Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture – 21
Contribution of Tail in Static Stability and Neutral Point.

Dear friends, welcome back, in our previous lecture we have discussing about stability of an UAV so, what should be the CG location of an UAV, so that the system behaves stable, in fact, we are also discussing about what should be limits of this CG traverse, so that the system still remain stable, right.

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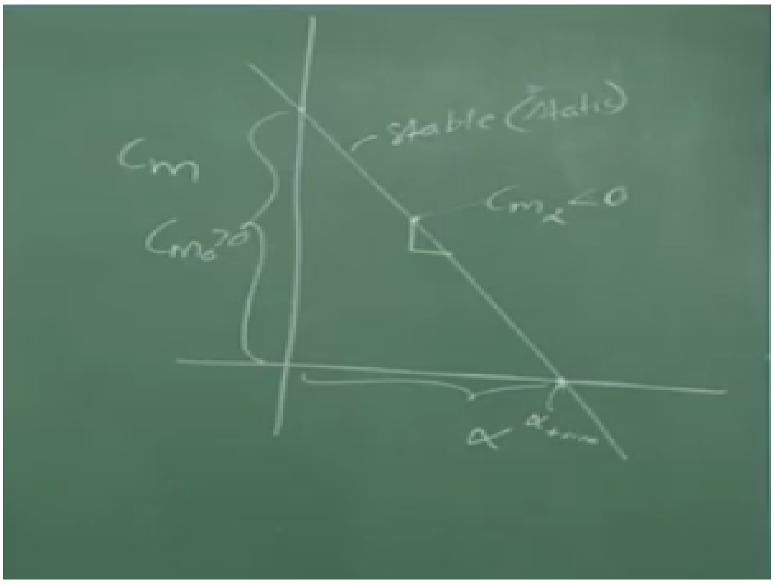

So, we witnessed that the CM with variation of angle of attack for this UAV should have a negative slope, so this is considered as stable system, stable and it is static, staticalistic or static stability, so the negative slope itself does not guarantees that it is statically stable, this CM alpha is < 0, the slope here is < 0, it is the necessary condition but the sufficient condition should be CM0 should be > 0 here, right.

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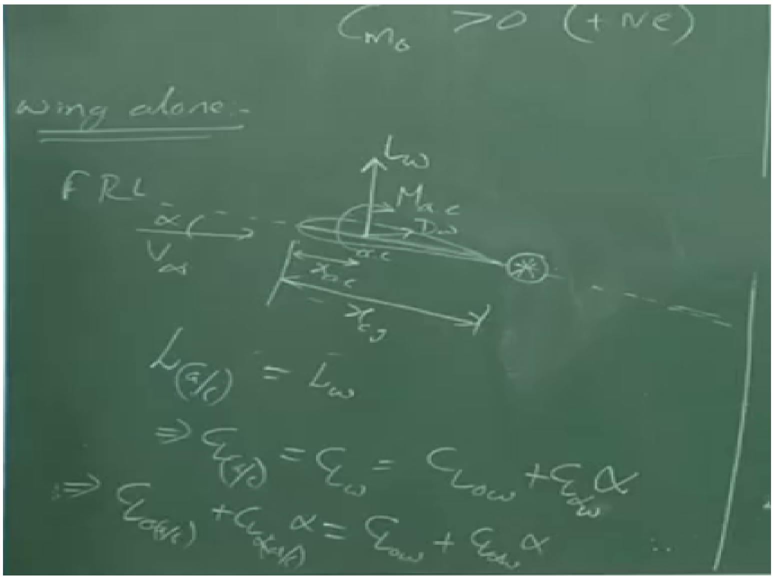

And the corresponding trim that you achieve is the positive angle of attack, alpha trim, okay so for a statical; static stability, so static stability in terms of longitudinal motion, for a system to be a statically stable during its longitudinal motion should be < 0 and CM0 should be > 0 right, negative slope should be negative, this y intersect should be positive, we considered various cases to start with wing alone.

Say, this is your wing and this is your chord line and for this case, we assume that wing incidence angle is 0; Iw is 0, which is the angle between the chord line of this wing and the fuselage reference line, so when we assume this, it means the chord line coincides with the fuselage reference line, right. Let this wing is at an angle of attack alpha, let this point be the location of aerodynamics centre.

