Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture - 08
Airfoil and Finite Wing, Various Wing Planform

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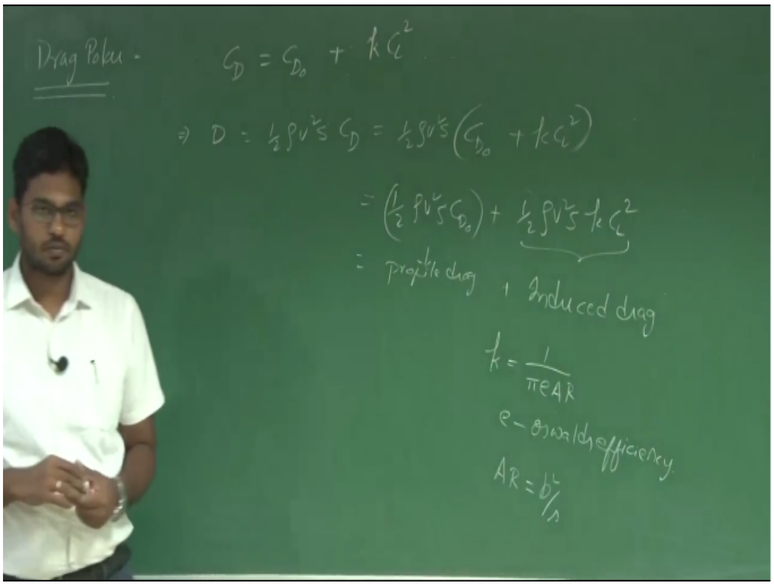

Good morning friends welcome back. In our previous lecture, we were talking about drag polar which is expressed as a summation of profile drag and induced drag right. So the total drag is 1/2 rho V square S CD which is 1/2 rho V square CD0+kCL square. This can be further expressed as a summation of profile drag as well as induced drag k CL square. So this term corresponds to profile drag.

So 1/2 rho V square S CD0 where CD0 is a profile drag coefficient and this particular term corresponds to induced drag and k is the induced drag correction factor which is=1/pi e AR where e is the Oswald's efficiency and aspect ratio is b square/s. Now the Oswald's efficiency here what does it mean? How it is going to affect the induced drag? So what the boundaries of this Oswald’s efficiency?

In our previous lecture, we witnessed the lift distribution along the span of this wing right.
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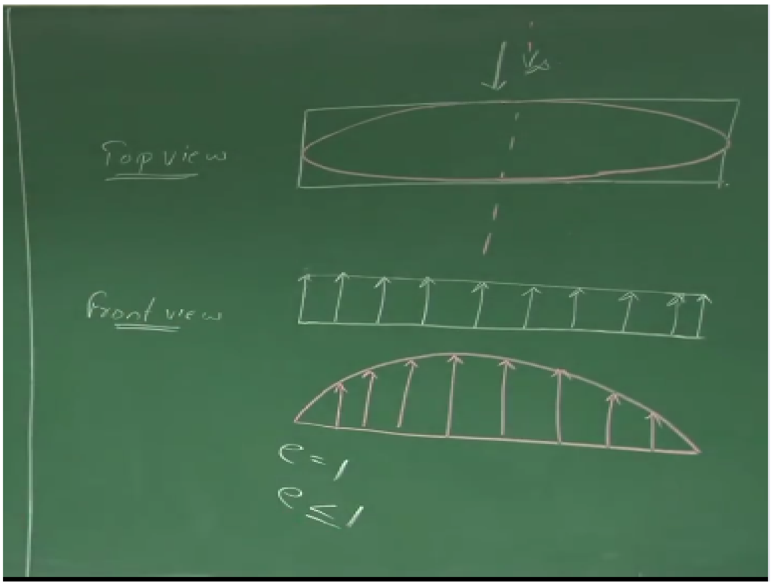

So for a rectangular wing, let us say this is your planform geometry okay. So you are looking this is your V infinity this is the top view right. So how the lift distribution along the span? This is your top view and this is your front view from the front side of the view right. So it is like you are facing the aircraft from the nose right.

Now if you look at the wing typically for a rectangular wing, this is how a lift distribution, ideally this is how it has to right because at each and every spanwise location you have an airfoil which gives a resultant aerodynamic force and component of that you will get as a lift right will contributes towards lift. So typically this is how you have I mean the lift distribution is for a rectangular.

Now let us say instead of this rectangular wing you consider an elliptic wing, it is observed that the lift distribution depends upon the shape of the wing right. So for an elliptic wing, you will end up having elliptic lift distribution right. For such lift distribution which is elliptic, the Oswald's efficiency factor is=1 and the corresponding induced drag will reduce. So the upper limit of this Oswald's efficiency is 1 that is for elliptic wing for neither any other shape.

This Oswald's efficiency factor talks about how if how good is your lift or it compares with that of an elliptic wing lift distribution. Let us say if you have designed a wing which is on elliptic, so this efficiency factor is a kind of figure of merit and that compares your current wing lift distribution with that of an elliptic wing lift distribution. So for any other geometry it is more or less < 1.

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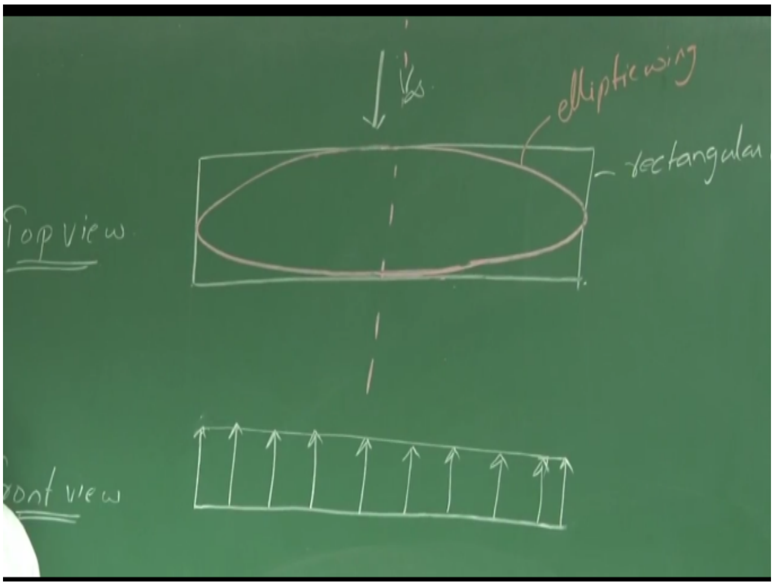

Instead of a rectangular wing now let us consider an elliptic wing. This is your rectangular wing. So how you got this lift distribution? At each and every location spanwise location, you have an airfoil with the respective pressure distribution right. Now how about the pressure distribution at each and every spanwise location of this elliptic wing?

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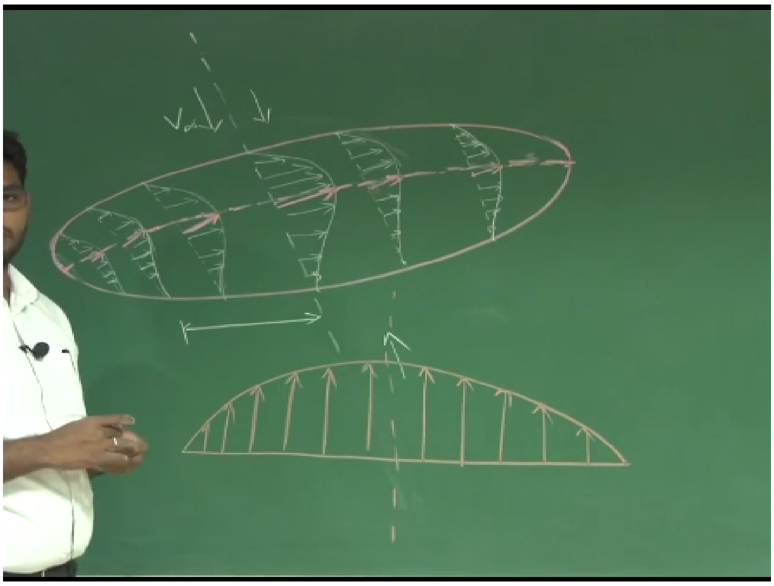

Consider an enlarged view of that elliptic wing right. Let us say the flow is along this parallel to this parallel to this fuselage reference line. So at each and every location, so the white lines here or the arrows here represent a pressure distribution at each and every location of this elliptic wing right. Now this pressure distribution is perpendicular to this planform that is coming out of this perpendicular to this surface of this planform right.

