

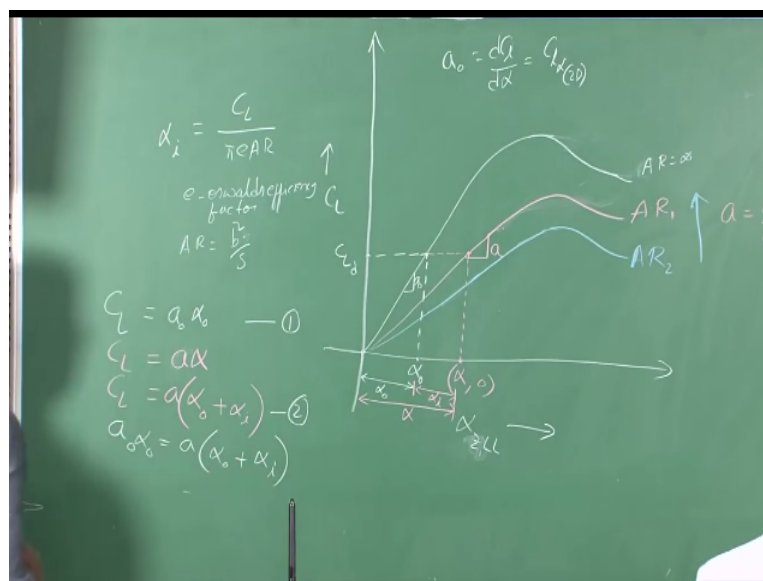
**Design of Fixed Wing Unmanned Aerial Vehicles**  
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**Lecture - 09**

**Interpreting Airfoil Data,  $C_L$  vs Alpha and Drag Polar, Selection of Airfoil**

Good morning friends. Welcome back. In our previous lecture, we were discussing about how  $C_L$  alpha of an airfoil and wing differs. When we say wing, we know it is of finite aspect ratio.

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So we have derived a relationship between the  $C_L$  alpha of airfoil and  $C_L$  alpha of wing. Let us say this is your zero lift line, angle of attack is defined with respect to the zero lift line of an aircraft and say this is your y axis represents your  $C_L$  variation and alpha is along the x axis here right. This is for aspect ratio infinite right say this curve represents the variation of lift coefficient with angle of attack for an airfoil right.

Now let us say this is for a finite aspect ratio wing which is  $AR_1$ , so this represents the  $C_L$  alpha curve for wing with aspect ratio 2 right, with aspect ratio  $AR_2$  right. Now say this is your increasing order of aspect ratio okay. Now let us assume we have used the same airfoil to build a wing right and what should be my angle of attack for airfoil to achieve a particular  $C_L$  and what should be the corresponding angle of attack for.

Let us say if I want to achieve a particular CL what should be the angle of attack that I need to trim an aircraft for an airfoil and for that of a wing right? So let us say this is your required CL say design CL or CL required. So for this particular curve say I require this angle of attack which is  $\alpha_0$ , so  $\alpha_0$  represents the angle of attack required by this airfoil to achieve this particular CL right.

So this is your  $\alpha_0$  right. Now let  $a_0$  be the CL alpha curve for this airfoil where  $a_0$  is  $\frac{dC_l}{d\alpha}$  of airfoil that is  $C_l$  alpha which is 2D. Now let us assume we have built a wing using this particular airfoil without any geometric twist and aerodynamic twist right. Now what happens if I want to achieve the same CL here right, I need to trim the aircraft at an angle of attack right.

Let us say this be alpha right, this point represents alpha, 0 right so this point represents 0, CL design okay. Now for a wing, we have an increased angle of attack compared to that of an airfoil. This increment is due to the induced angle of attack which we have witnessed during our previous lecture alpha i right where this alpha i according to lifting-line theory is  $\frac{C_l}{\pi e AR}$  where e is the Oswald's efficiency factor and AR is the aspect ratio  $\frac{b^2}{S}$ , both are non-dimensional parameters here right.

Now the CL design of the CL you can achieve by airfoil since it is a straight line you can assume this is in the linear domain of angle of attack or the variation of CL with angle of attack is linear up to this say whatever the CL that we have considered is in the linear domain okay. So now from the straight line equation we have  $y=mx+c$  that  $C_l$  is  $C_0$  here y intercept since we are defining this angle of attack with respect to zero lift line.

So  $a_0$  is the slope of this curve times the corresponding x  $\alpha_0$  here. So this is achieved by airfoil right. The same  $C_l$  I can achieve by the wing say  $a$  is the lift curve slope of this particular airfoil of particular wing, let  $a$  be the lift curve slope which is 3D lift curve slope where  $a = \frac{dC_l}{d\alpha}$  is  $C_l$  alpha 3D. So capital CL here represents 3D lift curve slope CL alpha whereas small  $C_l$  represents for 2D  $C_l$  alpha okay.

So this = slope of this wing lift curve slope of this wing\*by the corresponding x that is alpha here right. So this alpha is  $\alpha_0 + \alpha_i$  right, so say this is my equation 1 and equation 2 right. Now comparing 1 and 2 since we are producing that same design CL by using the

airfoil and wing, so the lift is same in neither the case, so from this equation 1 and 2 what we have is  $a_0 \alpha_0 = a \alpha + \alpha_i$  okay.

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The image shows a series of handwritten equations on a chalkboard:

$$a_0 = a \left( 1 + \frac{\alpha_i}{\alpha_0} \right)$$

$$\Rightarrow a_0 = a \left( 1 + \frac{C_L}{\alpha_0 \pi e A R} \right)$$

$$\Rightarrow a_0 = a \left( 1 + \frac{a_0 \alpha_0}{\pi e A R} \right)$$

$$\Rightarrow a_0 = a \left( 1 + \frac{a_0}{\pi e A R} \right)$$

$$\Rightarrow a = \frac{a_0}{1 + \frac{a_0}{\pi e A R}}$$

From here  $a_0$  is  $a \left( 1 + \frac{\alpha_i}{\alpha_0} \right)$  right. This implies  $a_0$  is  $a \left( 1 + \frac{\alpha_i}{\alpha_0} \right)$  right. Here is  $C_L / \pi e A R$  right. This is  $a_0 = a \left( 1 + \frac{C_L}{\alpha_0 \pi e A R} \right)$  right. So we can achieve  $C_L$  from this airfoil by trimming it at a particular  $\alpha_0$  right. So that means the  $C_L$  that I can achieve here is by  $a_0 \alpha_0$  from the airfoil  $a \alpha_0$ .

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The image shows a handwritten equation on a chalkboard:

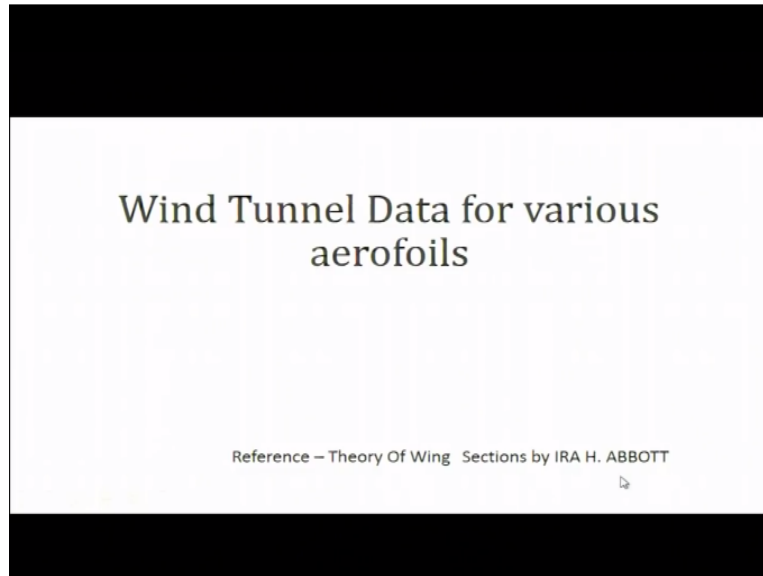
$$C_{L_{(3D)}} = \frac{C_{L_{(2D)}}}{1 + \frac{C_{L_{(2D)}}}{\pi e A R}}$$

This implies  $a_0$  is  $a \left( 1 + \frac{a_0}{\pi e A R} \right)$ ,  $a = \frac{a_0}{1 + \frac{a_0}{\pi e A R}}$  which is  $C_L \alpha_{3D} = C_L \alpha_{2D} / \left( 1 + \frac{C_L \alpha_{2D}}{\pi e A R} \right)$ . So this is one valid in the subsonic flight results right. This is the relationship between wing  $C_L \alpha$  and airfoil  $C_L \alpha$ . Now let us look at some of

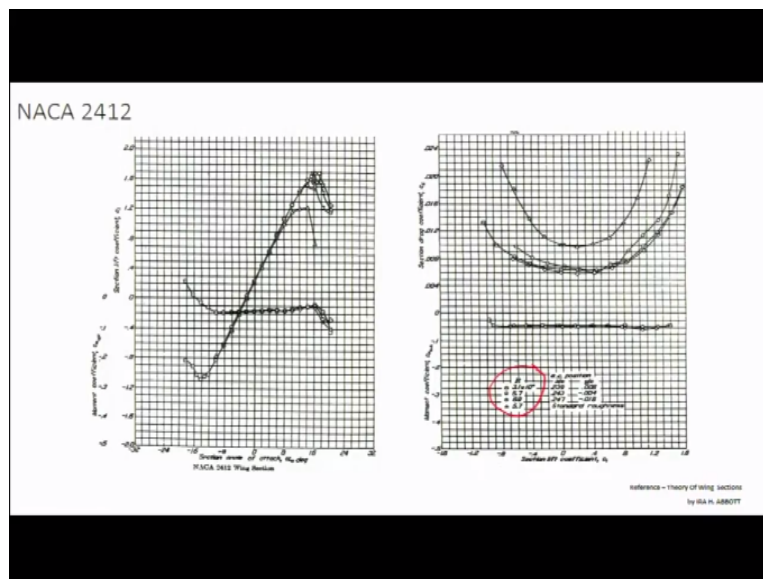
these airfoils and their lifting characteristics right. So this data that we are going to present is obtained from the wind tunnel test.