And the corresponding distance with respect to leading edge of this wing is x AC right, and say, the CG of this system say, if it is a UAV, wing alone UAV; let us assume, wing alone UAV one side of this way, right, say if you have a motor here, so assuming that the CG is located somewhere at this location, right. Say, this is your CG and the corresponding distance is x CG with respect to leading edge of this wing, right, so this is your x CG.

Now, due to this flow, there will be lift which is acting perpendicular to free stream, which is lift of wing and there is drag; drag of wing, in the same time, we are considering a generalised case where the wing also have a pitching moment about aerodynamics centre that is moment about aerodynamic centre, M, ac.

Now, the total lift of aircraft is lift of wing, right, which implies CL of aircraft is CL of wing here, which is CL0 of wing + CL alpha wing, here the wing is; the wing angle of attack as well as the aircraft angle of attack or the reference angle of attack is considered as same, right so CL alpha * alpha, here CL alpha is for the wing, right. So, the aircraft CL0 = CL0 of the wing since because; why because the lifting surface is wing alone here in this case, right.

So, the aircraft CL0 is considered as wing CL0 and the aircraft lift curve slope = lift curve slope of the wing, where you can express this CL of the aircraft as CL0 of the aircraft + CL alpha of the aircraft * angle of attack which is = CL0 of wing + CL alpha of wing * alpha, so by comparing the constant and the coefficients of alpha, you will arrive at the wings CL0 will be = the aircraft CL0.

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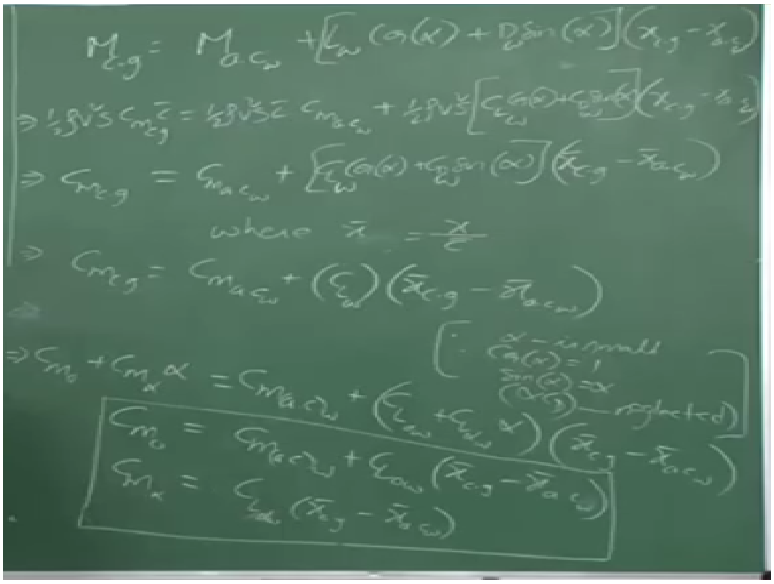

At the same time, the lift curve slope of the wing = CL alpha of the aircraft. Now, consider the pitching moment about CG, let say if I want to write moment about CG, so moment about CG = moment about aerodynamics centre which is already acting about the aerodynamics centre + what it should be; the force multiplied with the distance, so what is the normal force to this chord line or the fuselage reference line here?

Lw cos alpha + D sin alpha x CG – x AC, why because the pitching phase considered as positive, now let us say if the CG is ahead of AC, this terms become, this term, contribution from this term is negative, so this; so this term will always remain positive, right, so the pitching moment if the CG is ahead of the aerodynamics centre, so the lift will contribute towards pitch down moment.

Any change in angle of attack increase the lift that increases your pitch down moment, so this sin convention is automatically taken care like by assuming pitch up is positive and pitch down is negative, if the distance, if the CG is ahead, it will be negative, if the CG is behind that means this quantity is larger than this quantity, so this will be, this term will be automatically positive, right.

This is 1/2 rho V square S * CM CG * C bar = CM about 1/2 rho V square S, so please make a correction this is moment about aerodynamics, this is moment about the aerodynamic centre of the wing, right, so CM AC of wing, please add a subscript w here, 1/2 rho V square S C bar * CM about aerodynamic centre of the flow + 1/2 rho V square S, if I take it out, CL cos alpha + CD sin alpha * x CG – x AC, right.