Assume that it is perpendicular to this and now we can get a resultant force and a component of that perpendicular to this free stream is the lift at that particular span location right, at particular span location that is your lift right. Now say if you join, if you get the locus of this lift distribution at each and every spanwise location, then typically this is how it looks like say either you are looking from this direction or this direction right.

This is the lift at a particular spanwise location right. Now for an elliptic wing, it looks like an elliptic lift distribution right. So for a rectangular wing, it looks like a rectangular lift distribution right. So what do you infer from here? The planform geometry plays a key role in deciding the lift distribution over the span right.

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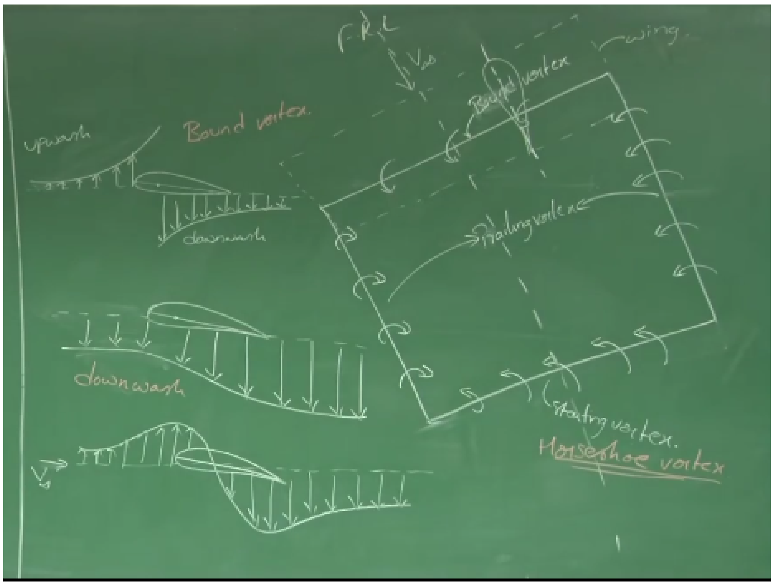

According to lifting-line theory, the wing can be replaced by a bound vertex and the associated flow field can be replaced by a trailing vertex, two trailing vertex and this vertex architecture is closed by a starting vertex. So this entire vertex architecture is known as horseshoe vertex right. So this bound vertex will create an upwash ahead of the wing and downwash behind the wing.

So this is by bound vertex whereas the trailing vertex will create a downwash throughout the span of the wing right. This combined effect like upwash and downwash will alter the local angle of attack at a particular spanwise location right. So we have derived it yesterday, this see this is your reference line for a fuselage reference line. Let us consider a spanwise location here. So you have an airfoil at this particular location right.

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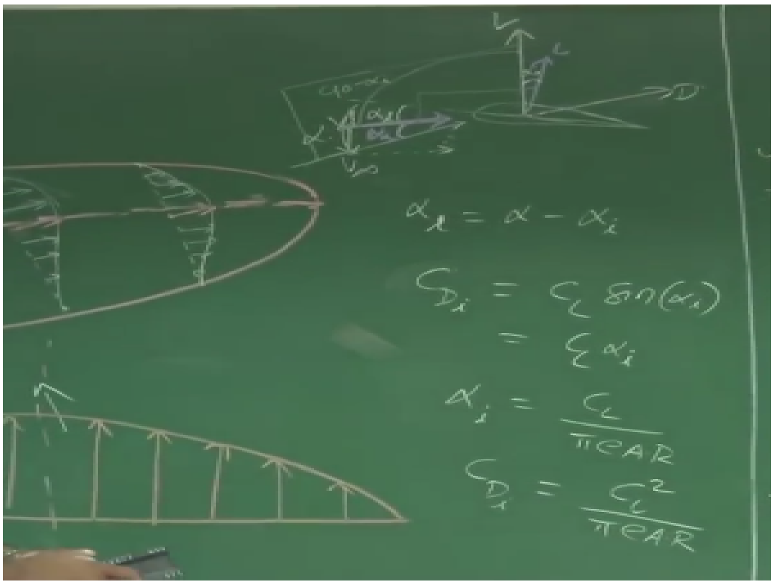

So let us say this is your airfoil at that particular spanwise location and which is at an angle alpha with respect to free stream velocity of the entire wing right but due to this upwash and downwash effect the local angle of attack at this particular span location changes right. So this is induced by a downwash right. This change in angle of attack is because of the downwash.

So this is your actual V infinity, this is V infinity at that particular location, V infinity prime okay. So the effective angle of attack for this airfoil is alpha effective ef or the local angle of attack here is alpha l, so this alpha l is = alpha-alpha i. Now the total lift of the entire wing is perpendicular to the free stream velocity and but this is the local velocity because of this induced angle of attack alpha right.

So the local lift will be perpendicular to this particular quantity right. This is your local lift, say this is the direction of your drag. This local lift will also have a component along the drag of this entire wing and the component is=say CDi=CL*this angle is your alpha i right CL*sin alpha i. Why? Because this angle is 90 degrees, these two are perpendicular V infinity and capital L are perpendicular and this local lift and this alpha or V infinity prime is perpendicular right.

So what is this angle? What is this angle? This is 90-alpha i. What is this angle? This is 90-alpha i. Since this is alpha i right this is 90-alpha i and you have this as 90, you have this as 90-alpha I, so 90 - of 90-alpha i is alpha i right. So local lift * cos of alpha i is along the total lift direction and local lift * sin of alpha i will be along the drag direction, total drag of the wing. Now since alpha i is small, we can assume CDi is=alpha i.

So CDi is = and again we know from the lifting-line theory alpha i is=CL/pi e AR, so CDi is=CL square/pi e AR. So induced drag correction so induced drag coefficient is=CL square/pi e AR. Now this factor e is the Oswald's efficiency.

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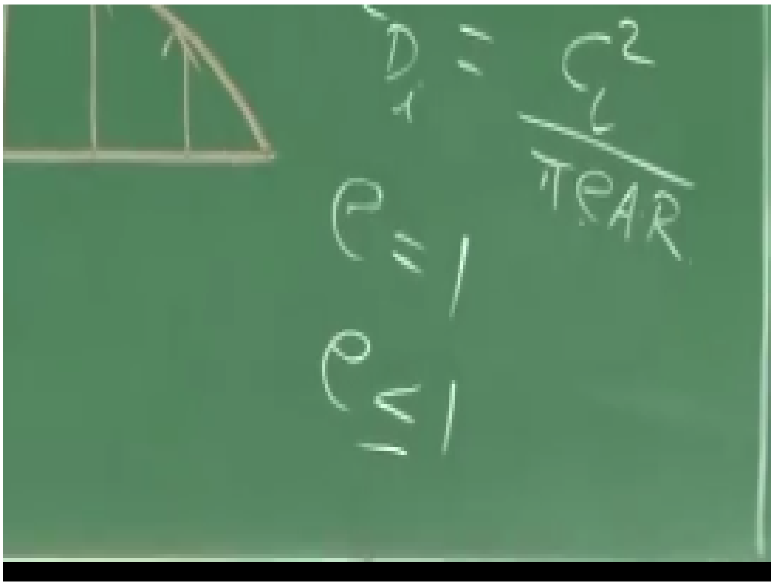

And from this theory it is observed that for an elliptic wing you have minimum induced drag with Oswald's efficiency factor is=1. So this is the upper limit of this Oswald's efficiency factor right. For any other planform, e will be<1. In general, you can say e is always< or =1. So will the lift of the airfoil and the lift of the wing, wing is made up of series of airfoils right, will it be same or not?

