Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture - 07
Lifting Line theory, NACA Airfoil Nomenclature

Dear friends. Welcome back. In our previous lecture, we were talking about Wing Planform geometry and how to find the mean aerodynamic chord and the aerodynamic center for a given wing, right.

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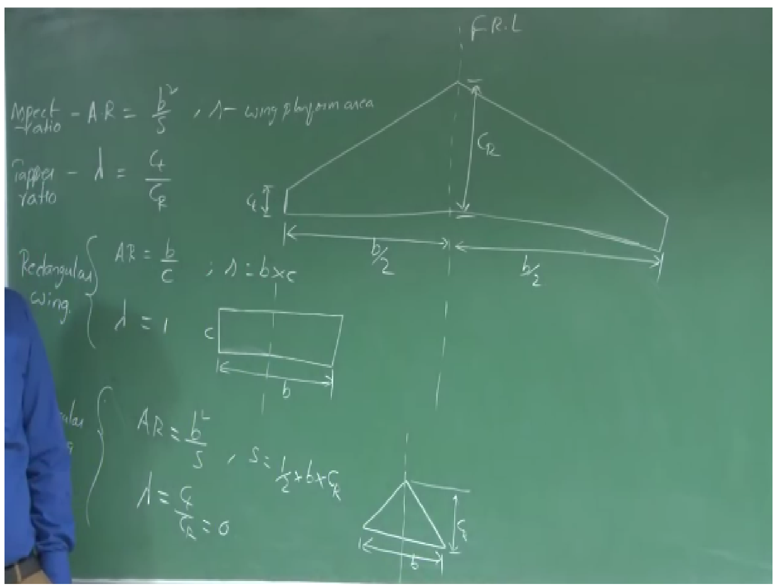

So, this is considered as fuse loss and reference loss. Assume that this is a wing a semispan of a wing and you have a root chord and you have tip chord and this be b/2 which is semispan. So we have defined some non-dimensional parameters called aspect ratio here which is b squared by S, where S is a Wing planform area. And lambda is a tapered ratio; aspect ratio and tapered ratio. Lambda = Ct/CR. CR is a root chord and Ct is a tip chord.

And we also witness that for a rectangular wing aspect ratio is this b/c and the respective surface area will be sorry; planform variable will be b*c bar, and for rectangular wing tapper ratio is 1 because Ct = CR for a rectangular. So this is for Rectangular wing. And for a Triangular wing what we have is Aspect ratio = b squared/S and area of a triangle is 1/2*span*height is CR, correct. And lambda, Taper ratio is Ct/CR which is 0 for a proper triangular wing.

Now the main important thing is at a later stage we will represent the entire wing by means of mean aerodynamic chord and the aerodynamic center with respect to forces in moment acting at the aerodynamic center. So while performing stability analysis and while sizing the wing and say for this particular wing what should be the corresponding tail size, right if you are designing a conventional aircraft, right. So during the process we represent both wing and tail by means of this mean aerodynamic chord.

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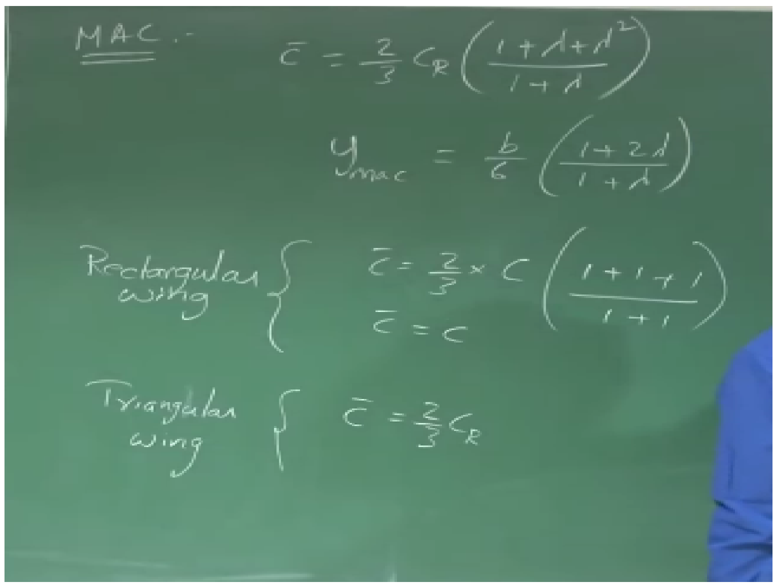

So how do you find mean aerodynamic chord, MAC is c bar = 2/3 CR*1+lambda+lambda square/1+lambda, right that give mean aerodynamic chord. Say, this is your c bar and the y mac location of this bar y mac is given by b/6*1+2 lambda/1+lambda, okay. So consider a rectangular wing. What will be the c bar? What is lambda, lambda is 1 in that case. So for rectangular wing c bar = 2/3*CR is C.

For a rectangular wing we have lambda=1; substitute 1 in this equation, so what you have c bar=c which is a CR an Ct in that case. What about for the triangular wing, 2/3 of the CR root chord, right. So for a triangular wing it is c bar=2/3 of Cr. With this mean aerodynamic chord how do you find the aerodynamic center?

