Introduction to Finite Volume Methods-I
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Lecture – 14

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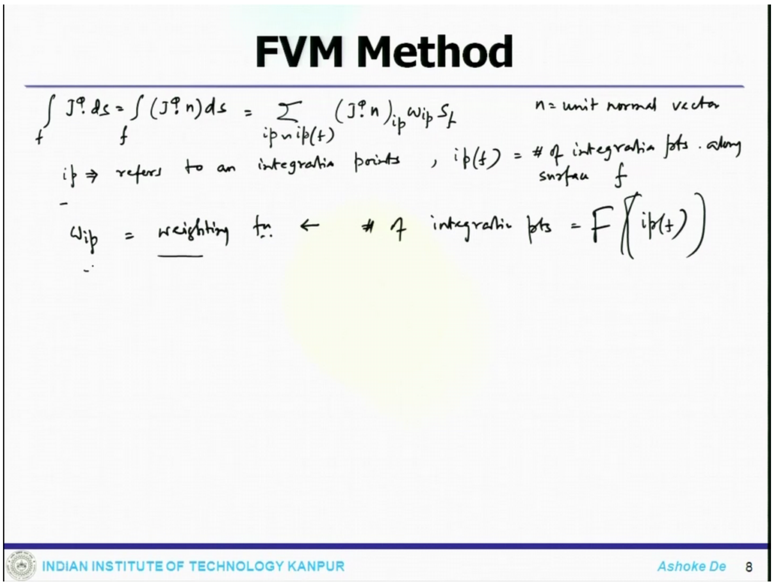

So, welcome to the lecture of this finite volume method. Now, we can actually approximate how many number of points you can have.

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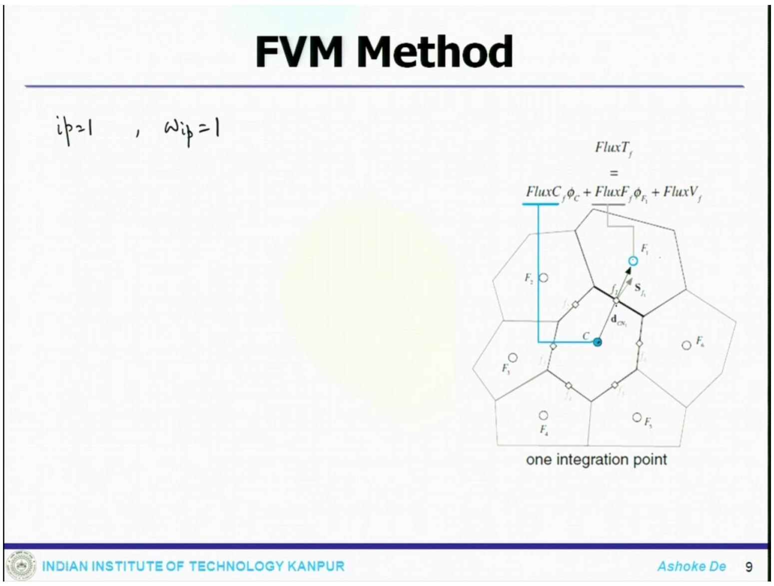

And so, for example, let us say you get this particular C element and you have the faces 1 2 3 4 5 6. This is the centroid of that element and your neighboring F 1 to get this flux integration I have one integration point. So, as I have mentioned that i p refers to the integration points and i p f is the number of integration points along the surface f.

So, this is a particular surface, let us say which is connected with F 1 and I have 1 integration points. So, that means, in this case i p is 1 and also w ip would be 1. So, I get a nice calculation for the flux calculations. Now, one can have multiple integration points. If you have multiple integration points, then what happens?