So, if I non-dimensionalise this, moment about CG of the aircraft = moment about aerodynamic centre of the wing + CL cos alpha + CD sin alpha * x bar CG – x bar AC, AC of the wing sorry, this is AC of the wing, x AC of the wing, right, where bar CG or x bar = corresponding distance non-dimensionalise by mini aerodynamic chord. So, here we talk about this alpha, let us say, if this is an equilibrium state, then the resultant moment is 0, right.

We talk stability about an equilibrium right, so this alpha whatever the stability equation or the moment equation that we are going to write, you want to see how this behaves under the disturbance and moreover assume the angle of attack here is the small angle of attack because the disturbance ultimately, we are going to encounter the small distant, when we talk about stability, we talk about an equilibrium, right.

In the disturbances assume to be very small here, so say let alpha be the delta alpha was the actual disturbance where initially, you are flying at 0 angle of attack right, say and now say, there is a disturbance that is that itself is alpha which is very small, right, CL of wing, there is another correction here, this drag of wing and this L of wing and CD of wing, right please make this correction.

CL of wing * x bar CG – x bar AC of wing, since alpha is small or alpha itself is a perturbation in angle of attack and say, cos alpha is 1 and sin alpha is alpha and the product alpha * CD which will be very small is neglected, right, so this pitching moment of the entire aircraft can be expressed as CM0 + CM alpha * alpha right, this is for the entire aircraft, I am not giving this subscript again CG or the aircraft here.

So, from now, we will consider this CM0 is for the entire aircraft and CM alpha is also then correct, so this is CM AC of wing + CL0 of wing + CL alpha of wing * alpha X bar Cg – X bar AC of wing, so again by comparing the constant and coefficient, what we have is; CM0 = CM AC of wing + CL0 of wing * X bar CG – X bar AC wing and CM alpha = CL alpha of wing * X bar CG – X bar AC of the wing.

So, these are the 2 conditions that we derived from these moments; by moment equation, now for a statically stable system, we witnessed that CM alpha has to be < 0 and CM0 should be > 0. First, let us look at necessary condition, right, CM alpha has to be < 0, right, so for this to be < 0, CL alpha is always positive, right, this difference between the aerodynamic centre and the CG should be negative, right.

That means that CG should be if this quantity need to be negative, the CG should be ahead of the aerodynamic centre, right. So, even for the; in the first video, we witnessed the flight of the flying wing, so in that case, there is no tail, right, it is a wing alone configuration, so for the system to be stable, what we have done is; we place the CG ahead of the aerodynamic centre, we know how to find the aerodynamic centre, right.

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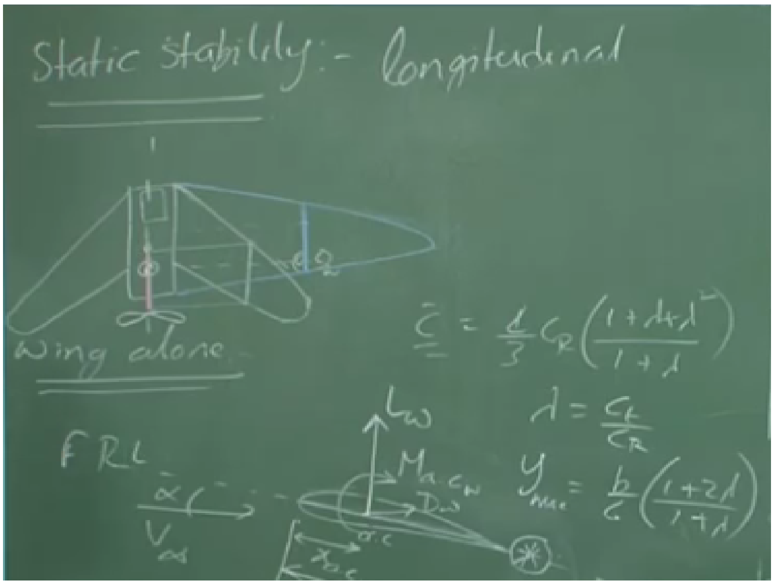

For a given planform, we can find out by using this expression, C bar = 2/3rd CR * 1 + lambda + lambda square / 1 + lambda, where lambda is the taper ratio, right, so let us draw the planform of the UAV that we; okay, assume that this is the UAV that we have learn in the first lecture, now we know how to calculate the aerodynamic centre C bar here, mean aerodynamic; sorry aerodynamic chord.