Let us look at if not what is the relation between lift of the airfoil and lift of the wing.

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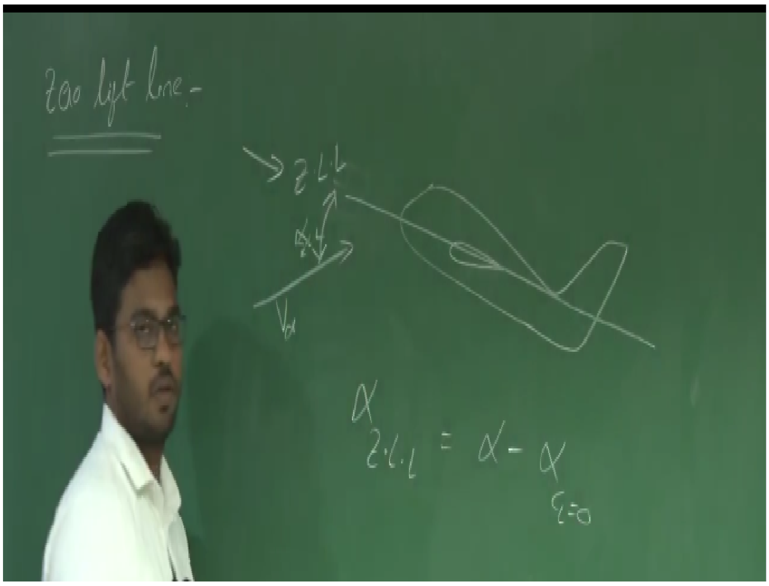

Now let us now define what is called zero lift line right. Say I have an aircraft. I say this is my zero lift line right. What does it mean? If the flow is along this zero lift line, then there is no lift produced by this aircraft right understand. Now let us define angle of attack with respect to this zero lift line. Say if the flow is in this direction, so this is my alpha of zero lift line right. So this is my V infinity, alpha with respect to zero lift line.

Now how to relate the angle of attack with respect to chord and angle of attack with respect to zero lift line right. Alpha of zero lift line is = alpha-alpha at CL=0. Now let us make a wing out of this airfoil and see how the lifting characteristics of airfoil are related to a wing right.

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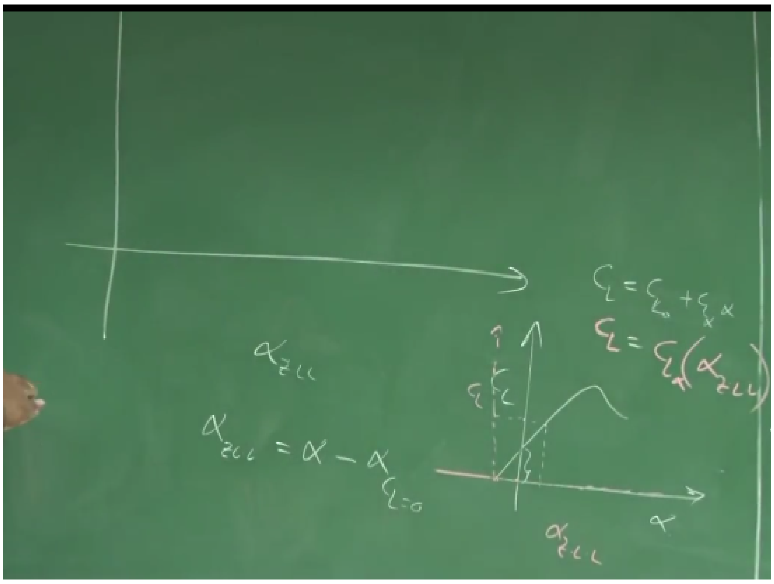

So let us consider a typical lift coefficient variation with angle of attack right. Now let us assume this angle of attack is defined with zero lift line right where alpha zero lift line is =

alpha with respect to chord or fuselage reference line-alpha at CL is = CL right. Let us assume, now what is this why - here because like we are actually shifting the axis y axis here.

For example, if this is my CL and alpha with respect to chord line right, now with respect to zero lift line what happens is my alpha zero lift line starts when will be 0 when CL is=0 right. So this is again the same axis y axis here right. So for example with respect to chord, this is your so for the same CL okay if you are measuring the angle of attack with respect to chord, we express CL is=CL0+CL alpha*alpha.

This is the normal representation of lift where you have lifted zero angle of attack right, alpha =0 and y =mx+c that is how you are going to express this and you can also express the CL with the help of zero lift line right where I mean you can express see we are trying to find out the same CL when we define the angle of attack with respect to zero lift line right. So zero lift line is like when the V infinity is along that zero lift line or along the reference line then there is no lift produced by this aircraft.

So that means you are actually shifting this alpha see here when alpha is=0 there is certain lift but in this case when alpha is =0 there should not be any lift, lift should also be 0. So according to this, so what you have is CL alpha, see the slope still remains the same, only the definition of alpha changes here zero lift line okay. The slope still remains the same here okay. Now consider a case where we are definition this lift with respect to zero lift line right.

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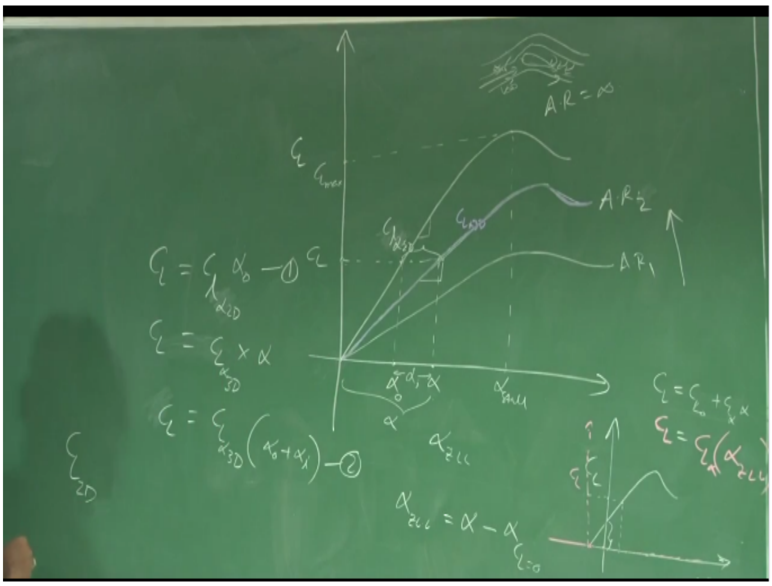

Now say typically this is how the airfoil CL variation with this angle of attack right which was defined with respect to zero lift line data's. Now we know this is your alpha stall where you can achieve your maximum CL, CL max right. So beyond this what happens? So this is your angle of attack and this is your free stream, this is your alpha. Say at alpha stall what happens is the flow separates, flow separation happens over this airfoil.

Major part of the flow will get separated from this airfoil because of this there is a sudden drop in CL beyond this alpha stall right and the drag also increases because of the flow, because of the flow separation you will get pressure drag right. So now coming back to this, now let us say this is your a0 or Cl alpha 2D because it is for an airfoil. I say Cl alpha 2D is for airfoil. So this is for an infinite wing.