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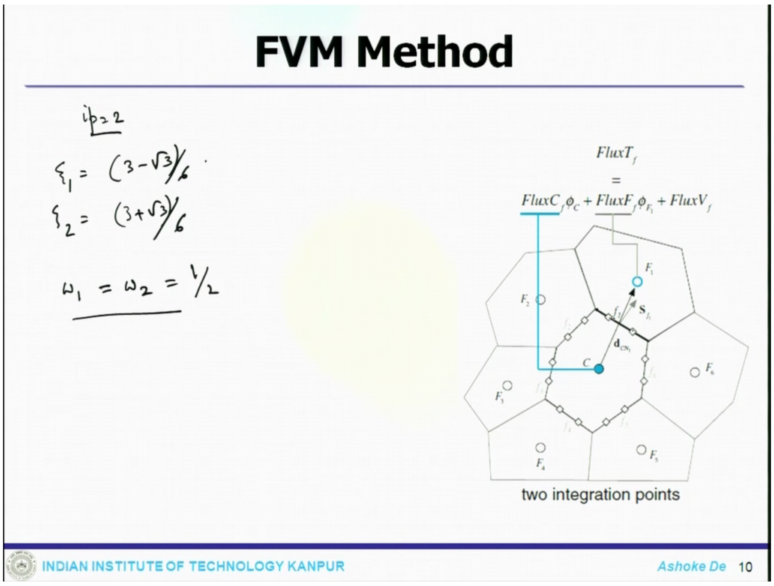

Let us say, you have 2 integration points along the surfaces ok. So, every connecting surface will have 2 integration points 1 and 2. So, in this case, i p is 2. So, if i p is 2, now my weighting functions are going to be different and then, I can get 2 different numbers x i 1 and x i 2. These are the and waiting function w 1 equals to w 2 equals to half.

So, the previous case I had one integration point. So, w 1 was 1 this case I have 2 integration point. So, the waiting function would be half and half and this xi 1 and xi 2 dictate the distance from one particular point to f 1 and then other side f 2. So, they are the distance.

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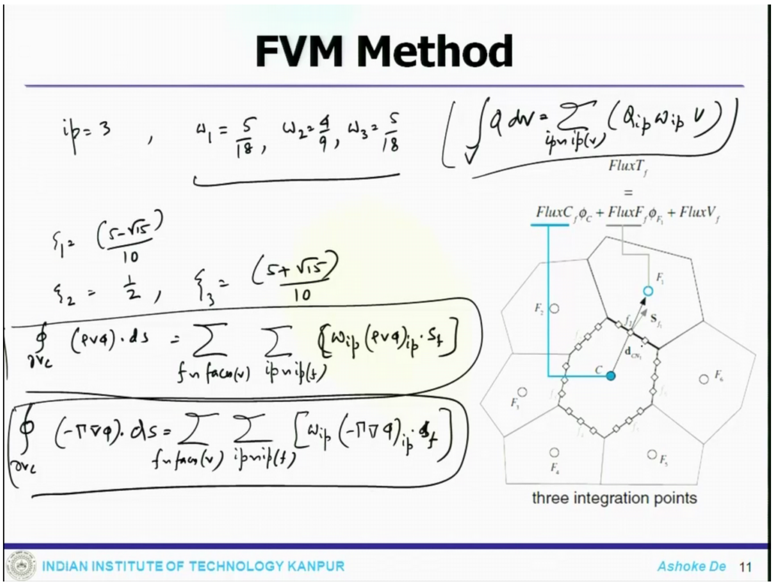

Similarly, you can have 3 integration points. So, that case, my i p would be 3. So, if you have 3 integration points, i p would be 3. Then, obviously, my waiting function w 1 would be 5 by 18 w 2 would be 4 by 9 w 3 is again 5 by 18.

So, if you look at that waiting function, they are equally sort of distributed then the distances would be x i 1 is 5 minus root 15 divided by 10 x i 2 is half and x i 3 is 5 plus root 15 divided by 10. So, these are the distance from one point to the other points ok. So, essentially what I got is that, rho v phi dot ds. It get me back the summation of f this is faces over v. So, i p over i p f w i p rho v phi i p dot s f.

So, that is what you get and similarly, for gamma del phi dot ds, that is my diffusive term. So, you get f faces over v and i p i p over f w i p minus gamma del phi ip dot s f. So, this is what you end up getting and the source term you can always calculate by doing the integration like, the source term you can always get q dv equals to summation of i p over v q i p w i p v. So, that is the way you get the source term integrated. So, then you can and i p and all these will depend how many integration points you have over the surface, ok.