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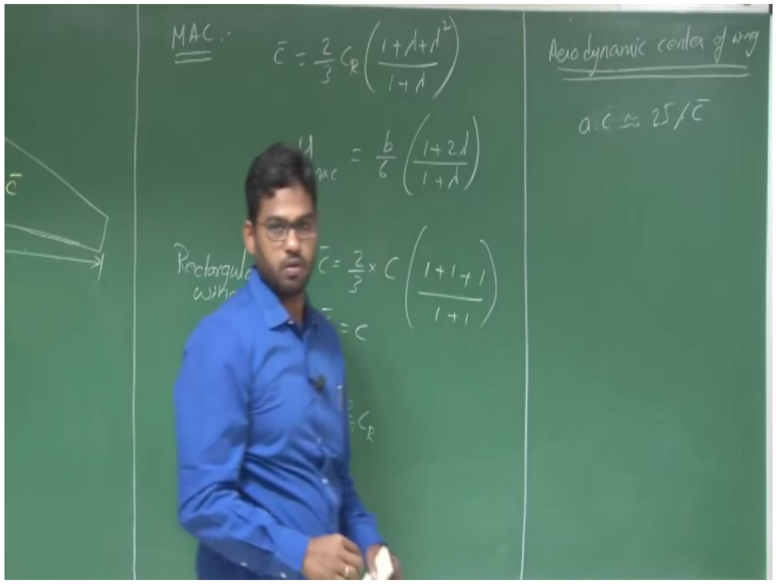

So Aero dynamic center of wing. So we are talking about subsonic UAV, right. So we can safely resume this ac aerodynamic center will be approximately at 25% of mean aerodynamic chord, right.

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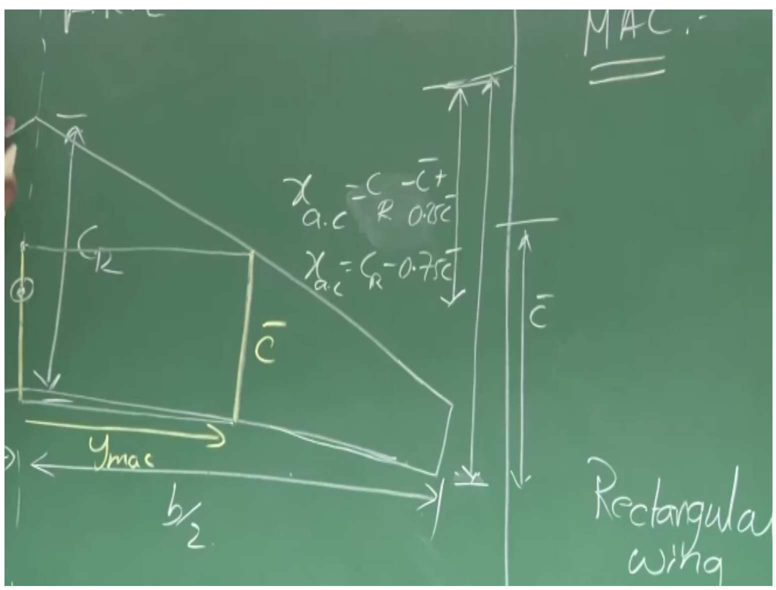

So once you know the mean aerodynamic chord you can project this mean aerodynamic chord onto your root chord, right. You can actually project this mean aerodynamic chord onto the root chord and take 25% of this mean aerodynamic chord to locate the aerodynamic center. So since you know the root chord, so say this is your root chord and you know c bar, say this is your c bar, so what you can do is you can find what is your mean aerodynamic chord x of ac that represents aerodynamic center of the with respect to the leading edge of this root chord, okay.

X ac=, so x ac=CR- c bar + 0.25 times c bar. This CR-0.75 c bar. So is the relation with which you can calculate or you can locate your aerodynamic center with respect to leading edge of this root chord. We are also talking about span wise distribution of airfoils.

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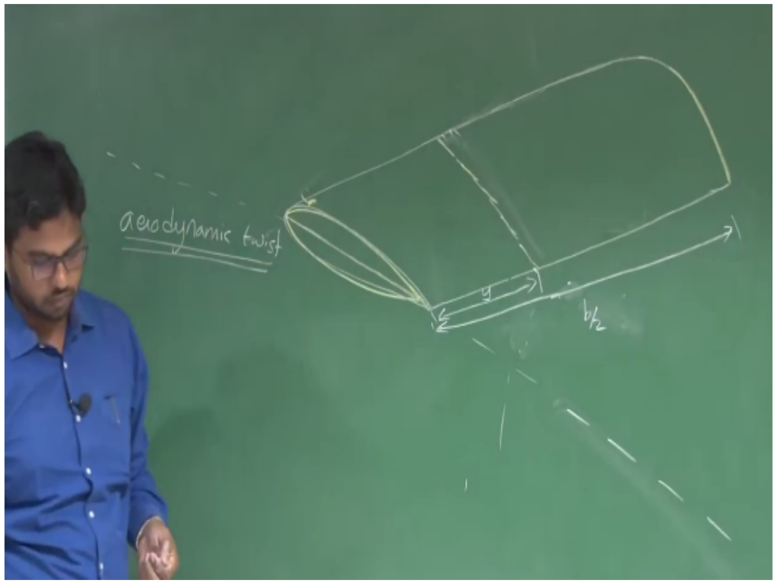

Say this is my wing. Now let us say this is my CR a root chord; the line connecting the leading edge and the trailing edge of the root. Say this is my center line or fuse loss reference line, okay. Now consider a span wise location y or say the tip consider the tip airfoil which is at a distance b/2 with respect to this root chord or from the fuse loss reference line. Now if the tip airfoil let us say I project this tip airfoil into the root.