So the reference that you can consider for this is Theory of Wing Sections by Abbott.

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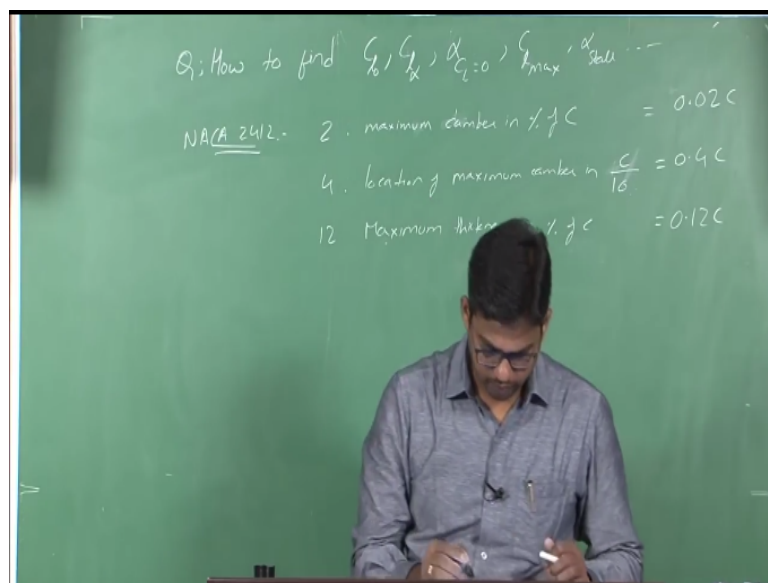


So the first airfoil here we also discussed this during our earlier example. It is a NACA 2412 airfoil. Now the question is how to calculate  $C_L$  right. How this data is obtained? You can see here, so this is obtained for different Reynolds number which means assume that you have used the same airfoil but you have performed this test at different velocities inside a wind tunnel.

So let us dedicate one lecture to figure out how to measure this  $C_L$ ,  $C_D$  and  $C_m$  from wind tunnel test right or say how a wind tunnel test is performed, how these measurements were taken for airfoil as well as wing. Now here let us consider any one of this Reynolds number say it is 3 million Reynolds number here in the first, so the corresponding marker here is they are circle.

So we need to follow the circle data to achieve the corresponding characteristics at 3 million Reynolds number right. So now the question is how to find what  $C_{L0}$ ,  $C_L$  alpha, alpha max, alpha stall,  $C_L$  max right, alpha at which  $C_L$  is 0.

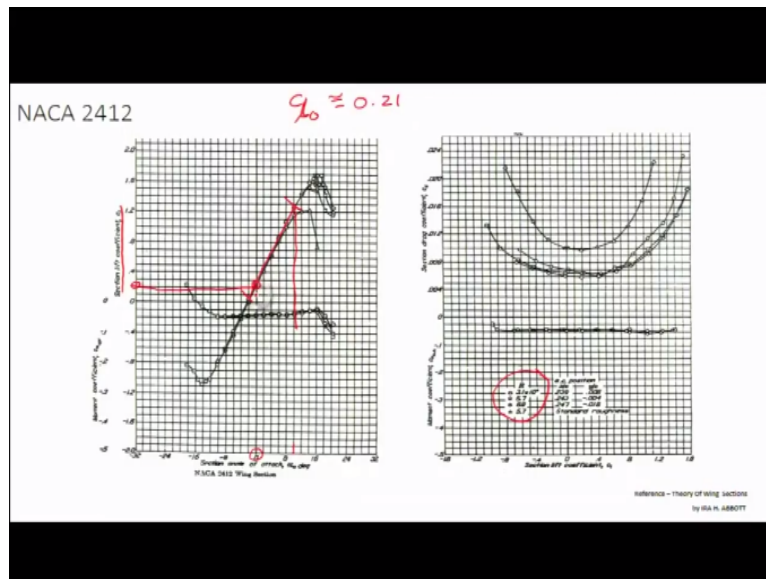
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How to find  $C_{L0}$  of airfoil,  $C_L$  alpha of airfoil and alpha at which  $C_L$  is 0 and  $C_L$  max of airfoil and alpha stall and so on right. So given the data let us see how to get these parameters. Now coming back to this 2412 data. We know what is 2412 right, just a quick recap. The 2 represents the maximum camber, maximum camber in percentage of chord right, 4 represents location of maximum camber in tens of chord that is  $c/10$  and 12 represents maximum thickness in percentage of chord.

See so this is=to maximum thickness location is maximum thickness is  $0.2c$  for this particular airfoil and corresponding location is  $0.4c$  and the maximum thickness is  $0.12c$  which is 12% here. Now what is  $C_{L0}$ ?  $C_L$  at which alpha is 0.

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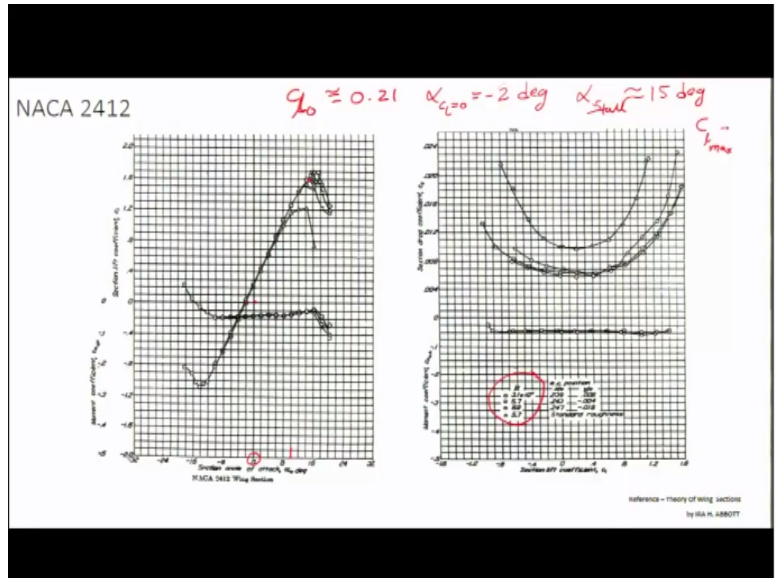


Now let us look at the angle of attack at which so this is your zero angle of attack. You can see here, now let us take the corresponding value of  $C_l$  that should be your  $C_{l0}$ , so the corresponding value is something here which say so this y axis represents the section lift coefficient here  $C_l$  right. So this is your 0  $C_l$  and there is an increment to 0.4 here that you can see so till that increment there are 4 steps which means this each line corresponds to 0.1  $C_l$  here.

So the corresponding  $C_{l0}$  from here we can notice it as  $C_{l0}$  so which is 2-dimensional,  $C_{l0}$  is 0.21 approximately right and say how to find out the  $C_l$  alpha here right. So we will find out  $C_l$  alpha while solving some of these examples okay. Now let us say how far this  $C_l$  alpha is linear, so for this particular curve which is achieved by performing this test at 3 million Reynolds number.

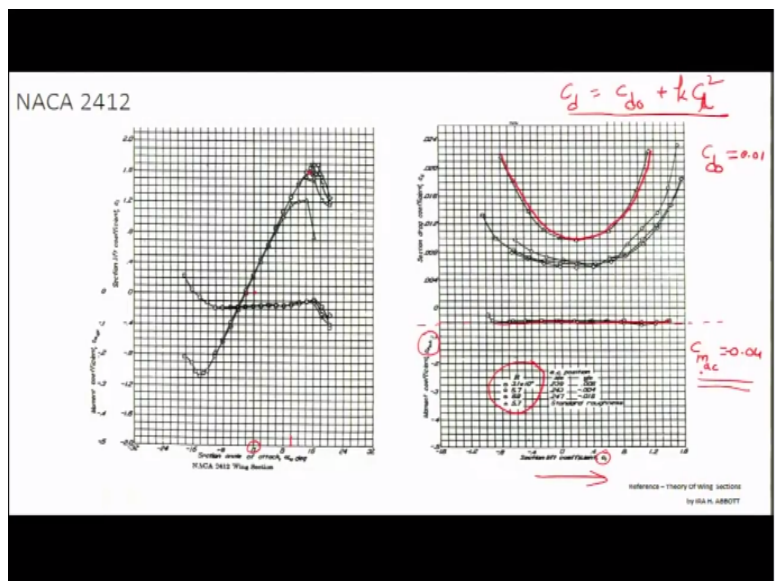
So up to here so up to here it looks pretty linear which is approximately 10 degrees angle of attack right, up to 10 degrees angle of attack this curve looks pretty linear which you can model it as  $C_l = C_{l0} + C_l \alpha \cdot \alpha$  right. So this particular regime you can consider the linear variation of  $C_l$  with angle of attack and now we have to look at alpha at which  $C_l$  is 0. This is your origin say 0, 0. I am sorry.