Now if I make a wing out of it with a finite aspect ratio. This is how typically it will look so aspect ratio is=say aspect ratio 2, aspect ratio 1. So this is the increasing order of aspect ratio okay. Now what is happening, the slope is changing right. Here you can see there is a steeper slope right, a lesser steeper compared to that of airfoil. Now consider let this be CL alpha 3D for AR 2 okay with the finite aspect ratio.

Now look at Cl for 2D case, let the angle of attack be alpha right for this particular case. For the wing made out of the same airfoil, the same CL is achieved at a higher alpha right. So this is alpha for airfoil say alpha 0 and this is your alpha right. This is the alpha for wing. This is alpha 0 for airfoil at which you are attaining the same CL okay. Now this is your induced angle of attack alpha i. The change is because of this induced angle of attack okay.

So now the Cl of this airfoil at this particular alpha I can express it as slope of this Cl 2D*the corresponding y=mx since I mean we have defined this angle of attack with respect to zero lift line here. So Cl alpha 2D*alpha 0, so this alpha 0 okay now the same Cl if you want to find out from the wing right this=CL alpha 3D or the wing with finite aspect ratio right*this, this is your alpha, so this is=CL alpha 3D=alpha 0+alpha i.

So equating this equation 1 and 2 because both are representing same value of CL right. So Cl 2D sorry Cl alpha 2D, here it is Cl alpha 2D please make a correction.

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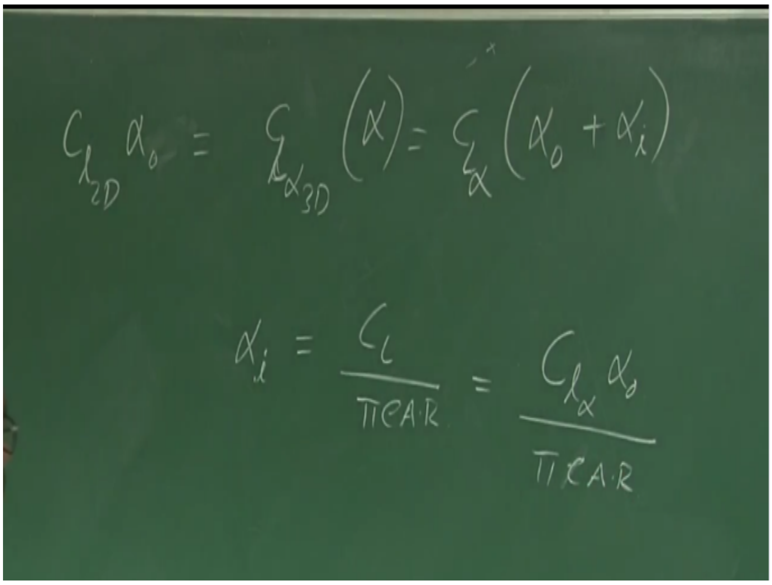

So Cl alpha 2D is=Cl alpha 2D*alpha 0 is=CL alpha or CL alpha 3D*alpha that is CL alpha. So the capital L represents for 3D case, small l represents for 2D case. So I am not explicitly writing 2D and 3D from so I am dropping these subscripts here. CL alpha*alpha 0+alpha i, alpha i is Cl/pi e AR, express it as Cl alpha*alpha 0 of the airfoil right/pi e AR. The same CL can be achieved from the airfoil as well as the wing.

The change is the angle of attack at which you are achieving the CL right. For an airfoil, you can achieve it at alpha 0 right.

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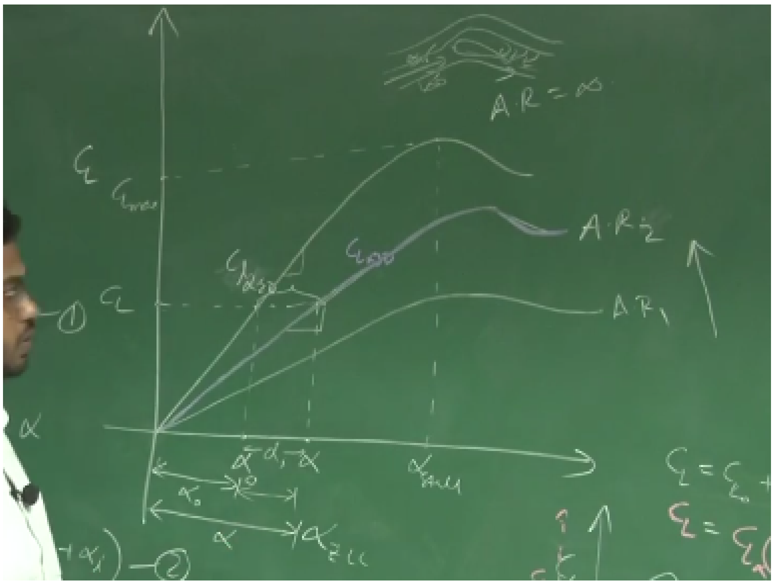

This is your alpha 0, this is your alpha at alpha so this is the induced angle of attack right. This induced angle of attack is changing the slope of this 3D and 2D. So this CL I can achieve it from the airfoil as well from CL alpha*alpha 0 right if I substitute this in that equation.

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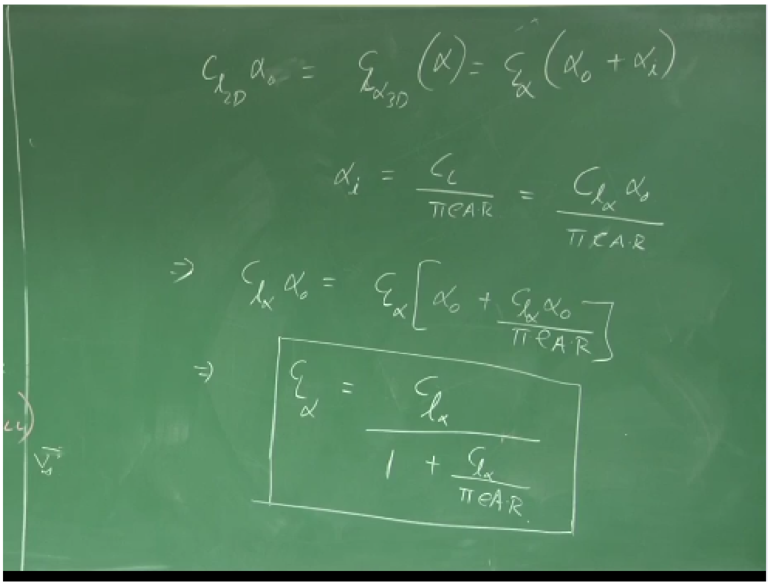

So Cl alpha*alpha 0 is = CL alpha of wing*alpha 0+Cl alpha*alpha 0 of wing of airfoil sorry/pi e AR. This implies CL alpha is=Cl alpha of airfoil/1+Cl alpha of airfoil/pi e AR. Now this is one of the important result. So this is well valid in the subsonic regimes right. So the 3D wing CL alpha which is made out of the airfoils with Cl alpha 2D right is related by this equation, Cl alpha/1+Cl alpha/pi e AR.

So if I have an elliptic wing then e becomes 1. So this is clear right. So the same CL can be achieved from the airfoil and the wing made out of this airfoils right. So as the aspect ratio changes, the CL alpha curve changes right. Let us say for the 2D case, it is the maximum right. Let this point be the CL and the corresponding angle of attack at which it is achieved is alpha 0 and Cl alpha represents the 2D case which is the airfoil Cl alpha right.

