you convert this PDE to discretized system, this is where you apply your numerical technique like FVM or FDM and get this system. Now there will be couple of variable arrangements, one can let us say for example, what we have chosen I can have a geometry like this, and inside that I can distribute them in this kind of triangular element ok.

Now, one option is that how do you basically the this is called the arrangement how do you store your variables so, that will dictate a lot. Now in this particular arrangement one can wish to store his variable like at the cell centered. So, one can store the variable at cell centered. So, in that case it would be called cell centered arrangement; that means, whatever variable that we will be solving for they will be stored at the cell center. And so, for whatever we have discussed, we have discussed keeping the values at the cell center only. Now one may; that means, when I look at this cell center. So, I have a particular element like that over which I will get the integration of my equations and get it. So, this is called the cell centered arrangement.

Now, one may wish to store at the this points at the vertex points. So, if you wish to store at the vertex point, this would be vertex center centered arrangement ok. So, there are two different approaches one can adopt, one is the cell centered arrangement another is the vertex centered arrangement. Now accordingly if you devise your finite volume scheme then this will be called vertex centered FVM sorry this will be called vertex centered FVM and the other one could be called the cell centered FVM.

Now, if you see the difference. So, far we have only discretized our PDEs considering a finite cell and got a discretized equation. Now while doing that we did not bothered to think about where to store the data or the where to store the variable. Now, we slowly moving towards that while doing the discretization, you need to have this arrangement and why are they important? Because my physical problem: if you think about a flow through a channel my physical problem needs to be transformed to a computational grid ok.

So, there when I transform my physical problem to this computational do domain, I need to look at the arrangement of my variable. So, whether it would be cell centered or at the vertex center and accordingly my discretization process need be taken care of. So, when I apply this process, before that this arrangement is very important, but what are the typical pros and cons with this kind of arrangement?

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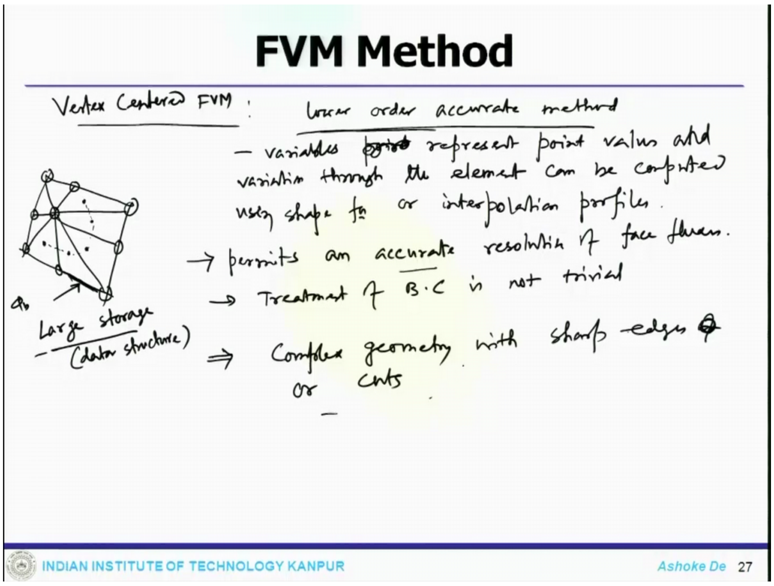

Let us first look at this vertex centered FVM. So, what you do? You have this particular arrangement. So, here you store the data at these points, the vertex points and if you have the centroid, then they would be connected like that. Now, what it is essentially the one of the biggest disadvantage of this vertex centered arrangement is that, it is actually get you a lower order accurate method. So, the vertex centered system or the finite volume actually gets you back the lower order accurate method, and why because in this case the variables or the variables point values; because the variables represent point values and variation through the element through the element can be computed using shape function or interpolation profiles ok.

But this actually gets you a lower order accurate system, but one of one very good advantage of this vertex centered arrangement is that the, this allows or permits; It permits an accurate resolution of face fluxes. Since the values are stored at the vertex center so, that allows you to permits to get an accurate resolution of the face fluxes. But that is not only the advantage apart from that these are having lot of having other shortcomings like your if you think about in programming the storage requirement is quite large; that means, your data structure.

So, you have to think about your programming point of view also, your data structure is quite large. So, the requirement would be large that means, you require high memory. Second problem is that if you have like if you think about this particular face and let us say you have specified a boundary condition here phi b, then the treatment of the boundary condition treatment of boundary condition is not trivial. See the other case when you have specified a value it was much easier to compute when you have stored things at the cell center.

So,. So, this should be a another cons of this vertex centered, and also along with that you may have some more complications like when you have complicated geometries with let us say complex geometry with sharp edges or cuts. So, if you have that kind of situation, then this vertex centered arrangement is not very healthy. Overall if you look at this particular arrangement, it does not give you too much of cost effective algorithm compared to the other one.