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Now this is your origin, now you need to find out alpha at which CL is 0 that means this is the point where CL is 0 right, this is the point where CL is 0. So the corresponding angle of attack for this is approximately -2 degrees right so alpha at which Cl = 0 is -2 degrees. This particular airfoil stalls at an angle of attack approximately 15 degrees right. Alpha stall is approximately 15 degrees and the corresponding value of Cl is 1.6 so Cl 2D max is 1.6.

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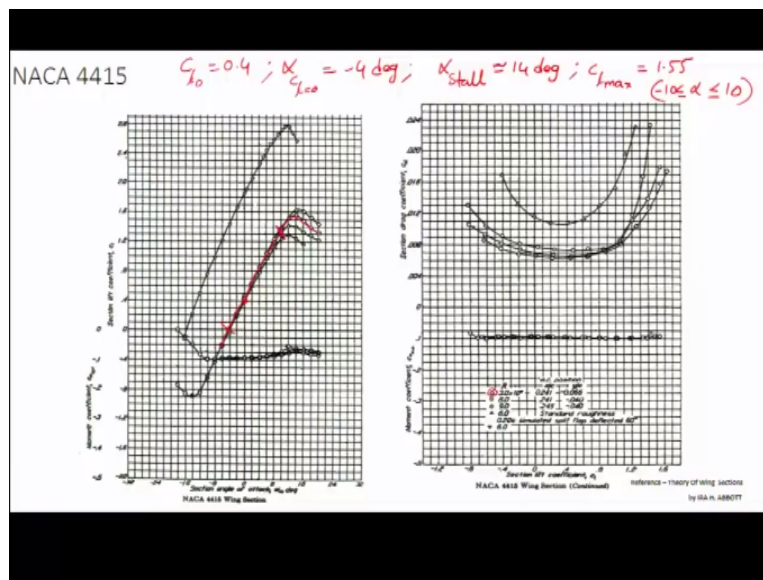


$C_d = C_{d0} + k C_l^2$  right. Now this particular plot represents the variation of Cd with Cl, see it is almost parabolic variation. So this particular equation or this particular formulation will definitely fit this data Cd whatever the data wind tunnel data that you have taken here. Now what is Cd0? The profile drag here which is at Cl is=0. So what is Cl0 here? So this is your Cl0 and the corresponding drag coefficient is 0.1 approximately.

So  $C_{d0}$  of this airfoil is 0.01, see this is the moment about aerodynamic center, variation of moment about aerodynamic center with  $C_l$  right although the lift changes, when the lift changes when the angle of attack changes lift coefficient is changing when the angle of attack changes right. So here this we can infer as the angle of attack changes and as the  $C_l$  changes, the pitching moment almost remain constant.

See it is a constant value now, although there is a variation in  $C_l$  that value of this pitching moment remain constant at about  $C/4$ . This is the corresponding aerodynamic center for this is  $C/4$ , so this moment is about  $C/4$ , quarter quad of that airfoil. So the value of this is about -0.04, so  $C_m$  about ac of airfoil is 0.04 right, -0.04. Please make a correction it is -0.04 which means it is a positively cambered airfoil. So for positively cambered airfoils, the  $C_m$  ac is negative right.

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Now let us go to the next data set which is 4415. It is again a four series airfoil, so you can see here so this is the variation of  $C_l$ ,  $C_m$  with  $\alpha$  and here to the right you have  $C_l$ ,  $C_d$ ,  $C_m$  variation with  $C_l$  right. Here what will be the  $C_{l0}$  for this airfoil,  $C_{l0}$  of this airfoil is 0.4. So let us look at the next airfoil which is also a four digit NACA 4415 airfoil right.

So we know the maximum camber is 0.4 c here and the location of 0.04 c and the location of maximum camber is about 40% of the chord and the maximum thickness is 15% of the chord right. Now for this airfoil what will be my  $C_{l0}$ ? So we need to select first the Reynolds number of our interest. Let us say 3 million is our current interest, 3 million Reynolds number is our current interest.

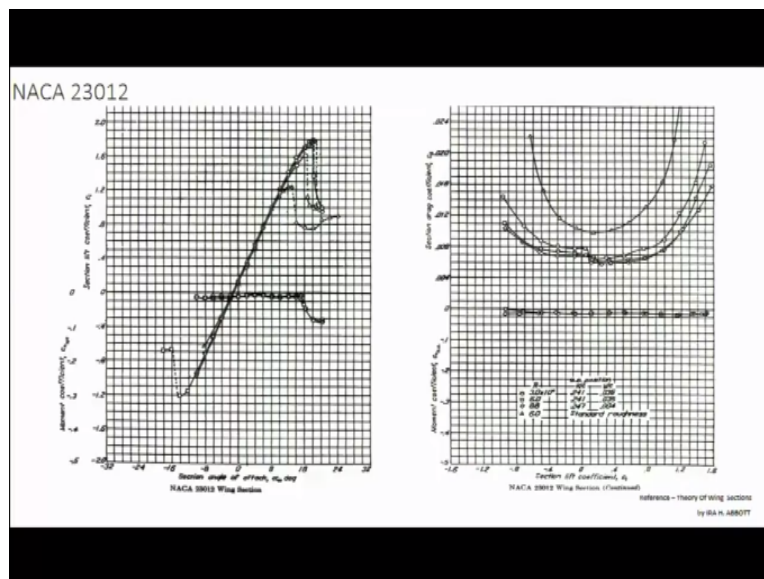




center?  $C_m$  about  $a_c$  is  $\approx -0.1$  approximately right. So you can see here with angle of attack, in the linear regime of angle of attack the  $C_m$  almost remains constant with angle of attack right in this linear domain.

So the aerodynamic center above we can consider this moment reference point is the aerodynamic center, why because the pitching moment is remained almost constant with angle of attack in this particular regime. This is the  $C_m$  variation with angle of attack.

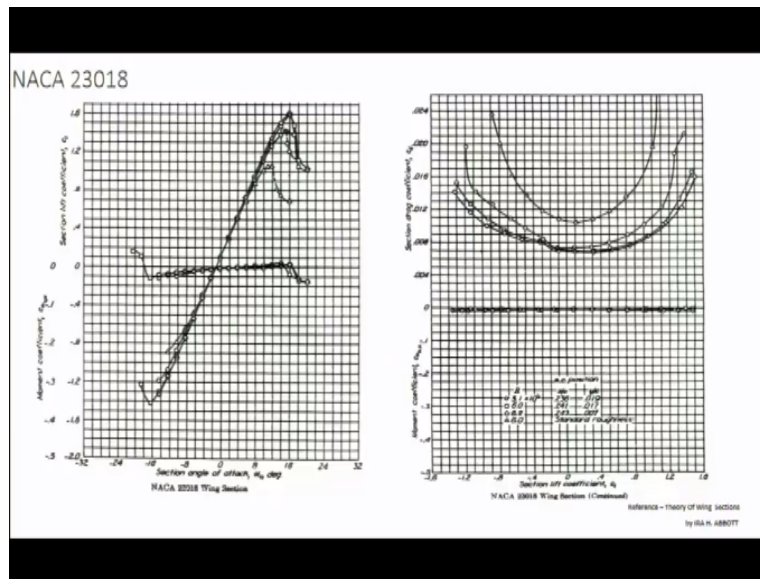
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Similarly, there is a five series airfoil 23012, you can also find  $C_{l0}$ ,  $C_l$  alpha, so the  $C_{l0}$  for this airfoil is about 0.1 and  $C_l$  alpha is about 6.45, will figure out how to find  $C_l$  alpha when solving some examples. The  $\alpha$  stall in this case is about 15 degrees and  $C_l$  max is 1.5 and the corresponding  $C_l$  max is 1.5 at  $\alpha$  stall of 15 degrees and  $C_m$  about aerodynamic center is -0.0125.

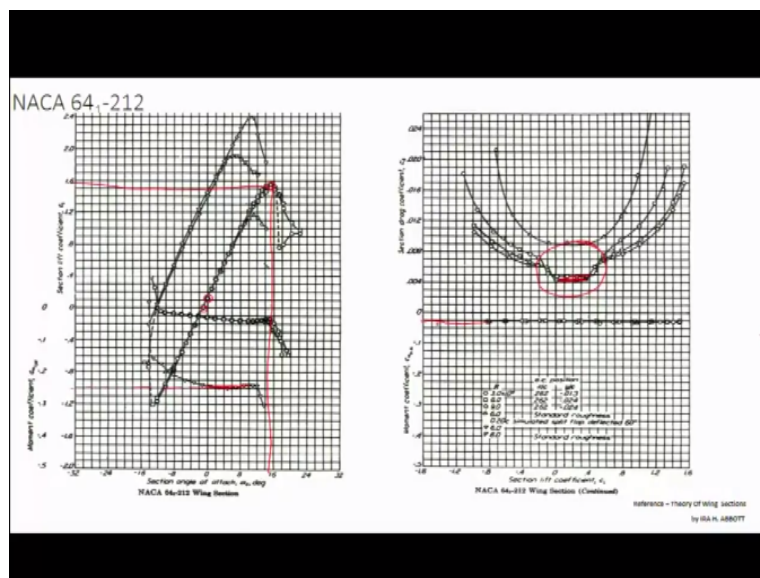
So we are talking about 3 million Reynolds number, data at 3 million Reynolds number and  $C_{d0}$  is 0.07 and  $\alpha$  at which  $C_l$  becomes 0 is -2 degrees in this.