Say, this is my C bar that means, at this particular location, the chord represents this C bar, right, where lambda is lambda = CT/ CR, so we have done enough exercises and solved enough problem to know to calculate to figure out the mean aerodynamic chord right. Once you have this mean aerodynamic chord projected to the root chord, right and then this will be your mean aerodynamic chord, right.

Since you cannot measure this distance here directly, so with the fuselage reference line, you will have some, I mean fuselage reference line is will be helpful for you to actually measure the physical distance, right even with the model but here you do not know because although, you can figure out what is Y; Y of M AC right, mean aerodynamic chord, which is V/ 6 * 1 + 2 lambda/ 1 + lambda.

So, although, you can but it does not make any sense because the CG that you will measure you will of course make the lateral CG balanced, right but the longitudinal CG is the thing that you have to figure out, right or say by the end, I need to place this longitudinal CG or the CG along this longitudinal axis should be ahead of this aerodynamic centre, right. Now, let us that is the reason why see, since the lateral CG is balanced, the longitudinal CG you will be measuring is about this particular axis of fuselage reference line.

So that is the reason why I was balancing that wing alone right, wing the fly, the wing that we used to demonstrate the stability, right, concept of stability in the previous lecture, so I was trying to shift the centre of gravity ahead in behind by placing some dead weight ahead right, so I am trying to figure out where the CG is located of the particular to be, right by balancing at a particular location along the longitudinal axis, right.

So, I will be; I can easily figure out what is the CG location on the longitudinal axis, if I have this mean aerodynamic chord on the longitudinal axis, it becomes easy for me to meet the system stable, right. So, in order to do that I need to project this mean aerodynamic chord on to the root chord, right and the 25% of this for the subsonic flow, right, subsonic flights, consider the 25% of this mean aerodynamic chord.

And that particular location which is at 25% of mean aerodynamic chord is your aerodynamic centre, approximate aerodynamic centre AC, AC of wing, right. Now, you should make sure that this CG should; for this wing alone UAV, it should be ahead of the aerodynamic centre, you should design the system in such a way that CG should be ahead, you need to place your battery somewhere here because it is like m1, x1 there is a distribution; additional distribution here.

And the wings are having the sweep right so, this CG will be will also be behind in most of the cases, right, if you do not design it properly. See, why we are pushing, why we are giving the sweep in first place, why cannot we simply use a rectangular one? If we use a rectangular wing or a tapered about midpoint, here we are giving a sweep at the same point, without any sweep, we can still have a taper, right.

But the issue is the aerodynamics centre will lie on the same line, see if you have wing which is tapered, same wing plan, what you called same wingspan with the same taper, if you have wing like this, say this is your aerodynamics centre, so mean aerodynamic or and say, this will be your aerodynamic centre in that case but the CG in some cases, you will not be push, you will not be able to push I mean, beyond certain limit.

The aerodynamic centre is too closed to the leading edge, there is not much mass distribution here, so it will automatically the CG will automatically shift behind and then what you have to do is to extend a rod and mount a motor shear, so that you are adding a mass ahead of the leading edge to shift the Cg ahead and there are certain disadvantages for this during landing and all.

So that is the reason why we trying, see the sweep for lateral stability is secondary reason but primary for a wing alone configuration is like pushing your aerodynamic centre behind this CG. Coming back to this, the CG should be ahead of this aerodynamic centre for CM alpha to be negative why because CL alpha of the wing is positive, it is a finite, positive and yes, at the same time, when you make this CG ahead this particular term here will be negative.

Because CG is the ahead of the aerodynamic centre, now, this negative and CL0 is positive, so this becomes negative if we use a cambered air foil or camber air foil; positively cambered air foil, the CM AC of the wing is also negative, so the CM0 of the aircraft becomes negative which leaves you with the only option to trim it at negative angle of attack but we are not interested in that solution, right.

So, to make this positive, what I do is; I will increase this CM AC, increase when I say it is like towards 0 or positive, I want to make this CM AC positive, right.