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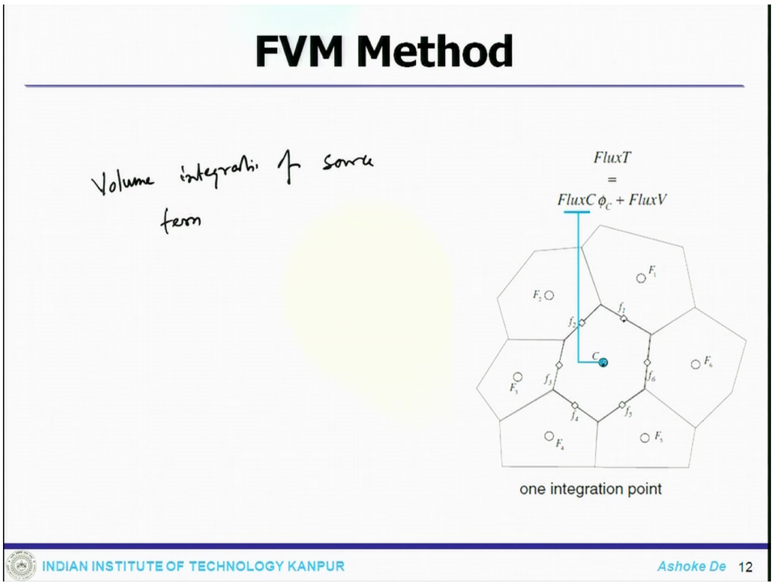

Now, you can get the similarly, the volume integration points with 1 integration point. So, basically the volume integration of source term.

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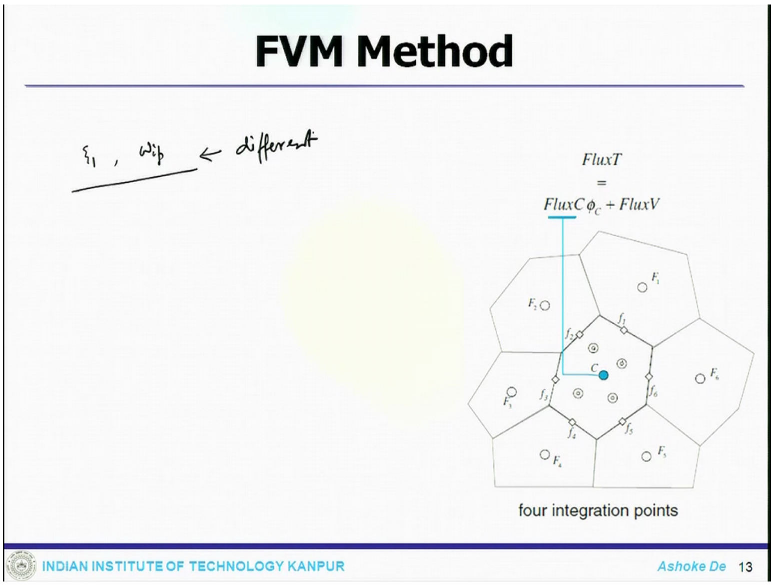

Similarly, you can have one point you can essentially, one point means here, you could have 4 integration points. Now, other case you had the points along the surface. Now, when you try to get the volume integration, you need multiple points inside the element of the volume. So, you could have 1 point to integrate, you could have 4 point to integrate ok. So, or you can have more. So, if you have it, then accordingly, your calculation of these x i and w i p they will be different. So, and you can also, I think you can have the 9 integration points.

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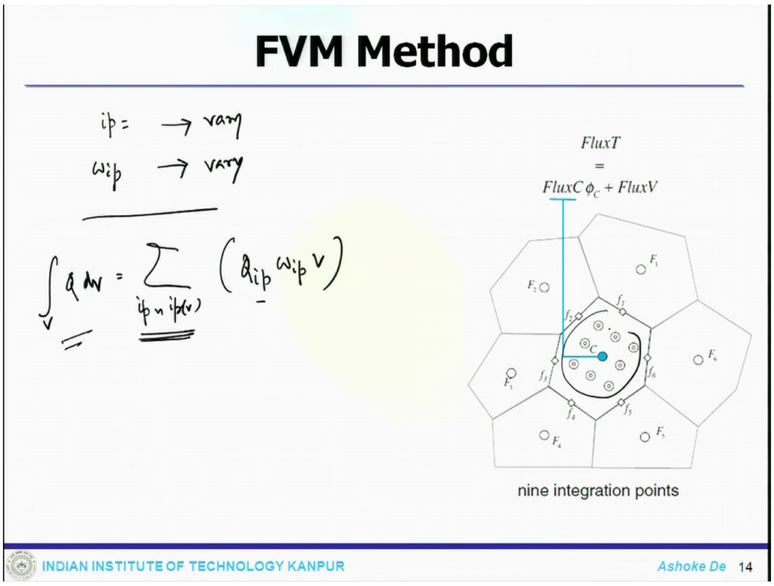