And now you have a wing with the finite aspect ratio and the same CL is achieved at an angle of attack alpha which is higher than alpha 0 which is in fact the summation of alpha 0+alpha i, so this additional angle of attack is because of the induced angle of attack right. Now by assuming the linear equation here right why CL is=CL alpha of 3D wing*alpha right and the same can be achieved by an airfoil the same CL right at a lesser angle of attack say alpha 0.

Now equating these 2, you get CL alpha=Cl alpha 2D/1+Cl alpha 2D/pi e AR.

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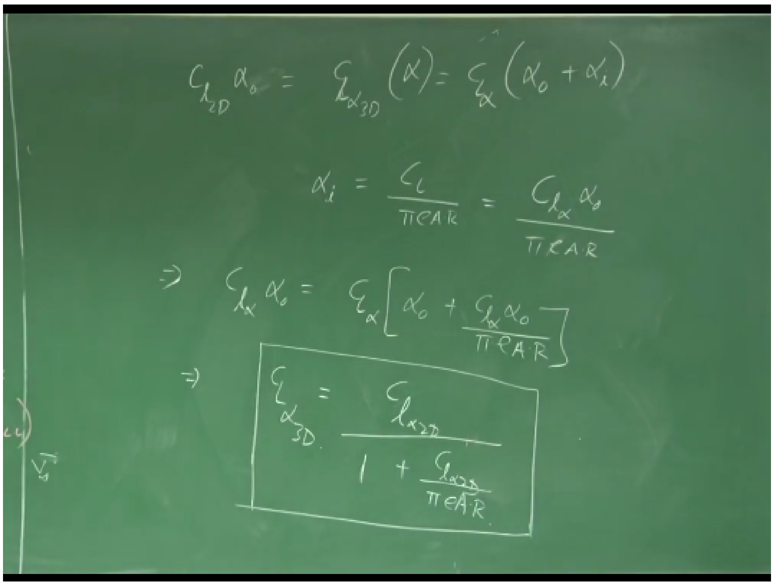

2D, 2D and 3D to complete this equation and now let us look at some of the examples where you have the 2D data that is airfoil data and you have a planform and you need to find the corresponding CL alpha of 3D wing right.

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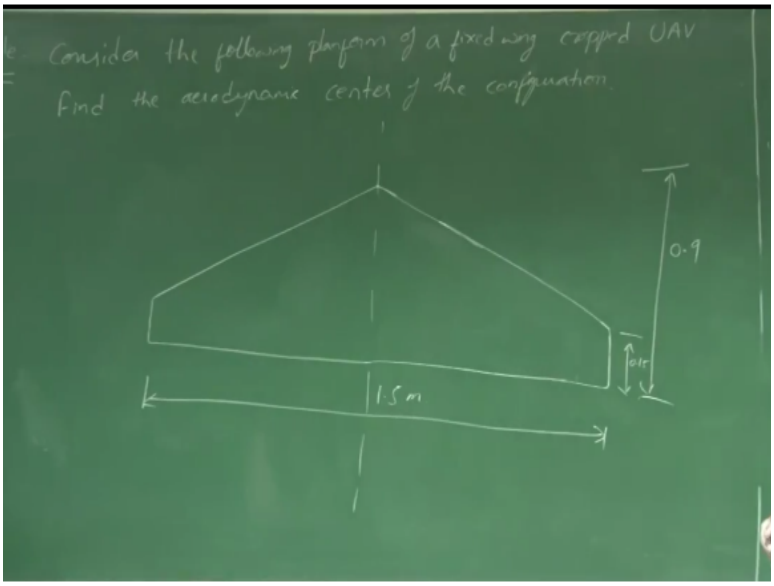

Consider the following planform of a fixed wing cropped delta UAV. Find the aerodynamic center of the configuration? So let the root chord of this UAV be 0.9 meters, the tip chord is 0.15 meters and the span is 1.5 meters right. So the planform geometry of this fixed wing cropped delta UAV is given with the root chord of 0.9 meters and tip chord of 0.15 meters and the span of 1.5 meters. We need to find the aerodynamic center of this configuration.

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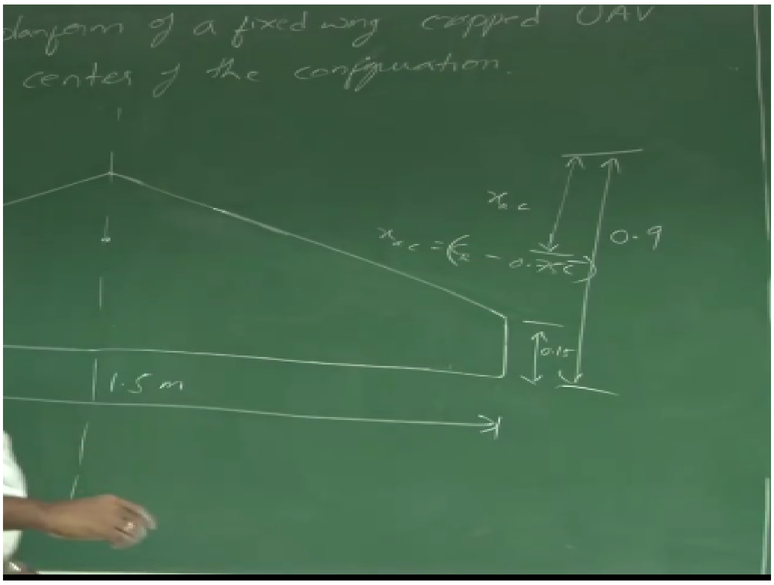

So aerodynamic center xac is=we derived it right, say this is your xac so xac is=CR-0.75 C bar right.

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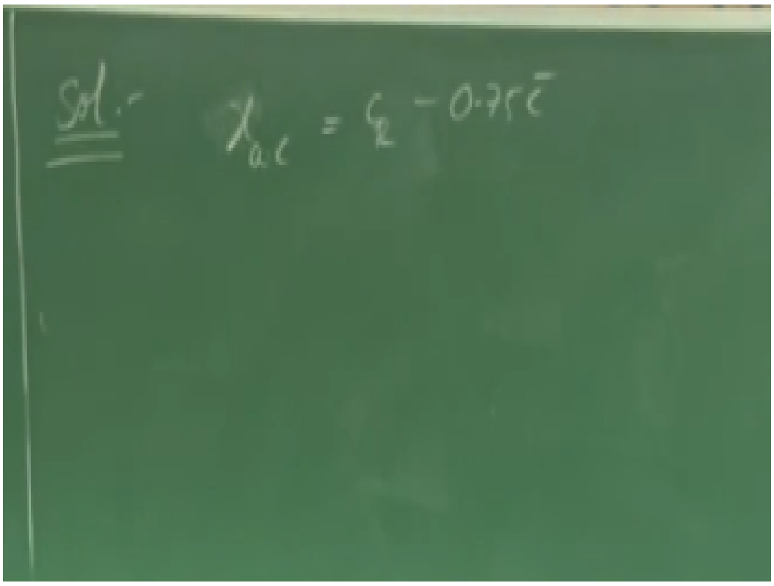

So what is xac, CR-0.75 C bar, so I need to know what is C bar here.