If this tip airfoil which is present here say, okay. If the chord of this tip airfoil is above the chord of this root airfoil right, then we call it as Positive Geometric Twist. Say if this tip airfoil is below this; if the chord of the tip airfoil, if a; if it is oriented in this direction, Say, if this is your tip airfoil so say if this is your chord and say if this if the chord is below this root chord, right. If the angle is below the root chord then you call this as negative twist, okay.

So this is basically you are giving a physical rotation to an airfoil at the tip, right. You can have a constant twist as well as variable twist wings, okay. Now say at a span wise location, okay this is b/2, say; let that location be y, right. So at this location; say if this; the shape of this airfoil is different compare to the shape of the root chord airfoil. That means, say this is a cambered airfoil and at the particular location I started using a symmetric airfoil.

So although the chord line of this of the airfoil present at this location is in the same plain as the chord line of this root chord, right. But the profile changes because I changing the airfoil and the corresponding upper and lower coordinate of this particular airfoil at different compare to that of the airfoil that we used at the root, right. So because of this although there is no physical rotation that is given because of the change in the airfoil the wing looks twisted, right.

You call it has aerodynamic twist right, so aerodynamic twist is to change the lifting properties along the span wise location of this wing, right. Let us look at a wing here. It is a rectangular wing. In our previous lecture we saw a tapered wing about the mid chord and in this case it is a pure rectangular wing. You can see, so this is a side view you are looking at right. So these particular structural members are known as ribs and you have sparse here, right. So this is the skeleton of a typical wing. It is about a meter span.

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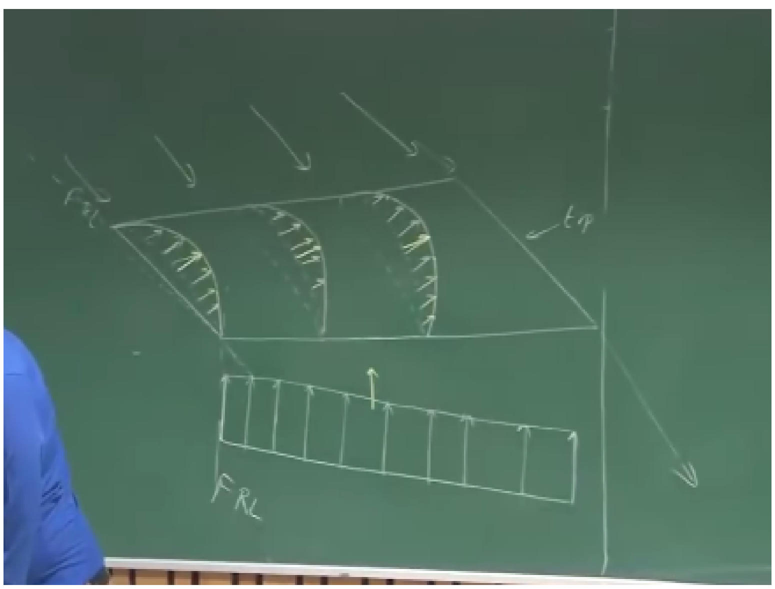

So let us consider a rectangular wing. Now, at each and every location you have airfoils; let us say there is no twist and there is no geometric as well as aerodynamic twist. So we have same airfoils oriented in a similar way, like the orientation of this is exactly what we have is a root chord, right. That means a chord of each airfoil lies in the same plain. Okay. Now we witnessed earlier like there will be pressure distribution on airfoil, right.

Say what will be the pressure distribution here say. When there is a flow you have pressure distribution here, right. And say on this wing again you will have similar pressure distribution because the chord is not changed, right. So at each and every location you have similar pressure distributes. Now you will have a resultant component of this at each and every point. Here say maybe somewhere here, somewhere here.

Since it is not tapered it will be lie on a same perpendicular plain to the wing span, right. Now if you look at from this direction or from here, what you can say is, see if this is my wing for a rectangular wing the left distribution will be something like this. So we can also talk about lift distribution along the span wise location. So we have lift distribution at a particular cross-section that is because of the flow or airfoil.

And we can talk about lift distribution along the span wise location, right. And it is mainly depending upon the planform of the wing shape of this wing. Now see, do not you see these are the tip, this is the tip you will have a wing symmetric about this fuse lose reference line, say this your FRL; this is your FRL, okay Fuse Lose Reference Line, right. So you have a symmetric wing and with the symmetric distribution. But what happens at this tip?

So this is the tip of this airfoil; wing right and you have a tip airfoil there. And similar to that of any other airfoil along the span this tip airfoil will also have this pressure distribution, right. So you know you have a greater section on an airfoil you have a greater section on the upper surface compare to that of lower surface. That means you can say low pressure on upper surface and higher pressure on bottom surface.

So due to; what happens, will there be a flow that; so there is a sudden ending, right. The wing is ending abruptly here. Now what happens at the particular point, the flow from the bottom surface will try to curl here curl towards an upper surface at the particular location. Now when you look at the tips which are the extremes of the wing, so, see this is my Fuse Lose Reference Line, okay.

And near the tips when there is a flow, so on this airfoil as well there is a greater section on the upper surface comparatively lesser section on the lower surface, which means there is a higher pressure on the lower surface and lower pressure on the upper surface. Now since this wing is abruptly ending what happens the flow near the tips will try to curl from the bottom surface to higher surface top surface, right; that is from higher pressure to lower pressure.