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And there is another five series airfoil 23018. You can see the pitching moment is constant almost constant,  $C_m$  variation with  $C_l$  is constant here if you can see and the corresponding value is almost 0 here 0.01 say 0.01. The  $C_{d0}$  value for this particular 23018 airfoil is approximately 0.007 which is almost same as 23012 and 23018. So although there is an increase in the thickness ratio  $2/c$  but there is no increase in drag coefficient, profile drag coefficient.

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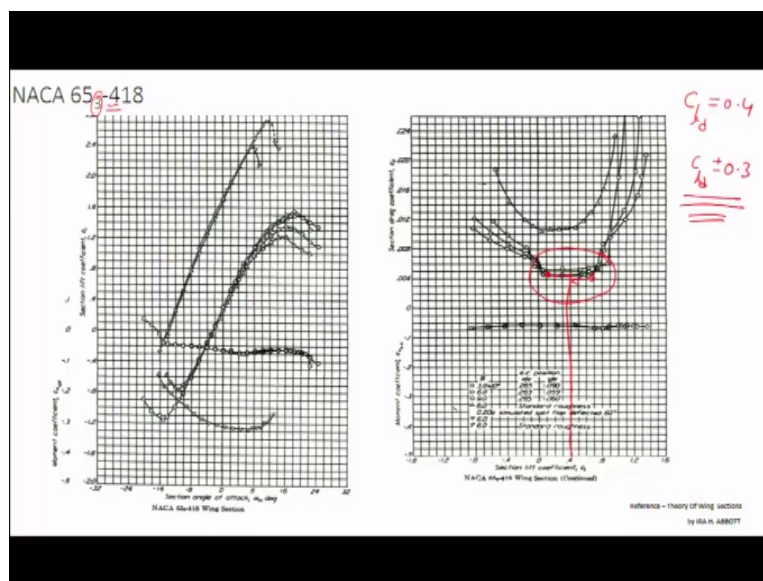


Now comes the six series airfoil. See there is a huge difference, the main thing that we can notice is that drag bucket. This is what we discussed in our earlier lectures. So in our previous case where the  $C_d$  versus  $C_l$  if you look at the drag polar plot, there is no significant drag bucket, you cannot witness a significant although for some portion it may remain constant almost constant but not exactly the same as what we have witnessing at six series airfoil right.

So right from here we do not see a significant drag bucket but for the six series airfoils you can see the  $C_d$  almost remains constant with variation of  $C_l$  or angle of attack here and similarly you can find out for this 641-212 airfoil, you have  $C_{l0}$  is 0.1 and alpha at which  $C_l$  it becomes 0 is -1 degree and  $C_l$  max is approximately 1.5 and the alpha stall is 15 degrees, this is 1.5 and corresponding alpha stall here is approximately 15 degrees right.

And  $C_m$  about ac is -0.025, so the value that you get  $C_m$  at different alpha is this is  $C_m$  at alpha 0 right. This is  $C_m$  at  $C_{l0}$ .

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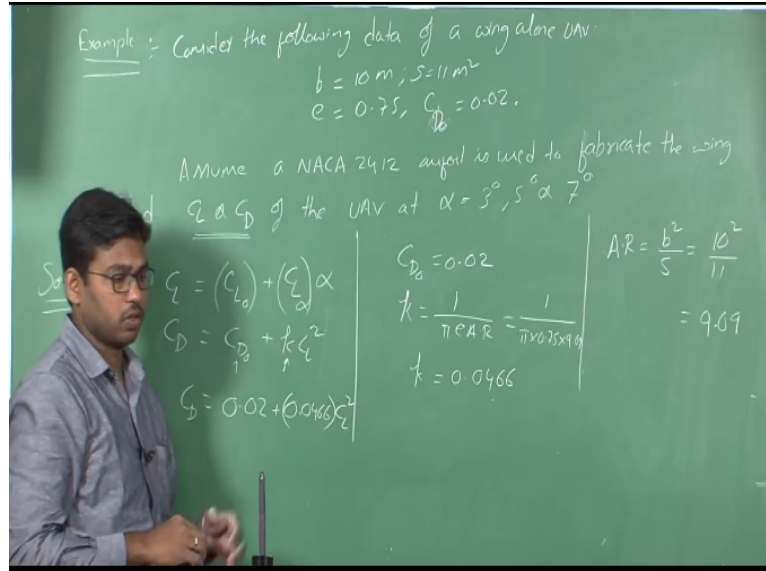
Now similar drag bucket we can also observe with 653-418 airfoil. Now let us look at this significance of this subscript 3 here right. Now from the nomenclature 65418 we can find out what is the design  $C_l$  for this particular airfoil which is  $C_l$  design of this airfoil is 0.4 right, this 4 represents in tens of I mean  $4 \times 1/10$  represents a corresponding design  $C_l$  of this airfoil.

Now let us say take this 0.4 here, consider 0.4  $C_l$  and say now the subscript 3 represents the regime in which  $C_d$  remains constant right for about this  $C_l$  design which is  $\pm 0.3$  of the  $C_l$  design, so in this regime  $C_d$  will almost remain constant. Let us see whether it is true or not. So  $0.4 - 0.3$  is 0.1 approximately, so up to this point the  $C_d$  is almost flat that is  $C_l$  design  $-0.3$  and then say  $C_l$  design  $+0.3$  is 0.7.

So we can also consider this regime as a constant  $C_D$  regime right, so this is a significance of this drag bucket here right that happens, there is a significant drag bucket for 6 series airfoil

and the nomenclature will help you to select the corresponding airfoil based upon your CL design. Let us take up few examples to find the relationship between wing CL alpha and airfoil CL alpha right.

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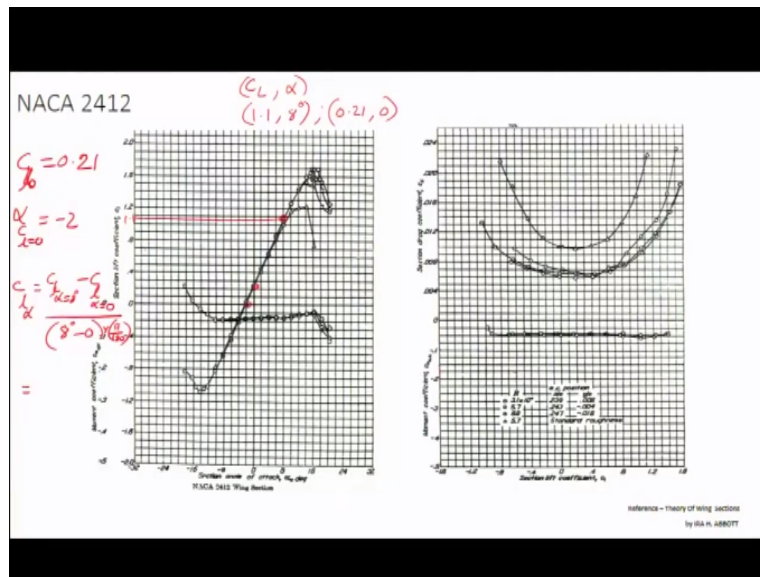
Let us take the first example. Consider the following data of a wing alone UAV. So there is some dimensional data is given here which is b is 10 meters, planform area is 11 meter square and the Oswald's efficiency is 0.75 and CD0 of the wing total UAV is given as 0.02 right. Assume a NACA 2412 airfoil is used to develop this UAV or since it is a wing alone we can say develop this wing.

Find CL, CD of the UAV at alpha is=3 degrees, 5 degrees and 7 degrees. When you say CL, CD here is for the entire UAV right and now how to solve this question? So CL is=CL0+CL alpha\*alpha of the wing right and what is CD, once you have CL if you CD0+k CL square right, you can just drag polar of the wing and you can find the corresponding CD at this particular CL right.

So what is this CD0 is given right, so CD0 is given as 0.02 and k, can we find k here? Induced drag coefficient factor 1/pi e AR right. So we know we have span and planform area, we need to find what is the corresponding aspect ratio. What is aspect ratio here? AR=b square/S=10 square/11 that is 9.09 right. So 1/pi e AR is so 1/pi\*0.75\*9.09 that is 0.0466 right is the value of k.

Now you know what is k, you know what is CD0. What is CD0 here?  $0.02+0.0466*CL$  square. Now for different values of CL you will get the corresponding CD, so when do you get different values of CL when you have different angle of attacks say 3 degrees, 5 degrees, and 7 degrees here but you need to find out what is the CL alpha of this wing and CL0 of this wing right. For that the information given is this is made out of 2412 NACA 2412 airfoil.