So, it depends so all these points. So, i p can actually vary accordingly your w i p will also vary and. So, your integration that you have got here like for the source term that Q dv, this is your volume integration of that. So, that will have the i p v q ip w i p v. So, this actually represents that how many points I have inside that volume. And, that is why this loop goes from number of points inside the volume. So, the difference remains between flux and the volume is that in the flux calculation. The number of points, they are varied along the faces, when you come down to the source term integration which is a volume integration, the number of points varying within the cell ok.

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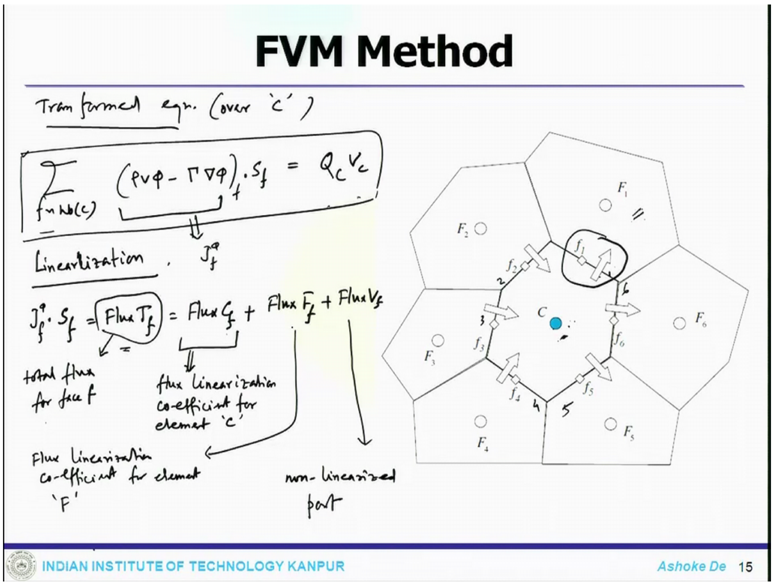

So, after getting all these done, if you look at that particular element C and you have all these neighboring elements and they are connected with the faces. So, my transform equation, transformed equation become obviously, over element C that would become like f nb of c rho V phi minus gamma del phi dot S f equals to Q dot Q c V c ok. So, that is my transformed equation.

So, what I started off? I started off a differential equation of steady state equation of the scalar transport equation. Now, I have written algebraic system. This is nothing, but the integration over the faces because, these are the fluxes. So, that means, it will go from 1 2 3 4 5 6. So, all these fluxes to be integrated. Now, another thing that one may wish to do is the linearization ok. So, you can linearize the system and so, how do you linearize the fluxes? You had this total flux J f dot S f. So, you can actually, you can say, this is the total flux T f, that is total flux and that can be realized as the flux C f plus flux F f plus flux V f.

What do they stand for? So, this essentially means, my total flux for face f if I consider, f 1, then my total flux should be this, is essentially my J f phi, that is the that includes both the component ok. So, that has both convection and diffusion. So, this represents total flux. What is this? This, this term actually represent the flux linearization coefficient for element C; that means, this is the element which I am interested in and trying to get the equation integrated over that. So, when I do the linearization because, this is the total flux along this face and this face is connected with face F 1.

So, one contribution comes from the linearization coefficient of this particular element along this face. So, the second is that flux linearization coefficient for f or you can put element f for element f. So, if you are talking about this flux. So, mind it, that we are talking about total flux for a particular face.

So, the total flux linearization at f 1 must be different from f 2. But if you look at only one particular face, again if you look at this equation where you have the total system which is integrated to the linear system, I mean algebraic system you have a loop over all the faces around that particular element. For this particular example, you have to have this loop go over from 1 to 6 because you have 6 faces.