So this curling will actually affect the span wise flow; will actually induce this span wise flow. Ideally the flow has to be along the chord, right but this curling due to the finiteness of due to a finite wing will induce a span wise component of velocity to this flow, right. So how to model this? How to model this phenomenon?

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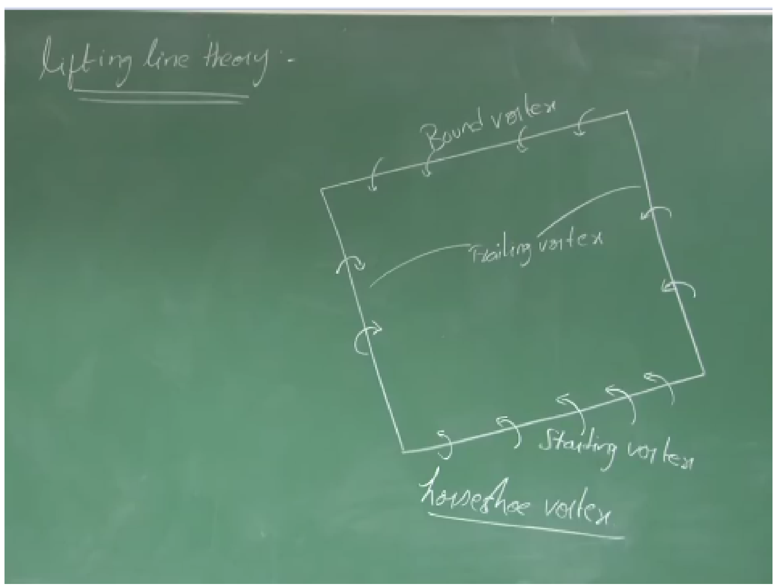

So lifting line theory. So lifting line theory talks about lift distribution along the span of a wing, right. So this whole setup like the wing and the trailing edge; trailing vertexes are replaced by a Bound vertex and associated with to Trailing vertex. So how the flow will be at this particular tip? So the wing is replaced by a Bound vertex and at the tip, what happens? How the flow; direction of the flow will be? From bottom to top, right.

The flow velocity direction will be from bottom to top; in counter clockwise direction. So at this tip it will be along the clock it will be in the clockwise direction, right. So and these two vertex are connected by a Starting vertex. So this entire vertex architecture that represents the lift distribution on the wing is known as Horseshoe vertex. So this Horseshoe vertex is used to model the list along the span or model the lift of a finite (()) (21:52). This vertex architecture, right.

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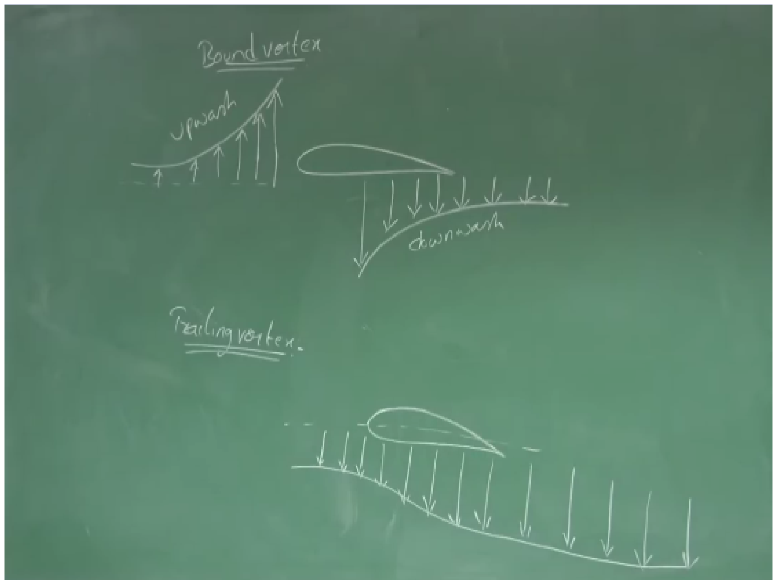

So let us consider this Bound vertex. What it is going to, how this is going to influence the flow or the wing, right. Let us take the sideway here. So this Bound vertex will induce an upwash ahead of the wing and downwash behind the wing. So assume that it tries to push the flow down behind the wing and push the flow up ahead of the wing, right makes a move up. Whereas this Trailing vertex will create a downwash along the span wise throughout the span of the wing. So this combination, this upwash and downwash combination;

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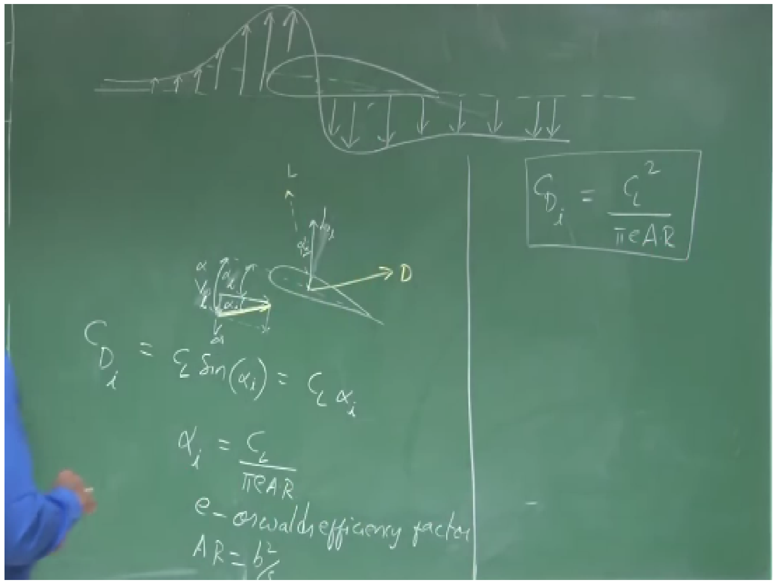

So this upwash and downwash combination will result in a flow field something similar to this. So this alters the local angle of attack at each and every span wise location, right. Now what happens is, let us a consider a span wise location along the wing. Let this be the angle of attach; I mean airfoil which is at an angle of attack v infinite; this is a direction of the actual flow, right overall flow.