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So if you go back to NACA 2412 airfoil here, what is CL0 of this airfoil which is 0.21, so CL0 I can write down here as CL0 of airfoil is 0.21 right and alpha at which CL=0 is -2 degrees which is this point this particular point, this point we have CL0 and what is CL alpha of this plot say this plot we can safely assume a linear till 8 degrees angle of attack right. So CL alpha 2D is  $C_L$  at  $\alpha = 8$  degrees -  $C_L$  at  $\alpha = 0$  /  $8$  degrees -  $0 * \pi / 180$ , you will get it in radians right.

So what is CL at 8 degrees? CL at 8 degrees is 1.1 approximately, so this particular point is 1.1, so CL, alpha you have at  $\alpha = 8$  degrees what you have is 1.1 and at  $\alpha = 0$  degrees what you have is 0.21, 0 so by using this since it is a straight linear equation you can find the slope of this curve right,  $y_2 - y_1 / x_2 - x_1$ .

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following data of a wing alone UAV  
 $b = 10 \text{ m}$ ;  $S = 11 \text{ m}^2$   
 $e = 0.75$ ,  $C_{D0} = 0.02$ .

a NACA 2412 airfoil is used to  
of the UAV at  $\alpha = 3^\circ, 5^\circ$

$$C_{L(2D)} = \frac{C_{L(8^\circ)} - C_{L(0^\circ)}}{\frac{8^\circ - 0^\circ}{\frac{\pi}{180}}}$$

$$= \frac{1.1 - 0.21}{(8) \left(\frac{\pi}{180}\right)}$$

$$= 6.374$$

$$C_{L(3D)} = \frac{C_{L(2D)}}{1 + \frac{C_{D(2D)}}{\pi e AR}}$$

$$= \frac{6.374}{1 + (6.374) \times 0.0466}$$

$$C_{L(3D)} = 4.9123$$

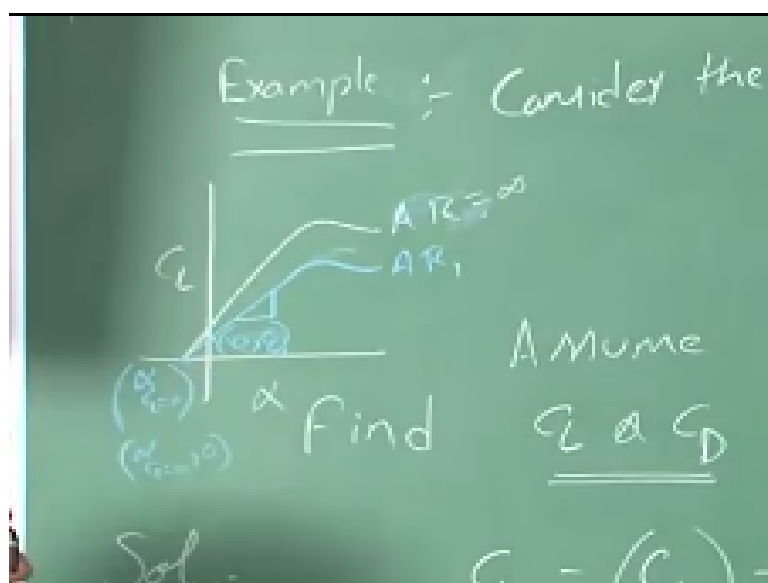
$C_{D0} = 0.02$   
 $k = \frac{1}{\pi e AR} = \frac{1}{\pi}$   
 $k = 0.0466$

$+ (C_L) \alpha$   
 $+ k C_L^2$   
 $+ (0.0466) C_L^2$

CL alpha 2D of this airfoil is=CL value at alpha is=8 degrees-CL value at alpha is=0/8 degrees, so per degrees I am converting into per radians. This is alpha is=8 degrees you have 1.1 approximately  $-0.21/8 \cdot \pi/180$ . So this turns out to be 6.374 right. This is your CL alpha 2D and other important yes once you have CL alpha 2D, you can find out CL alpha 3D. CL alpha of this wing is=CL alpha 2D/ $1+CL$  alpha 2D/ $\pi e AR=6.374/1+6.374 \cdot 1/\pi e AR$  is  $0.0466 = 6.374$  is 4.9123.

So CL alpha per radian of this wing entire wing is 4.9123 you have CL alpha here 4.9123 per radian right. Now you need to find out what is CL0 to find out CL at different alpha as well as CD at different alpha right. So how to find CL0? So the assumption that we make here is since we are making this wing out of this airfoil right.

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So we have  $C_l$  versus  $\alpha$  plot right this is for say airfoil okay and we are making this wing out of this airfoil, the assumption here is the  $\alpha$  at which  $C_l$  is 0 for airfoil will remain same for the wing as well right. So this is the point at which  $\alpha$   $C_l=0$  will remain same for wing as well as airfoil, this is the assumption.

So this is the  $\alpha$  at which  $C_l=0$  so this remains same for infinite wing and finite aspect ratio wing as well as infinite aspect ratio wing right okay. Now with this assumption, we know what is  $C_l$   $\alpha$  of this curve so we got  $C_l$   $\alpha$  3D and you know this point we need to find this point, so this particular point is  $C_l$   $\alpha$  at which  $C_l=0$ , 0 and this particular point is  $C_{l0}$ ,  $\alpha$  is 0,  $C_{l0}$ .

So you have two points, you know the slope like you do not know one of the point here, you can straight away find this  $C_{l0}$  right.

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$$C_{l0} = -C_l \alpha_{C_l=0}$$

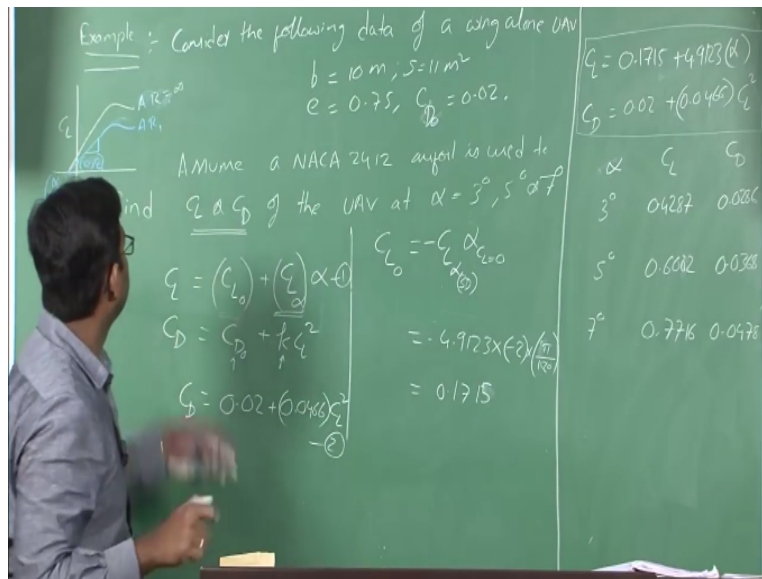
$$= -4.9123 \times (-2) \times \left(\frac{\pi}{180}\right)$$

$$= 0.1715$$

So this  $C_{l0}$  is  $C_{l0}$  3D is  $C_l$   $\alpha$  -  $C_l$   $\alpha$  3D \*  $\alpha$  at which  $C_l$  becomes 0 right that is what you will end up solving that equation right. So what is  $C_l$   $\alpha$  3D here is  $4.9123 * \alpha$  at which  $C_l$  is 0. So from the airfoil data we can find  $\alpha$  at which  $C_l$  is 0 is -2 degrees right. So -2 is it right? We considered  $C_l$   $\alpha$  as 4.9123 per radian so you need to convert this  $\alpha$  from degrees to radians. So this is  $2 * \pi / 180 = 0.1715$ .

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So now substitute this in this equation 1 say this is your 1 and say this is your 2 right, so from equation 1 what we have  $C_{L0}$  is  $0.1715 + C_L \alpha$  is  $4.9123 * \alpha$  right. So  $\alpha$  should be in radians here and  $C_D$  is  $= 0.02 + 0.0466 * \text{corresponding } C_L^2$ . Now we need to find this at 3 different trim conditions that is  $\alpha$  for 3 degrees,  $\alpha$  for 5 degrees,  $\alpha$  for 7 degrees.

Now if you substitute  $\alpha$  3 degrees what you end, so say this is  $\alpha$ , this is  $C_L$  and the corresponding  $C_D$ , say write  $\alpha$  3 degrees what you have  $C_L$  is 0.4287 is the  $C_L$  and the corresponding  $C_D$  is 0.0286 right. So at 5 degrees the  $C_L$  value is 0.6002 and  $C_D$  value is 0.0368 and  $\alpha$  at 7 degrees  $C_L$  is obtained as 0.7716 and  $C_D$  is 0.0478. So all these values are obtained by using these two equations right.

So what we have done, given a wing planform geometry and Oswald's efficiency factor as well as profile drag coefficient, so we figured out what are the  $C_L$   $\alpha$  of this wing and  $C_D$  of the I mean  $C_{L0}$  of this wing and then we calculated what is the corresponding lift coefficient at different angles of attack as well as the drag coefficient at different angles of attack. So it is given that this wing is made out of NACA 2412 airfoil right.