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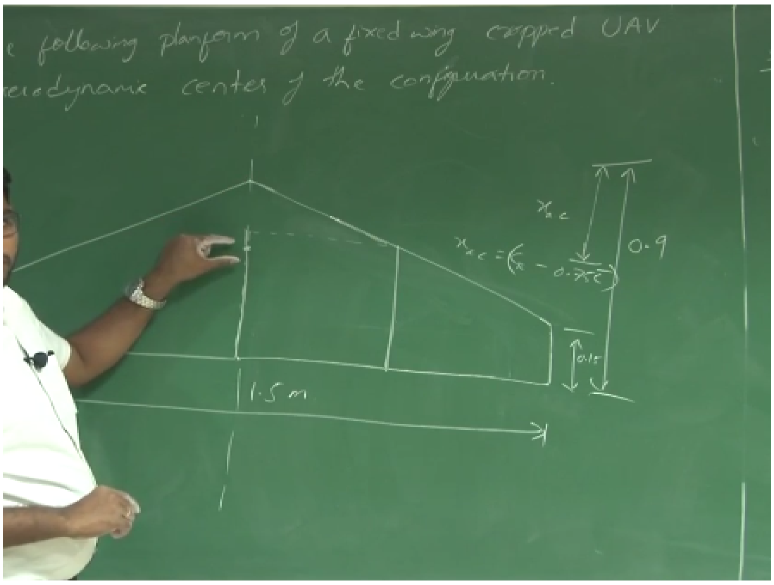

So say this is your mean aerodynamic chord right, project this mean aerodynamic chord onto the root chord and take 25% of this mean aerodynamic chord so that you will get the corresponding distance which is CR-C+0.25 C bar which is CR-0.75 C bar.

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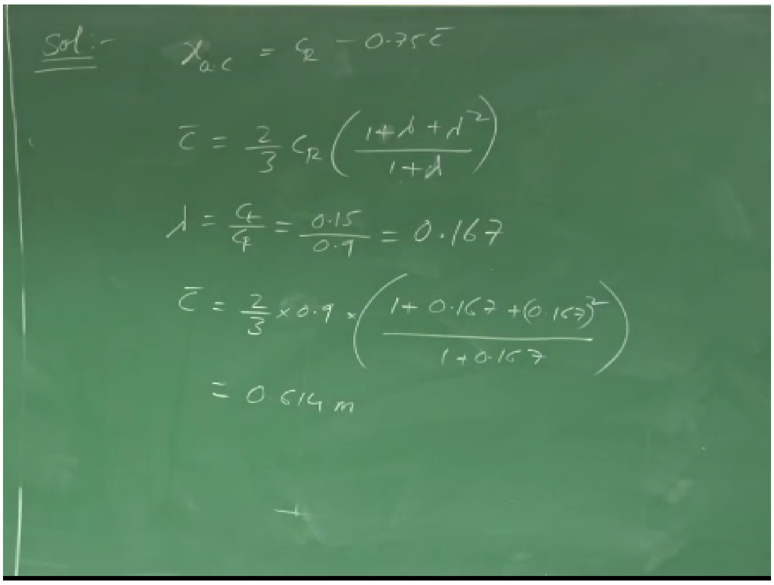

Now to find out C bar you need 2/3 root chord*1+lamda+lambda square/1+lamda. So what is lambda? Ct/CR which is 0.15/0.9 which is 0.167, 0.166 or 0.167 right. Now substitute this lambda in the above equation 1+0.167+0.167 whole square/1+0.167. So this =0.614 meters right. So the mean aerodynamic chord is approximately 0.614 meters.

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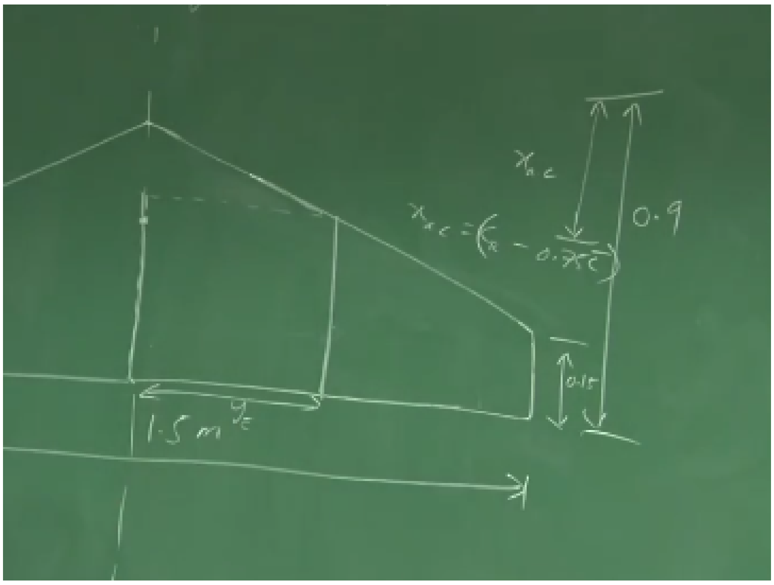

Now so if you are interested you can also find what is the y location of this C bar, y location of MAC.

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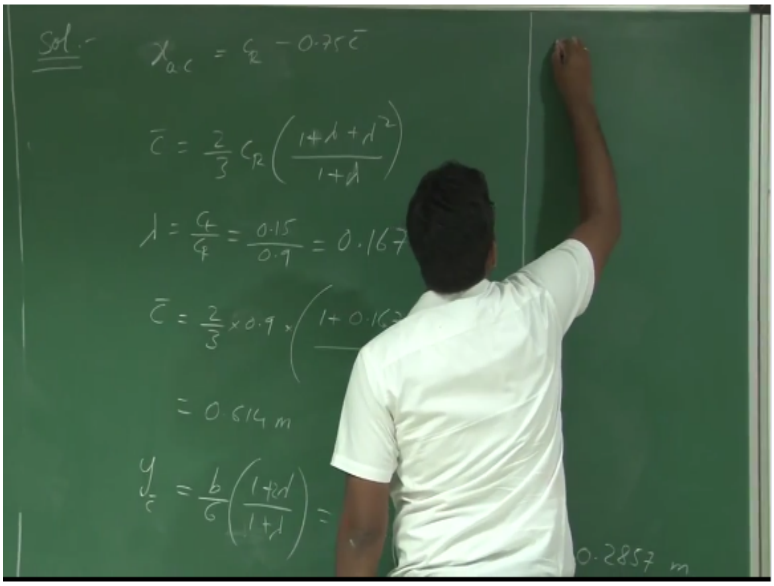

So let us quickly do it, y of C bar is=b/6 1+2 lambda/1+lambda is=1.5/6*1+2 times of 0.167/1+0.167=0.2857 meters approximately. It lies at approximately 29 centimeters from the root chord. What is our mean aerodynamic chord MAC?

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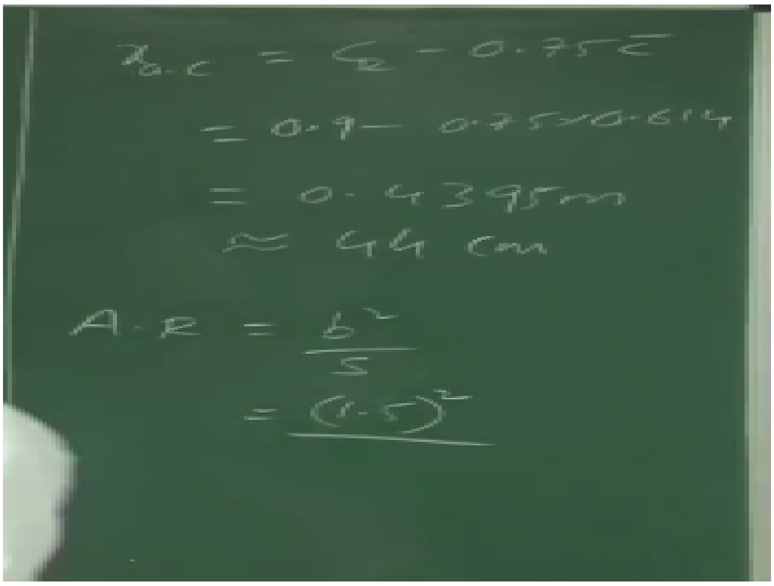

Location of this aerodynamic center xac is=CR-0.75 C bar which is 0.9-0.75*0.614, this = approximately 0.43 meters, 0.4395 meters which is approximately 44 centimeters. It is at the location of 44 centimeters from the nose leading edge of this root chord right. Now let us also find the aspect ratio of this configuration. Now what is this? What is the aspect ratio? It is b square/S that is 1.5 meter square/what is s for this configuration?