Now what happens is due to this downwash; so effectively this angle of attack decreases, right. There is something called induced angle of attack. These two are similar, right. This is a local velocity; direction of local velocity. And this will be your say this is my actual angle of attack. And this is my effective angle of attack or the local angle of attack, right. So this is, this quantity, this is the downwash and both are same these two are same.

So this is the downwash which is affecting the flow at a particular span wise location, right. So that causes an induced angle of attack, alpha i. Now; so the location lift will be perpendicular to this; so the actual lift direction says, this is my; so lift of the overall aircraft, right or overall wing which is perpendicular to the free stream velocity, right. But the local velocity is at an angle alpha i with respect to this free stream velocity.

So the local lift will be perpendicular to; v infinite local. So this will be the component. So this is the local velocity and the corresponding local lift, alpha lift at that particular location, local. And this is your drag, right which is acting along v infinite. So this local lift has a contribution along here, along the actual lift of the aircraft or the overall lift of the aircraft as well as along the drag overall drag of this aircraft.

So what is this CDi let us say is a coefficient of induced drag, right is equals to CL is a coefficient of lift at that location, right *; see this is your alpha i, right this into sin of alpha i; since these triangles will be very small what you can assume is CL*alpha i induced the angle of attack. Okay. Induced angle of attack is contributing induced drag, right. So the lift generated here is also contributing towards drag.

This is the penalty that we are paying that we will discuss very soon. CDi = CL*alpha I, right. So according to this lifting line theory, this alpha i = CL/pi e AR, Alpha i or the lift and induced angle of attack is related by alpha i A CL/pi e AR where e is the Oswald efficiency factor. And AR is a Aspect Ratio which is b square/S, correct. Now substitute this alpha i here, in this induced drag. So CDi=CL squared/pi e AR.

So the lift also contributes toward induced drag due to the finite aspect ratio. So for a 2D wing it is infinite; aspect ratio is infinite; so induced drag is almost negligible there.

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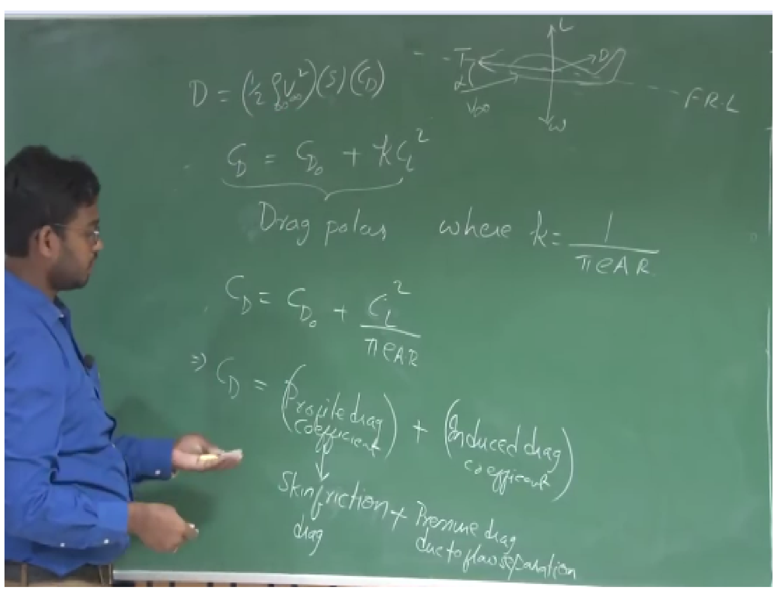

So what we need to talk is; what is drag? We already defined, it is a component of resultant aerodynamic force which is acting along the free stream velocity. Say this is your, this is your aircraft. This is your aircraft. And you have thrust which helps the aircraft to move at the required velocity. You have weight and say this is your v infinite and say this your Fuse Lose Reference Line or the Reference Line and this is your alpha.

What you have is lift and drag as aerodynamic forces. Now this drag is component of resultant aerodynamic force acting along this v infinite, right or parallel to free stream which is defined as 1/2 rho v square S*CD, right. So this is the dynamic pressure times to the planform area and non-dimensional drag coefficient. So this CD is given as CD naught can be expressed as CD naught + K CL square which is known as Drag Polar.

So this KCL square is nothing but CDi; where k=1/pi e AR. So here CL square/pi e AR, right. So CD is a summation of something called Profile drag or Lift independent drag and Lift induced drag, so you call induced drag, okay. So this is how the induced drag is related, right with respect to with CL and the aspect ratio. We will look at what is this Oswald efficient factor. We will come back to that.

So this Profile drag again is due to skin friction since we are in moving in a fluid and fluid is in contact with the surface of skin of this airfoil of the exposed surface of the aircraft, so it has some fluid friction so that is known as Skin friction drag + Pressure Drag due to flow separation. So pressure drag due to flow separation, right. And as you go to higher velocities there is something also called Wave drag right, that comes into picture.