Now, once you go down or come down to each of these faces now if you can look at each fluxes. So, along one particular face it needs to be linearized between this element and this element. So, one coefficient come from this another coefficient come from element F and then you have a component which is called the non linearized part so, you get 3 component ok.

So, the total flux along a particular face is decomposed in essentially 3 components; one component come from the element itself, the coefficient of the linearization. Second component come from the linearization coefficient of the neighboring or the connecting cell along that face and third coefficient is the non-linear part.

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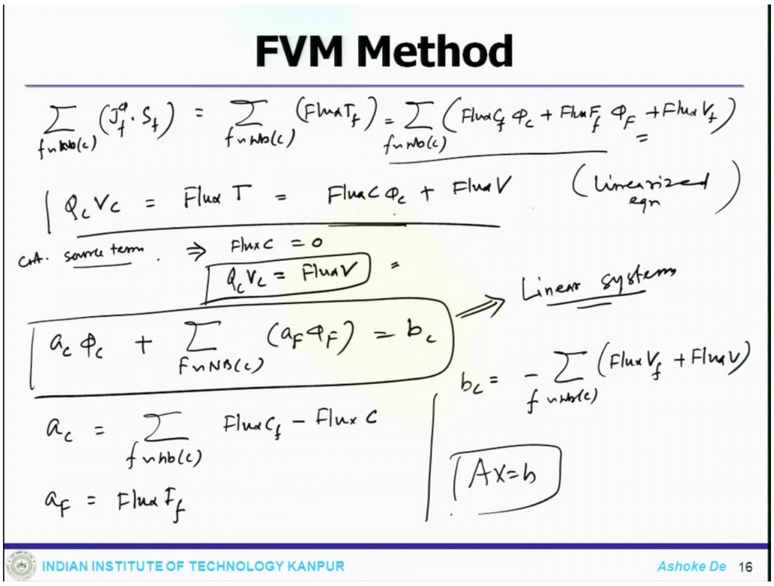

So, if you look at all this 6 faces, you will have this components ok. Now, if I essentially write down that equation, which is going for this element dot S f ok. So, this would be nothing, but my f the total flux ok. So, that means, this equation left hand side of the equation, I am writing at the total flux. So, this nothing, but the f over that element you have flux C f phi C plus flux F phi F plus flux V f; that means, if I come down to face 1, this would be flux C 1, I mean the coefficient of C 1 phi C coefficient of f 1 phi f flux V 1 ok.

So, for all the faces you can write down and then, I have the source term Q c and V c equals to essentially the total flux. So, if you write down the flux C phi C the flux F phi f. So, it is a linearized equation ok. Now, if you have let us say, constant source term, what happen? The constant source term So, I am again writing the source term. In this kind of fashion the contribution coming from the C element contribution coming from the neighboring element. Constant source term means flux C would be 0. So, sorry this should be this is flux V. So, my Q c V c is equal to flux V only ok.

Now, if you substitute everything back in this particular equation ok, if you substitute everything back there, you get phi c a c plus summation of this integration a F phi F equals to b c. So, essentially this expression and this expression both of them are, if you collect them together, you get this and what a c stands here? A c is f n b c flux C f minus flux C and a F would be flux F f and so, b c is nothing, but summation of f over this element flux V f plus flux V. So, what you have here? And here, if you put them together, then you get this. So, this is nothing, but your linear system and if you put them in a matrix form this is going to get you back the A X equals to B ok.

So, the thing is that over a particular element C you integrate your equation. So, the integral equation will look like that and once you compute all the fluxes and put them together, so you get back this system. Now, here you get a linear system like this, you cannot solve without having the.