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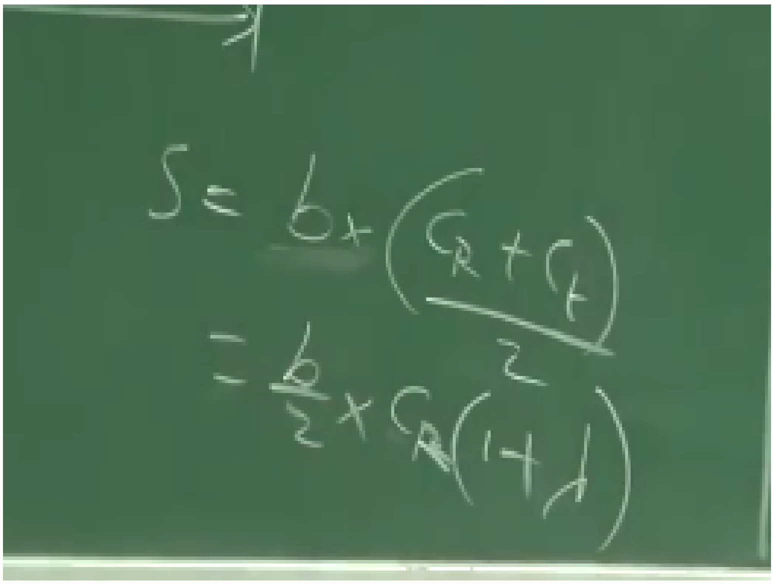

S is b*the total area of this configuration, b*CR+Ct/2, this=b/2*if I take CR out, this is 1+Ct/CR which is lambda, b/2*CR*1+lamda. So the area of this configuration is approximately 0.787.

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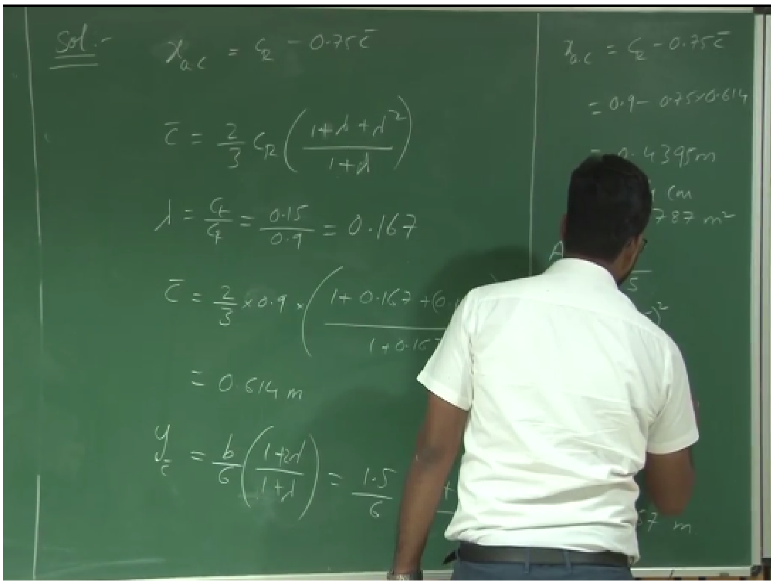

So S of this configuration is 0.787-meter square. The aspect ratio is 2.85.

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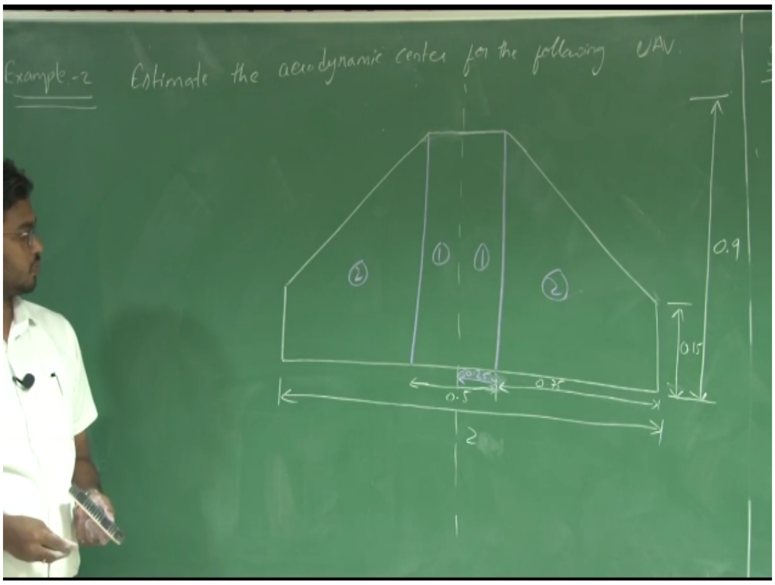

So estimate the aerodynamic center of the following UAV right. Let us assume this is a planform geometry. Now see here it is not a single taper right, it has multiple sections. See up to here it is a rectangular wing right, after that is a tapered wing has a cropped delta wing and this taper is above trailing edge here right. Now what we need to do is let us divide sections 1 and 2 right.

Now let this be 0.9 meters and 0.15 right. So that means the root chord and tip chord are same as previous example. So span of this UAV be 2 meters right and say the rectangular portion is 0.5 meters. Now say what will be this portion? This portion will be 0.25, the rest will be 0.75 right, 1 and 2. Now how to find the aerodynamic center for this configuration?

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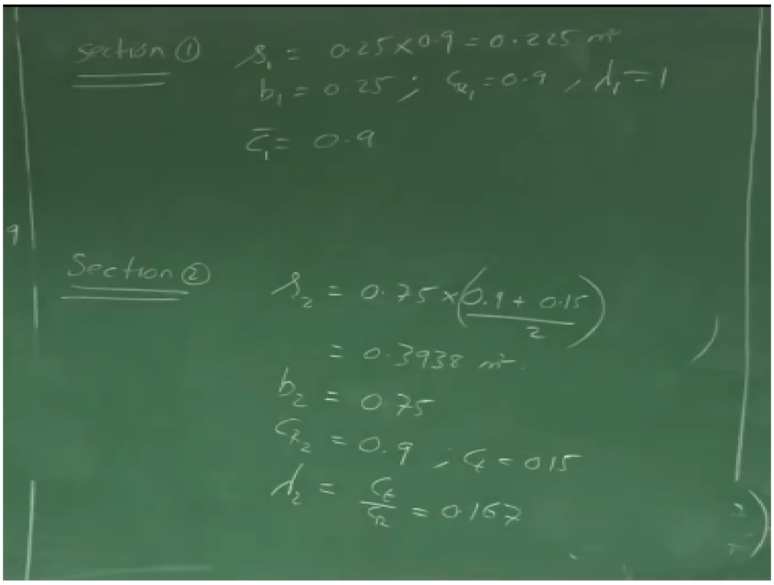

Now let us take section 1, what is the area of the section 1? Say s1 is CR*this span right. So this s1 okay let it be small s1 which corresponds to only this particular half right, the area that we are going to talk corresponds to only this particular half okay. So s1 here is 0.25 times 0.9=0.225 meter square and b1 here is 0.25 and what is CR1? Is 0.9. What is Ct? Since it is a rectangular wing right, the CR1 this section is rectangular, so CR1 is=Ct1 here right.