And in this lectures we are going to discuss only about low speed UAVs, so there is no need for us to discuss much about this wave drag here, right. So you have CD naught + CL square/pi e AR, right. So these are Profile drag coefficient in fact, Induced drag coefficient. So this induced drag and profile drag coefficient multiplied by ½ rho v square S will give the corresponding profile drag and an induced drag, right.

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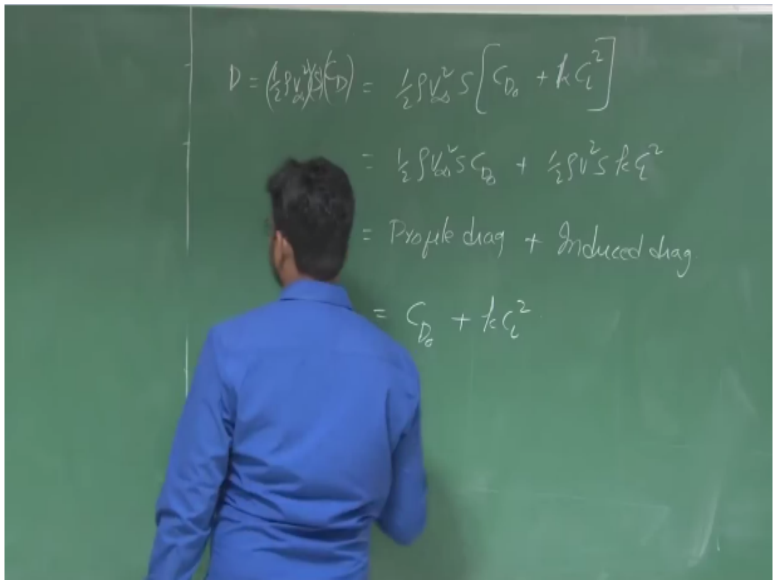

So CD or Drag = 1/2 rho v square S * CD = 1/2 v infinite square S * CD0+ k CL square = 1/2 v infinite square S * CD0+ 1/2 rho v infinite square S k * CL square, correct. So this particular term is Profile drag + Induced drag. Now we have expressed CD as a function of CL right. Here CD = CD0+ k CL square. Now we can see it is a quadratic function.

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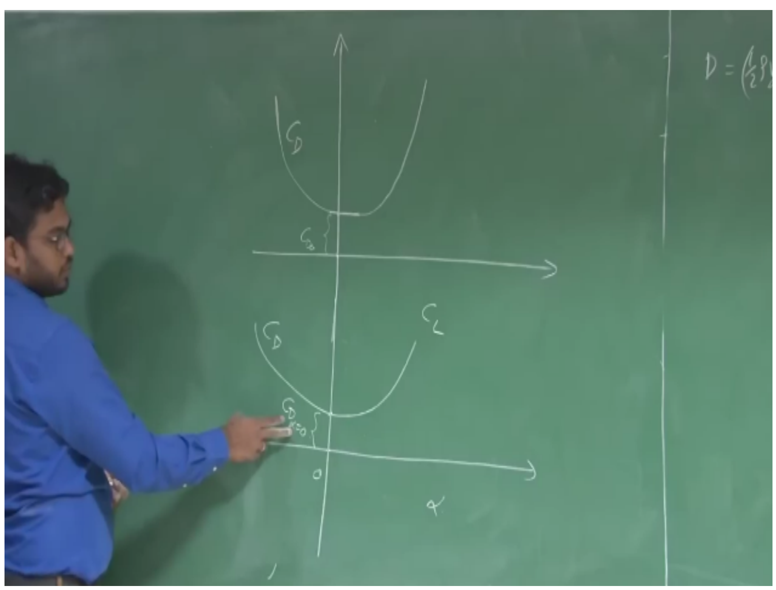

Now we can see it is a quadratic function. Now let us plot CD with CL. At CL=0 you have something called CD0. Typically, this is how CD versus CL plot look like, right. So there is something also called Drag bucket where the CD value almost remains constant for a range of CL values, right. So that particular regime for which this CD remains constant with CL is known as Drag bucket.

So since we mentioned Drag bucket it is worth discussing about various airfoils, various series of airfoil, right. So it may not exist explicitly for all the airfoils but there are some airfoils called laminar series airfoils where you can see such a drag bucket, right. And let us see how CD varies with angle of attack. So this is 0 that angle of attack 0 you have CD. And CD increases with angle of attack on either side, right.

So this particular point is CD when alpha = 0. This is not CD0, CD when alpha = 0. So you have to be careful about using this CD, then alpha=0 when CL=0.