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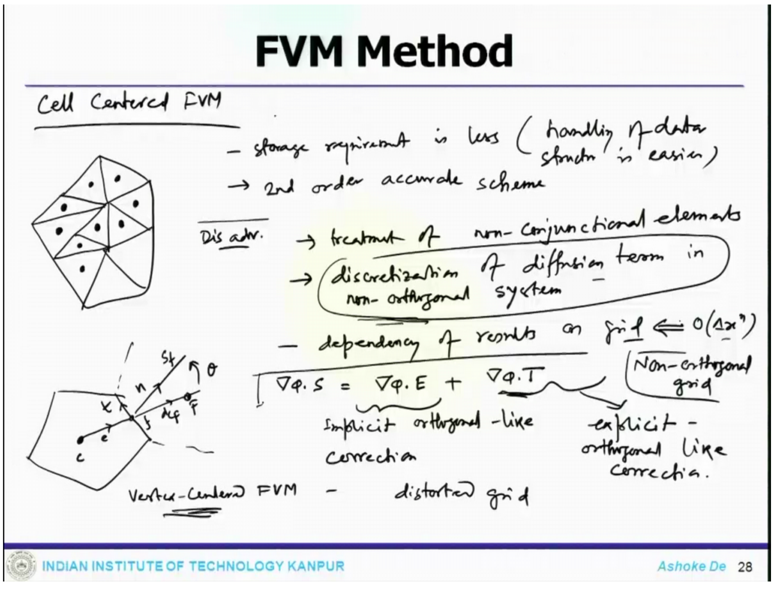

Now, if you look at the cell centered compared to the vertex centered. So, similarly if you look at your arrangements; so, here the arrangement would be completely different here you store at the cell center. So, values are stored at the cell center ok. So, you have storage requirement is less. So, essentially the handling of data structure is easier because only thing you need to know the connectivity amongs the neighboring cells ok.

Also this gets you back second order accurate scheme. So, in general the cell centered algorithm actually get you the second order accurate scheme so, that is a good advantage of this things. But obviously, it does not come with all sort of advantages, some disadvantages are also associated with that. One of the very prominent one is that essentially treatment of non conjunctional elements. So, that becomes a problem also the discretization of diffusion term in non orthogonal system ok.

So, as we move along we will see when at least when in the next lecture we will discuss about the mesh we will see when the system is not non orthogonal; that means, the orthogonality is not there, then how difficult or not trivial would be the discretization of this terms ok. Another thing is that obviously, there is a dependency of the result dependency of results on grid so; that means, the order of accuracy is also dependent on the grid spacing. So, if you have finer grid resolution.

So, then, but this particular issue of the non orthogonality issue of the diffusion term would be an biggest bottle neck and for example, if you let us say you have a element you consider one element like that, and you have a cell center c you come to f face. So, you goes like that e from here you goes to f this is n, this is t and d c F distance this is theta ok. So, this is the neighboring cell F this is your s f ok. So, typically the discretization term of the diffusion term would be del f dot s. So, del phi dot E plus del phi dot T.

So, this is essentially the implicit orthogonal like correction and this particular term is the explicit orthogonal orthogonal like correction. So, if you see that like the theta that increases the contribution of this term, would be more ok. So, that case this discretization will not or may not remain the. So, the remain is more robust. So, the robustness; so, this is a situation when the non-orthogonality will come non orthogonal grid or the system this will become.

So, the only situation where the performance of the vertex centered FVM will be superior over the cell centered on some sort of a distorted grid. That it is the only situation where you may have some superiority over cell centered arrangement otherwise in general the cell centered arrangement is widely preferred and has lot of advantage, but obviously, it does not come really of cost it comes with some disadvantages like non orthogonality discretizations of the diffusion term and all this that we will see later on and how that can be handled easily. Now, apart from that you have the implicit.

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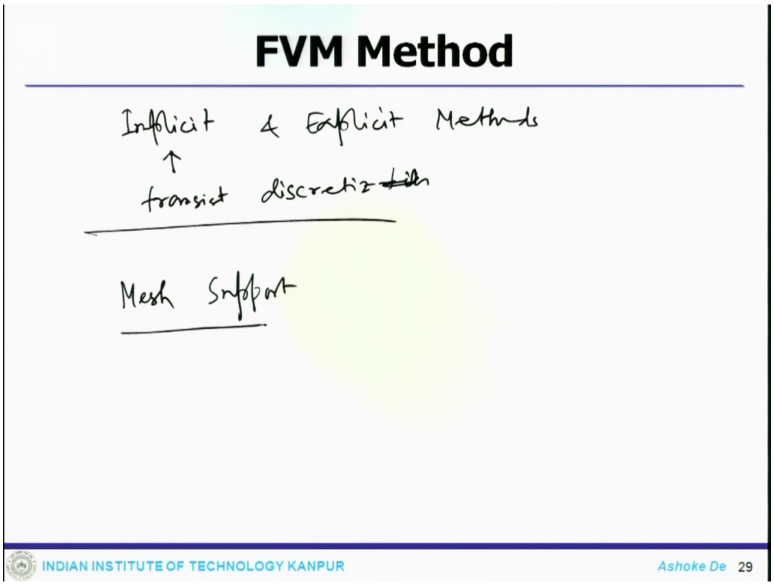

This already we have declared implicit and explicit methods. So, this is essentially associated with the transient discretization discretization. So, the solution algorithm the transient algorithm will have that dependency. Now in inherently what is important here is that the mesh support and that we will discuss in the next lecture.

Thank you.