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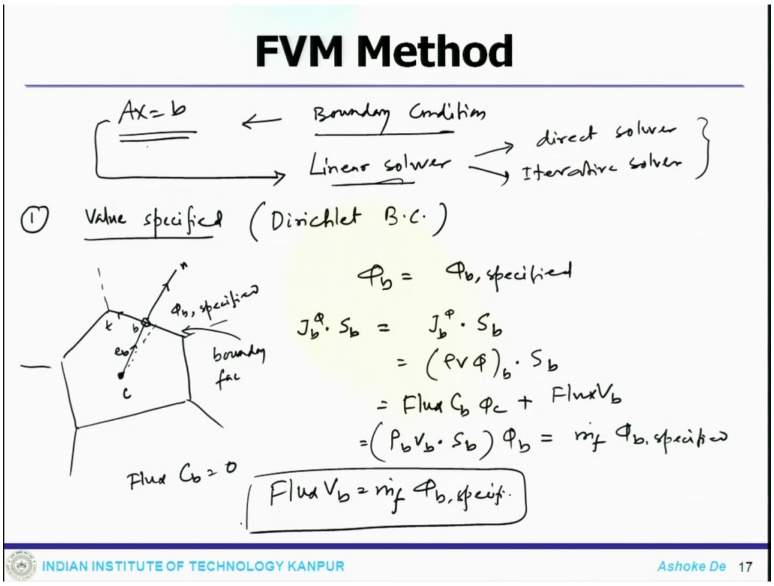

So, once you need to solve this linear system, the important ingredients are to have the boundary condition ok. One is the boundary condition which will fit to the system and then, the linear solver. Here, you can have direct solver, you can have iterative solver ok.

So, we have already discussed in a, I mean very quickly what kind of solvers. But, in details, we will do later on in a due course of time. But now, let us look at the boundary condition. How do you implement that? So, first, one important condition is that your have a the value is specified; that means, in other words, you say it is a dirichlet boundary condition ok. So, if you have a cell like this, what you had like and then you have a point here. Now, this is your c center and this is connecting and this would be the normal ok. And this is where your phi b this is the boundary surface this is t, this is specified ok. This is the point b and this is e b ok.

So, if phi b is specified, then I should have phi b equals to phi b specified ok. Then, at that particular face, this is the boundary face, this is the boundary face my J b phi dot S b. This is the, if you look at this integration, so, at that particular face, this will become J b phi dot S b. So, which would be if you just say the convective part rho V phi b dot. So, this is flux C b phi c plus flux V b. So, this will get you back rho b V b dot S b phi b which is nothing, but m dot f phi b specified. So, you can have flux C b is 0 and flux V b is m dot f phi b specified. So, you get that value.

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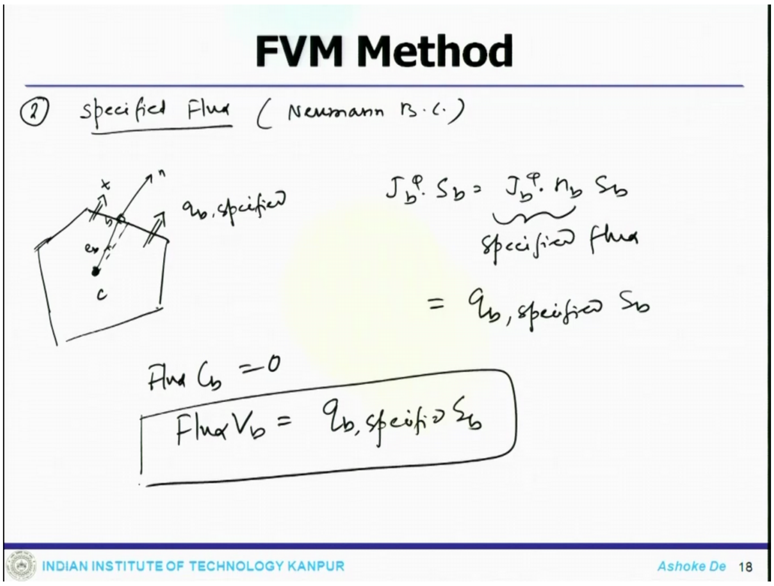

So, once you have that value, then now, the second type of boundary condition would be specified flux ok; that means, called the Neumann boundary condition. So, if you look at the same element here, this is the center. Then you come this is the normal this is e b, this is point b, this is t, this is where q b specified ok. So, once you do that, then your J b phi dot S b would be J b phi dot n b S b, ok.

So, this is your specified flux. So, which is nothing, but q b specified S b ok. So, my flux C b is 0 and flux V b equals to q b specified S b. So, these are the 2 type of conditions that you can specify at the face. So, now, we will continue from here in the next lecture.

Thank you.