So lambda of this particular section is =1 and C bar of first section is 0.9 right. What about C bar of second section? Section 2, so s2 =0.75*0.9+0.15/2 is =0.3938-meter square right and b2 is 0.75 and what is CR2 is=0.9 meters here which is Ct1 right, 0.9 meters and Ct is 0.15, so lambda 2 is=Ct/CR which is 0.167. So we did it from I mean it is same as previous example right. So the semi span of previous example is also 0.75 and we have root chord 0.9 and tip chord as 0.15. Now what will be the total area?

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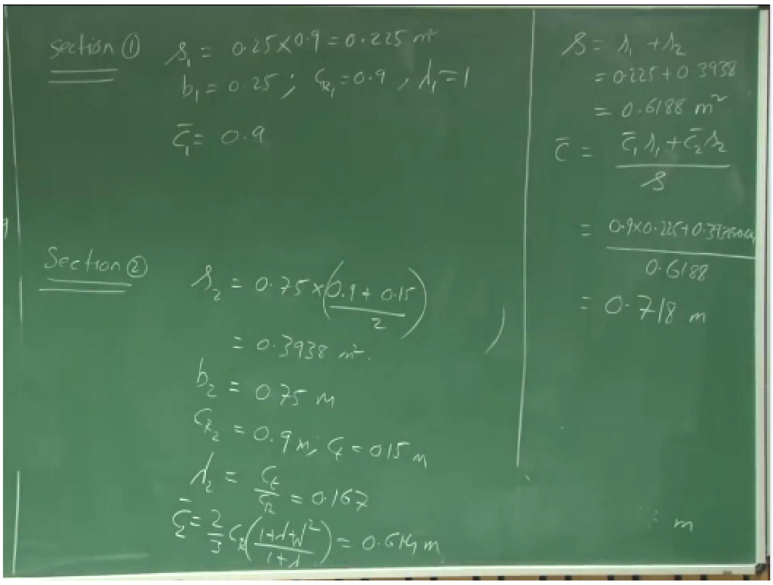

Say s1+s2 right total area of this half say s represents the half area of this wing right. This =s1+s2 which is 0.225+0.3938 which is = 0.6188-meter square. Now what is C bar for this configuration? C bar for the section 2, C bar is=2/3 CR*1+lambda+lambda square/1+lambda which is=0.614 meters right. So all dimensions are in meters here, all the lengths are in meters.

Now we have C bar 1 and C bar 2. To find out mean aerodynamic chord of this configuration, we can take the weighted average of which is C1 bar s1+C2 bar s2/s, so this=0.9*0.225 + 0.3938*0.614/0.6188, this=0.718 meters, this is the mean aerodynamic chord for this configuration. Now what will be the aspect ratio of this configuration?

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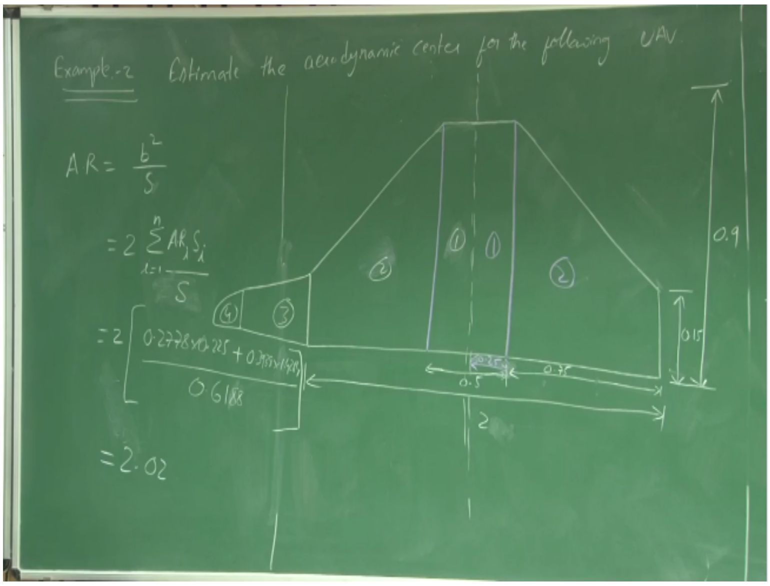

Aspect ratio can also be obtained by this weighted average right. So that is so from the definition if you remember it is b square/S which talks about both the sides right, although it is symmetric it talks about aspect ratio talks about the entire planform. Now since you are considering semi span and semi area here, so the effective aspect ratio for these multiple sections is twice sigma i=1 to n say n is the number of sections.

You can have one more section here okay, effectively you can have another section like this, you can have another section like that right, so sigma i is=1 to n Ari*Si/S. Then, it becomes this maybe 1, this maybe 2, this maybe 3 and again you can have a planform geometry and this may be 4 right. So you can have n number of sections right, there are different opposite at each and every stage right.

So for this configuration say 1 and 2 what you have here is two times of aspect ratio first portion is around 0.2778*0.225+0.3938*1.4284 otherwise/s1+s2 which is 0.6188, this =2.02 is the aspect ratio of this planform. Say if you have an elliptic planform like this right, so you will not be able to find what is the tip chord here.

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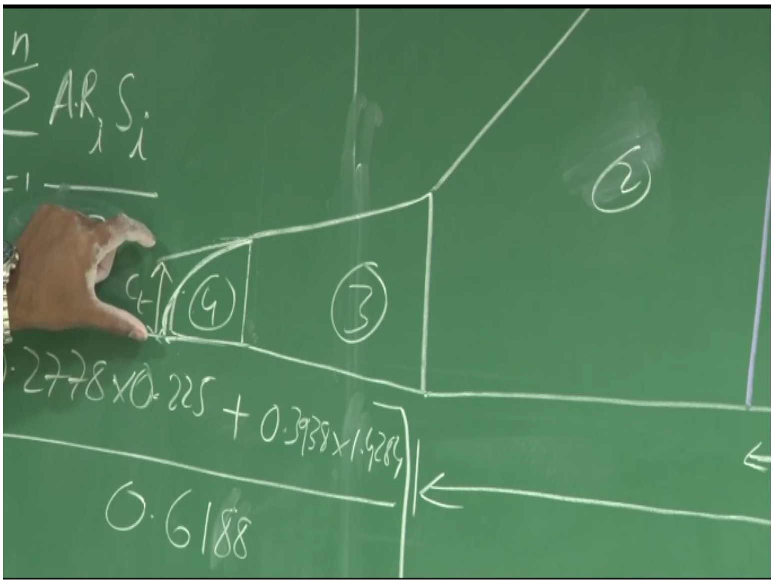

In that case what you can do is you can project this taper, you can extend the taper and consider this distance as your tip chord Ct right. For an elliptic wing so here that I mean taper ends right, after that you will get an ellipse here, so the tip chord can be you extend this taper of this edge section and find the corresponding length here at the tip right. So this chord length will be the tip chord length for the elliptic wing.

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Dear friends, to my right is the medium altitude surveillance UAV. So it has a push up configuration. It is a 3-blade pusher configuration with the T-tail and there is a twin boom. This T-tail is mounted on a twin boom, so let us look at the planform geometry of this wing. At least one so this first portion which was is in yellow color is a rectangular wing if you can notice this, the chord is around 0.3 meters.

So till this point it is rectangular wing, beyond this there is a taper up to this point and beyond this point there is an elliptic planform right. So to find the mean aerodynamic chord of this planform geometry, we need to adapt the methodology that we have discussed in example 2 right.

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So from this point if you want to find a taper ratio of this section let us say, what you need to do, you need to consider this as your tip chord right, this as your tip chord. Yeah consider this portion as your tip chord and you can find out the corresponding, so it is like you have to find the mean aerodynamic chord by considering this as 3 sections right. So first with the rectangular section and then a tapered and then an elliptic planform.