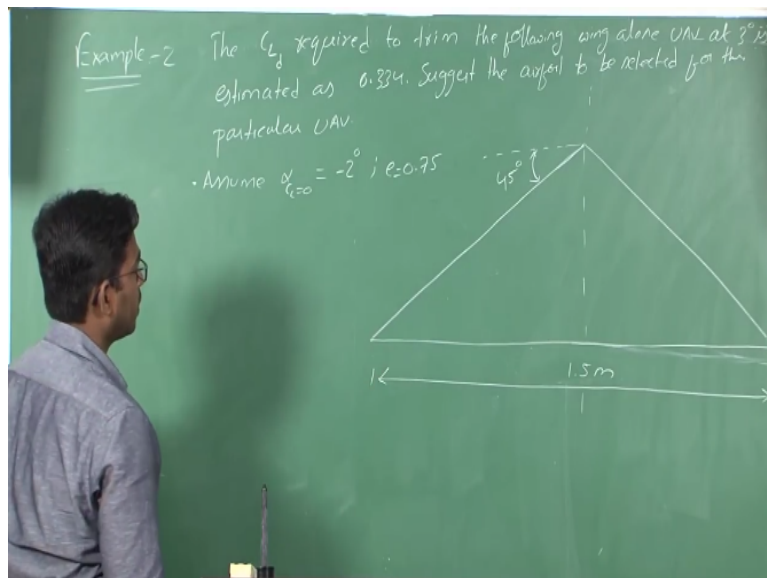
So from this airfoil data we have figured out what is the  $C_L$   $\alpha$  of this airfoil and  $\alpha$  at which  $C_L=0$  for this. So using those two, we found out  $C_{L0}$  of the wing, initially we figured out what is  $C_L$   $\alpha$  of this wing and then the  $C_{L0}$  of this wing again right. So substitute those two parameters in this equation, we will be able to find out the corresponding  $C_L$  and  $C_D$  at different angles of attack.

Now why we need to find CL and CD in the first case? Now we will appreciate as we progress, once we start discussing about the what should be the lift or design CL right to sustain at particular weight at a given altitude right, how to design the wing that means how to select the planform geometry, during that time you need to have a design CL, what should be your design CL right.

So when we discuss those concepts, you can appreciate this particular exercise and say which power plant you need to select that requires the information of CD right. So during various phases, we need to move at different velocities and different flight conditions. So which power plant to be used, how do you find? You first gather the information about what is the requirement of the system.

The requirement of the system is drag, so to calculate drag you need to find what is the corresponding CD.

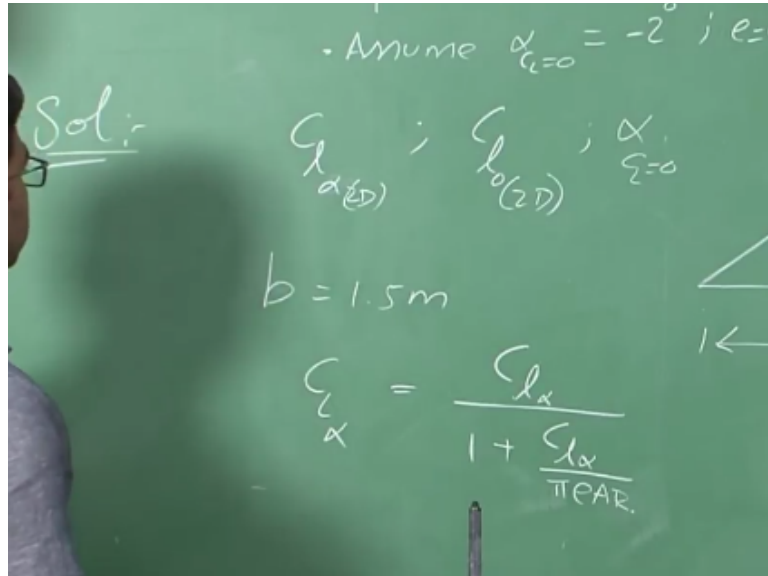
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Now let us consider this example 2. The CL design required to trim the following wing alone UAV at 3 degrees is estimated as 0.334. Suggest the airfoil to be selected for this particular UAV right and it is also given alpha at which CL is=0 is -2 degrees and Oswald's efficiency is 0.75. Now this is the planform that has been given, the UAV is with the triangular planform here right.

So we know what is the span of this and leading edge sweep of 45 degrees is given right, with this information we need to find out, we need to suggest an airfoil which means how do you suggest an airfoil?

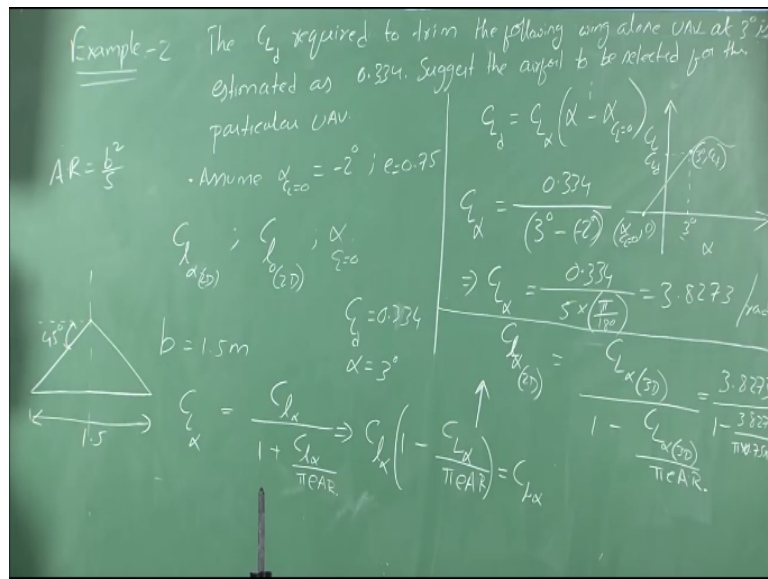
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In terms of lift and drag characteristics right, say lift characteristics here. We need to give what is  $C_l$  alpha 2D of that airfoil should be and say  $C_{l0}$  of that airfoil right,  $C_{l0}$  2D and say alpha at which  $C_l=0$  right. This is already given because for wing as well as airfoil it will remain same here that is the assumption that we have okay. Now first figure out what is given here right. The span is 1.5 meters right.

What is the relationship between  $C_L$  alpha 3D and  $C_l$  alpha 2D?  $C_L$  alpha of wing is  $C_l$  alpha of airfoil /  $1 + C_l$  alpha of airfoil /  $\pi e AR$ . So what you need is  $C_l$  alpha 2D right. How to find  $C_l$  alpha 2D?

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Planform right with the span of 1.5 meters and leading edge sweep of 45 degrees and with the leading edge sweep of 45 degrees. Now I need to find out what is  $C_L$  alpha 2D right of the airfoil from the wing data. So how to get wing data here in the first case? So  $C_L$  design =  $C_L$  alpha \* alpha trim-alpha at which  $C_L=0$  can we do this? What does this equation mean? Assume that this is the  $C_L$  versus alpha of your aircraft or the wing right.

This is your  $C_L$  versus alpha. The information that is given is  $C_L$  design,  $C_L$  design is given as 0.334 and this design is achieved at angle of 3 degrees at an angle of attack of 3 degrees, say at 3 degrees this is your  $C_L$  design. That means you know this point right and you have the information about this alpha at which  $C_L=0$  right this point corresponds to alpha at  $C_L=0$ , 0 you know this point that is 3 degrees,  $C_L$  design right.

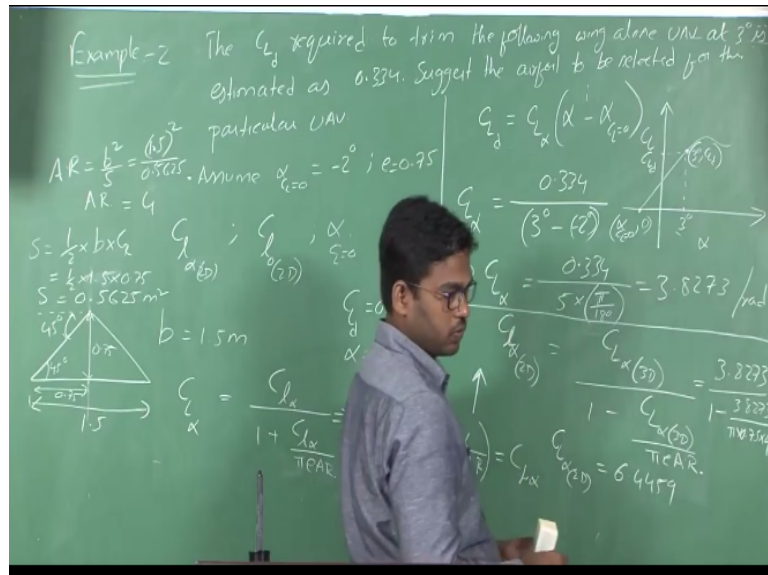
From here you can find out what is  $C_L$  alpha slope of this curve right. So  $C_L$  alpha of the wing is =  $C_L$  design is  $0.334/3 - -2$  degrees right this implies  $C_L$  alpha of the wing is  $0.334/5 * \pi/180$  which is in radians. So here it is in degrees, it is per degrees here it is per radians which is =  $3.8273$  per radian right. Now you have  $C_L$  alpha 3D. Now can we find out  $C_L$  alpha 2D from here. So how to do this? Can we manipulate this equation?