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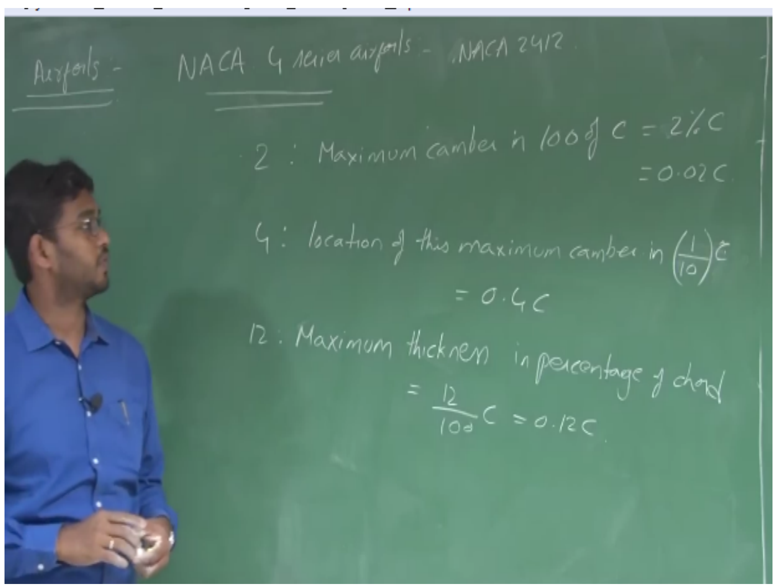

Let us look at Airfoils, right. We will again get back to that Drag Polar meanwhile let us look at various airfoil series that are for which the experimental data is available. So NACA is National Advisory Committee for Aeronautics have developed various series of airfoils and they have tested these airfoils for different Reynolds number, right. So the data first CL variation with angle of attack; CD the variation with angle of attack as well as the pitching moment coefficient variation with angle of attack is available, right.

Now, ultimately what we want the wing to generate a sufficient lift, right. Sufficient lift to overcome the weight, right. Now the planform you will decide from the machine requirements, but you need to select the corresponding cross-section as well called airfoil selection. What we will do in this course is we will try to see what is the designed CL and from there we will try to get to know what is the CL alpha of this wing

From the CL alpha of wing we will try to figure out what should be the airfoil characteristic. What should the CL alpha of airfoil? So to understand what is CL alpha of airfoil, CD naught of airfoil, and CM alpha of airfoil or CM about aerodynamic center of airfoil. So we need to understand various series that are available. So let us first consider a NACA 4 series airfoil which is famous example, right NACA 2142.

The first digit 2 talks about maximum camber in 100 of chord, that is equals to 2% of chord or percentage of chord, 0.02 of C. Let us see if you consider an airfoil say, this is my airfoil; if this is NACA 2412 airfoil then let us say if this is 1-meter chord then the maximum camber that I have is above 20 centimeters, right. And the second digit 4 represents a location of this maximum camber in 10 of chord.

Or location you will get when you multiply this second digit by C/10, that is equals to 0.4 C, right, approximately at 40% of chord. This maximum camber will present approximately at 40% of this chord. Say, this is your chord line joining, leading edge and the trailing edge and say at 40% you have this maximum camber. And last two digits use the information about maximum thickness of a 4-digit airfoil, maximum thickness in percentage of chord.

That is = 12/100*C= 0.12C, 0.12 thick. So if you have 1 meter chord you will have 12 centimeters thickness, right. All you can get is maximum thickness 12 centimeters. So you can design your spar accordingly. And say if you want to accommodate a fuel tank in the wing or if you want to accommodate batteries in the wing then you have to keep a upper cap on the battery size of the fuel tank size.

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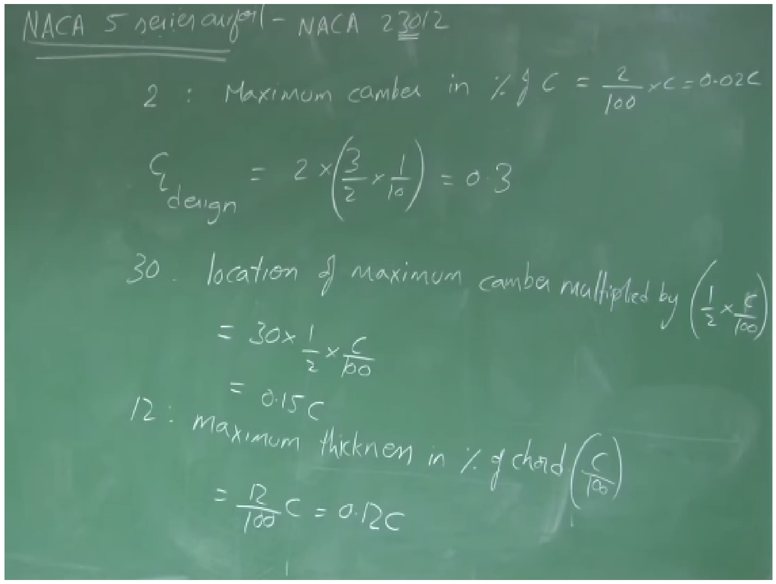

NACA 5 series airfoil, so example is NACA 23012. So the first digit here 2, first digit uses the information about maximum camber in percentage of C that is equals to 2/100 * C = 0.02C. So the first digit will also talk about design CL, right. So CL design is a design lift coefficient = 2*3/2*1/10. So c/2*1/10 is a factor that you need to multiply with this first digit of this 5 series airfoil to get to know about what is a design series of this airfoil.

And now 30 represent a location of maximum camber. So these two digits together location represents location of maximum camber multiplied by 1/2*100*C/100; multiplied by 1/2 * 100 of chord. So the location of maximum camber for this example is 30*1/2 0.15C. So if you consider a meter; if you consider chord as 1 meter that is 1-meter airfoil then the maximum camber lies at approximately 15 centimeters from the leading edge, right.

And last two digits gives you the information about maximum thickness in percentage of chord. This is C/100. So the maximum thickness is 12% of C. Now there is another series called 6 series airfoil also known as laminar series airfoil.

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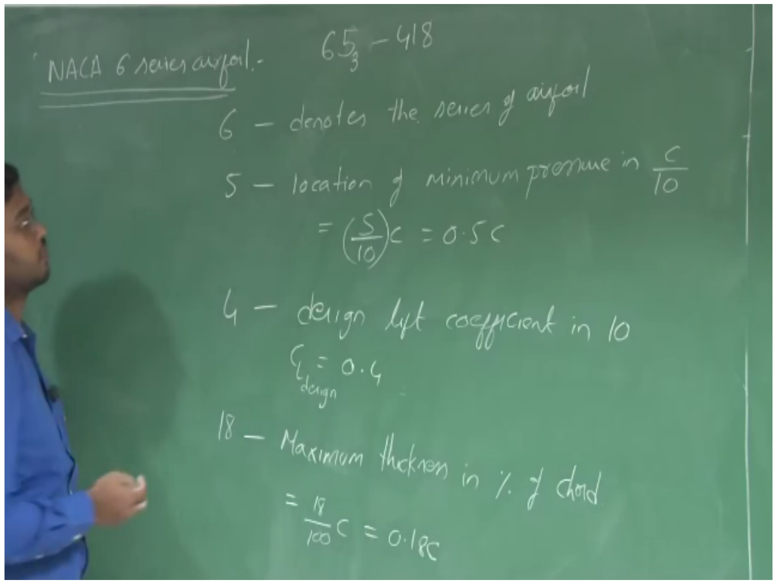

So example is 6 5 – 418, okay. So you also find some other digit as a subscript, right so this kind of airfoil, so let us see what is the significant of the subscript here. Now the first digit 6 simply denotes the series of airfoil, that means this airfoil belongs to 6 series of airfoil, right. And 5 denotes a minimum pressure location, location of minimum pressure in 10s of chord.

So the minimum pressure when the flow is completely attached will be at 0.5 C bar or C. 5/10 * C = 0.5 C. 4 talks about design CL, design lift coefficient sorry, in 10s. so design lift coefficient in 10 that is 0.4 is a design CL of this airfoil. And 18 here talks about maximum thickness in percentage of chord = 18/100*C =, so it is approximately 18% of chord. Maximum thickness for this airfoil is 18% of chord. Now what does these subscripts denotes, right what is a significant?

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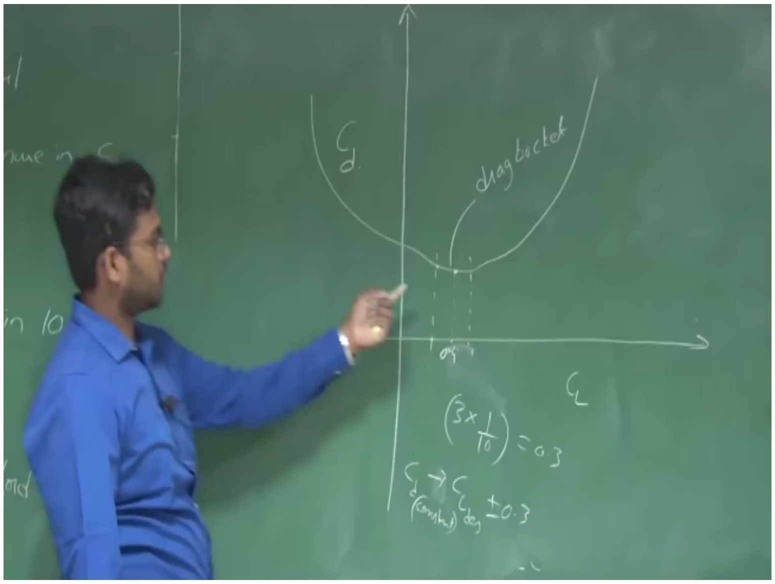

So we have designed CL, right. Say if this is CL coming back to this drag polar CL and CD; variation of CL and CD, okay. Now, say this is my CL design. In this case it is about 0.4. This is the CL at which I need to fly. We will talk about this design CL in the coming lectures right. So but assume that this is a CL at which I need to fly, right. If I fly at a particular velocity and with this CL right.

I will be able to generate enough lift, that is the understanding of what enough lift to sustain the weight at that particular altitude. So there is an understanding of this design CL for the time being. Let us keep it up to that. Now, so this design CL is achieved at a particular angle of attack, we know CL is expressed as CL naught + CL alpha * alpha at that particular angle of attack. So say, if you; somehow if you change the velocity due to maybe a head wind or tail wind suddenly the aircraft velocity or UVA velocity is change.

So because of which you need to again increase or decrease the CL depending upon the type of disturbance right, or the CL has to be changed right. So you need to either trim it higher angle of attack, bit higher angle of attack or bit lower angle of attack to achieve the respective CL, right. But this particular series of airfoil; if you look at this particular series of; even though you change your design CL to a limit to a; within a small vicinity this CD will still remain same; almost constant.

So that particular regime where the CD remains constant with even the CL changes or the alpha changes is known as drag bucket. So here in this case it is like 0.4 is your design CL. Now; so say this is your 0.4 now 3 here 3/10 is what 0.3 right. 3 multiplied by 10, 1/10 which is 0.3, right. So you have CL design + or – 0.3 the CD remains constant; CD remains almost constant. So 0.1 to 0.7 of CL the CD almost remains constant for this particular.

So this is the significant of this particular digit, right. So do you find it interesting; like you can increase L/D without increasing the drag. That means, you can trim it higher angle of attack.