This implies  $C_L$  alpha \*  $1 - C_L$  alpha /  $\pi e AR$  is =  $C_L$  alpha 3D right. If you take this to your left hand side or multiply this equation by this denominator right and then taking this second term to the left side, to the right hand side so what you have here is so this implies  $C_L$  alpha 2D is =  $C_L$  alpha 3D /  $1 - C_L$  alpha 3D /  $\pi e AR$ . Now substituting these values of  $C_L$  alpha 3D is

3.8273 and  $1 - 3.8273/\pi$  \* what is Oswald's efficiency factor 0.75 \* we need to find what is the aspect ratio of this wing.

So how to find aspect ratio of this wing? We know aspect ratio  $AR = b^2/S$  right. So we have the span, we need to find the area of this particular planform.

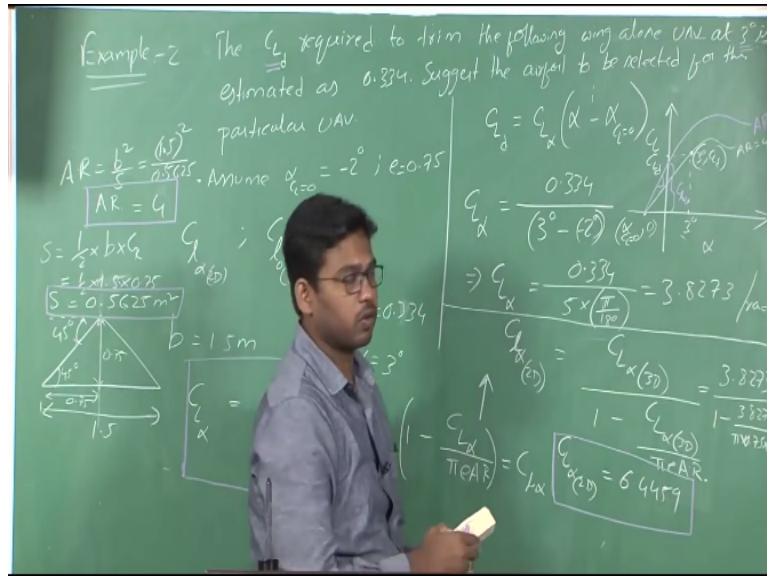
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So how to find the area?  $1/2 \text{ base} \times \text{height}$   $1/2 \text{ span} \times \text{say CR}$  root chord here, so what is the root chord here, so when say this angle is 45 degrees this is also 45 degrees, you know this span, you know this, you know this semi span which is 0.75 right. So  $\tan 45$  is opposite by adjacent, so this particular height will also become 0.75,  $\tan 45$  is 1 right. This particular height of this or the root chord of a triangular wing is 0.75.

So you have  $1/2 \text{ base} \times \text{height}$ , you can simply substitute  $1.5 \times 0.75$ . So  $S$  is 0.5625-meter square right. So the corresponding aspect ratio is 1.5-meter square/0.5625. This is 4, the aspect ratio turns out to be 4. Now substituting this aspect ratio in this equation, you can find out what is  $C_L \alpha_{2D}$ .  $C_L \alpha_{2D}$  is 6.4459. Now we are able to find what is the  $C_L \alpha_{2D}$  of this airfoil. Now we need to find what is the  $C_L \alpha_{2D}$ ,  $C_L \alpha_{2D}$  of this airfoil right.

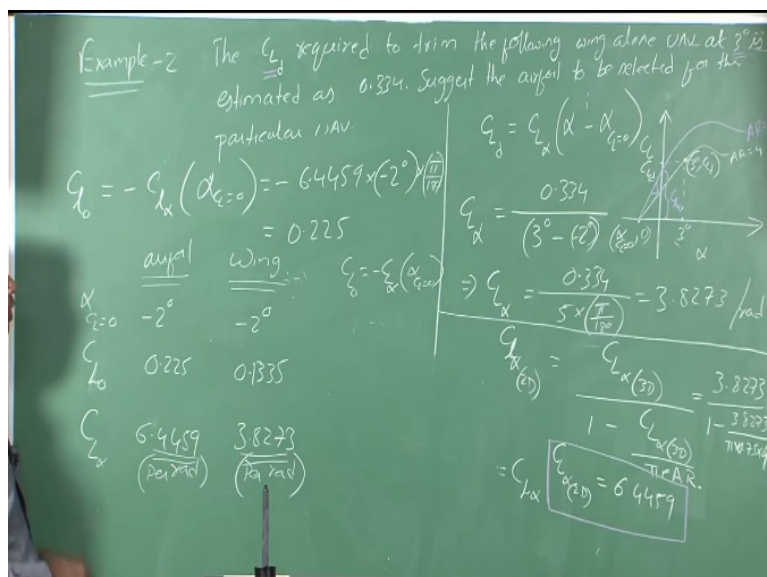
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Say this is my airfoil data right. Aspect ratio is infinite, here we have aspect ratio is=4 so this one. So now we need to find out this y intercept of this airfoil. We know the slope of this airfoil, just now we have figured it out as 6.4459. So we need aspect ratio to calculate that CL alpha, so we have figured out area from the given geometry is the relationship we have started with 2D Cl alpha and 3D CL alpha.

So we figured out 3D CL alpha based upon the design CL as well as the angle of attack which this aircraft has to be trimmed or that CL design has to be achieved. Now we have 2D CL alpha right, you know the slope, you have this point alpha at which Cl is=0 right which remains same for airfoil as well as wing that is the assumption. Now using the slope and this we can find out what is the Cl0 of that.

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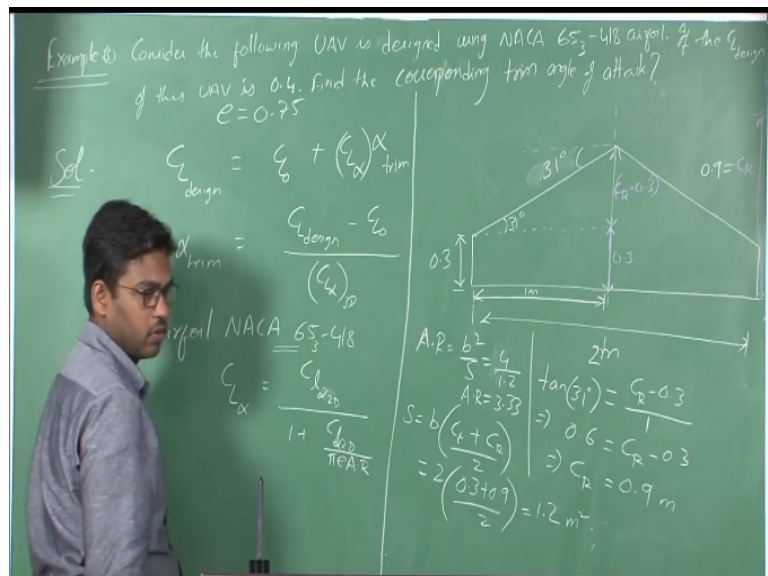


So  $C_{l0}$  of airfoil is  $= -C_l \alpha$  of airfoil  $\times$   $\alpha$  at which  $C_l$  is  $= 0$  which is  $-6.4459 \times -2$  degrees,  $-2$  degrees is given, is given in the question. So this is in degrees that the slope is in per radian so I need to convert it as I need to multiply this by  $\pi/180 = 0.0175$  right.  $C_{l0}$  is  $0.225$  for an airfoil and let us see what is  $\alpha$  at  $C_l = 0$  and  $C_{l0}$  of airfoil or say  $C_{l0}$  and  $C_l \alpha$  for airfoil and wing.

So for airfoil it is  $-2$  degrees and yeah here it will be same. What is  $C_{l0}$  of airfoil is  $0.225$ . What is the  $C_{l0}$  of the wing? So we can find out right that  $C_{l0}$  is  $= -C_l \alpha$  at which  $C_l = 0$ , you substitute the wing  $C_l \alpha$  you will get the corresponding  $C_l$  as  $0.1335$  right and  $C_l \alpha$  of the airfoil is  $6.4459$ , of the wing is  $3.8273$  these are per radian, per radian. See lower the aspect ratio lower is the  $C_l \alpha$  of your wing, you can observe that right here.

The same airfoil is used to build a wing which is of lesser aspect ratio wing, so the aspect ratio is about 4 here. So the  $C_l \alpha$  drastically reduced right. Now say if I increase the aspect ratio.

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Now let us consider example 3 here. Consider the following UAV is designed using NACA 653-418 airfoil if the  $C_L$  design of this UAV is  $0.4$  find the corresponding trim angle of attack? So the planform of this UAV is given which is of  $2$ -meter span and  $0.3$  meters tip chord and the leading edge sweep of  $31$  degrees right so we need to find the trim angle of attack right first.

See if the CL design is known then we can express the CL of this wing as  $CL_0 + CL_{\alpha}$  of the wing  $\cdot \alpha$  trim. So this  $\alpha$  trim is  $= CL_{\text{design}} - CL_0 / CL_{\alpha}$  which is 3D of wing and this  $CL_0$  is also for wing right. So what are unknown here? So  $CL_{\text{design}}$  is given which is 0.4 here and we need to find out what is  $CL_{\alpha}$  3D and  $CL_0$  of this wing.

So another information given is airfoil right. Regarding airfoil which is used here is NACA 653-418 right. Now we need to find out in order to find out  $CL_{\alpha}$  3D, we need to know what is the airfoil  $Cl_{\alpha}$  okay so as well as the aspect ratio and Oswald's efficiency. Say the Oswald's efficiency is about 0.75 right. So we need to find the aspect ratio of this wing. First how to find the aspect ratio?

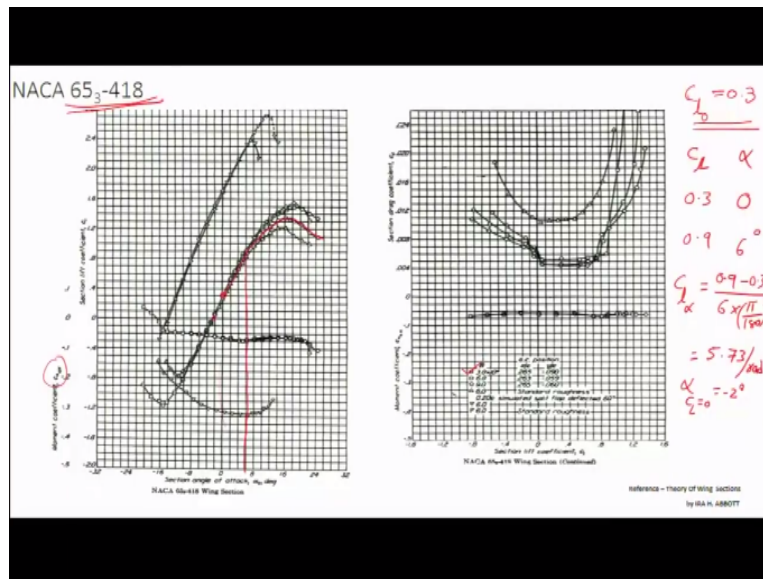
We know  $AR$  is  $= b^2 / S$ , so we need to find what is area here. So to find out area I need the root chord right, I need to know what is this root chord. So how to find this root chord? So say this is say up to this it is 0.3 right, say this entire thing is  $CR$ , the entire chord at the root is  $CR$ , so this is like  $CR - 0.3$  right. Now I know leading edge sweep of 31 degrees, I know the span here which is 1 meter right.

So we know aspect ratio  $= b^2 / S$  right, so to find this aspect ratio I have to know the area here, to find this area I need  $CR$  here. So  $\tan 31$  degrees is  $= CR - 0.3 / 1$ , so this implies  $\tan 31$  is approximately 0.6 is  $CR - 0.3$ . This implies root chord is 0.9 meters. So  $CR$  here is 0.9 right. Once I have  $CR$ , I can find out what is the area of this wing. What is area of this wing?  $b \cdot C_t + CR/2$  which is otherwise  $b/2 \cdot CR \cdot 1 + \lambda$ .

So  $b$  is 2-meter span  $\cdot 0.3 + 0.9/2$  which is 1.2-meter square right. So substituting this span 2 meter square and 1.2-meter square area will get to know what is the corresponding aspect ratio. So aspect ratio here is  $4/1.2 = 3.3$ , so aspect ratio again is a low aspect ratio wing here, it is a low aspect ratio wing UAV, so you know what is aspect ratio. Now what you need to know is  $Cl_{\alpha}$  2D. This is  $Cl_{\alpha}$  2D or say let us go back to this airfoil data NACA 653-418 right.

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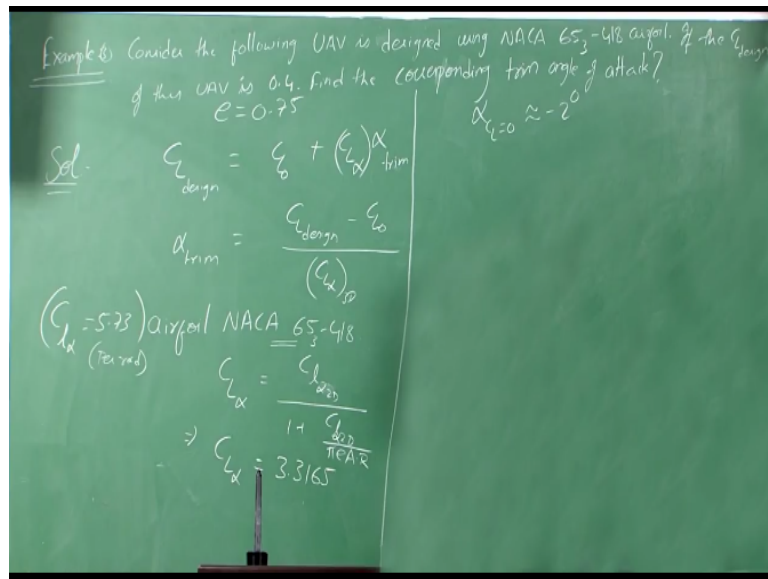
So this is the airfoil, so we can see here the variation of lift coefficient with angle of attack as well as pitching moment coefficient with angle of attack and this particular plot represents the drag polar and this is  $C_m$  versus  $C_L$  about  $c/4$  or the aerodynamic center here. This is about  $c/4$  okay. Now what we need to find out is  $C_L$  alpha of this wing first. So let us first consider what is  $C_L$  at  $\alpha = 0$  that is  $C_{L0}$  is 0.3. See this is  $C_L$  at which  $\alpha = 0$  right.

So this particular point  $C_L$  and  $\alpha$  for 0 degrees, this is 0.3. Now how far we can consider this linear here? So let us say if we look at this curve, so we are talking about this curve right, 3 million Reynolds number right. So it is with circular marker and see if you consider that curve here how far this is linear? So here about this point so maybe up to this point you may consider this as a linear curve right, straight line.

So the corresponding angle of attack is approximately 6 degrees because beyond which there is a change in slope right, it becomes nonlinear. Now at this 6 degrees angle of attack, the corresponding  $C_L$  value is 0.9. So how to find out the  $C_L$  alpha of this curve?  $y_2 - y_1 / x_2 - x_1$  that is  $C_L$  at  $\alpha = 6$  degrees is  $0.9 - 0.3 / 6 \times \pi / 180$ . So what is the answer for this? 5.73 per radian right.

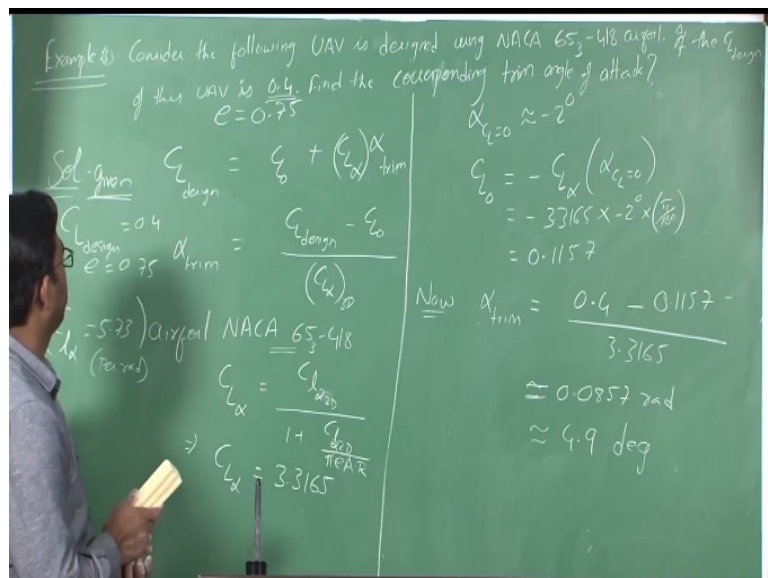
Now you got to know what is  $C_L$  alpha of this plot and let us also look at  $\alpha$  at which  $C_L$  is 0. So this particular point is the  $\alpha$  at which the  $C_L$  is 0 which is approximately 2 degrees right, 2, 4, 6, 8, got it. It is approximately -2,  $\alpha$  at which  $C_L$  is 0 = -2 degrees. Let us take this information and find out what is the  $C_L$  alpha 3D which is the  $C_L$  alpha of wing made out of this airfoil.

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Now from here we got to know using this NACA 653-418  $C_L$  alpha of this airfoil is 5.73 per radian, this is the  $C_L$  alpha of this airfoil. Now substituting this in this equation what you will get  $C_L$  alpha 3D is approximately 3.3165 right. So to calculate alpha trim, we need another variable here  $C_{L0}$  right or another parameter  $C_{L0}$  that we need to find out. So  $C_L$  alpha is 3.3165. Now how to find out  $C_{L0}$  of this wing?

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Again from the airfoil data, we have alpha at which  $C_L$  is=0 is approximately -2 degrees, alpha at which  $C_L$  is=0 is approximately -2 degrees which will remain same for wing as well as the airfoil okay. Now  $C_{L0}$  of the wing is  $-C_L$  alpha of wing\*alpha at which  $C_L=0$ , this= $3.3165 * -2$  degrees\* $\pi/180$  because this slope is wing per radian right. So the corresponding value of  $C_{L0}$  is 0.1157.

Now alpha trim can be estimated from this equation as  $C_L$  design is  $C_L$  design given is 0.4 that is  $C_L$  design given is 0.4 and  $e$  is 0.75 here, so this is like  $0.4 - C_{L0}$  which is  $0.1157 / C_L$  alpha of this wing which is  $3.3165 = 0.0857$  radians, so in degrees it is approximately 4.9 degrees right. So this is the angle of attack with which the aircraft will be trimmed to attain this particular  $C_L$  right.