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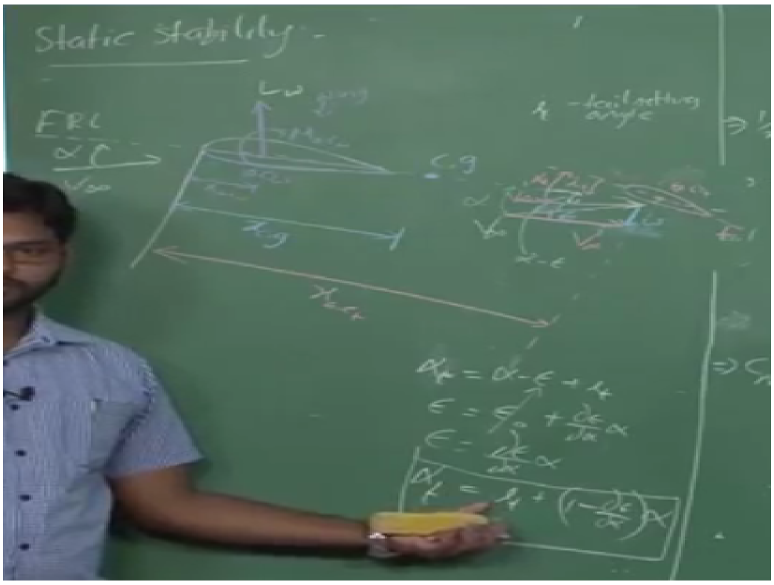

So, to make that CM AC positive, we have to select an air foil that gives positive CM AC, right, which is generally known as reflex air foil, for a reflex air foil, the trailing edge is with deflected upwards, it is like giving an elevator up, similar concept, we will discuss what is elevator that so, this at the same time this reflex air foils, you have to compromise at the CL0 and CL alpha right, so you need to properly select an air foil that gives you this CM about aerodynamic centre positive.

And this CG, the distance between them should be just sufficient enough to make it stable and at the same time, you should, I mean, it should have help towards CM0 positive, which enables you to trim at positive angles of attack, chord of the wing and in the first case, we are talking about wing alone, right now let us say we added tail to it, right, so draw this tail, okay, the sizes are not equal; wing and tail, right, definitely tail is smaller than wing right.

See, this is a chord line of the tail and this symmetric air foil that I am using here, right, so the chord of the tail, this is your tail say; say tail, this is your wing and this is your tail and say this chord of this wing is inclined to this fuselage reference line at an angle of I of T called tail setting angle, right, I of T is known as tail; tail setting angle, right. So, now there is V infinity and angle of attack with respect to fuselage reference line, FRL here.

So, similar to the first case, we have the aerodynamics centre of the wing, which is at a distance X AC of wing with respect to leading edge of the wing or the leading edge with respect to the root chord of the wing, right and now say, you have this CG of the centre aircraft located at a distance X CG with respect to the leading edge of the root chord of the wing, X CG, right.

And finally, say this is your aerodynamic centre of the tail AC of T, so this aerodynamic centre of the tail, these distances are measured along the chord line, parallel to the chord line, right, so this is your X AC of your tail with respect to leading edge of the root chord of the wing. Now, again we know this is lift of wing and I am not considering the drag, anyways, we are going to neglect them, neglect those terms.

So, let us talk about lift and by assuming the smaller angle of attack, L sin alpha will become L cos alpha will become L, lift of it. Now, what we are interested is; so this is your wing, lift from the wing and the same time, we will consider the moment about the aerodynamic centre of the wing, see V infinity ideally should be parallel to; right, whatever the wing is facing or the fuselage reference or the aircraft is facing should be same as what the tail has to face.

So, ideally it should be V infinity and this has to be alpha, right but we witnessed, we studied about downwash and upwash effect, right, so the combined effect will induce the downwash at the tail, so this induced downwash so, downwash is nothing but downward flow here, right here, you can assume that may not be exactly same but we can still assume it has a downward flow, right.

So, this because of this, downward component, this will that V infinity will or now be modified, right, see, if this is your downwash w, small w, okay downwash, so because of this, the flow at the tail is modified, so this is your V infinity prime and the corresponding angle this is epsilon which is induce because of this downwash right. Now, I can take a parallel to this right, this I can represent it as V infinity prime; V infinity prime at the tail.

And this particular angle is epsilon, right and this is my angle of attack of tail, what is alpha of tail here? See, this is epsilon, I should write epsilon in this colour because it is induced by the downwash right, so this is my alpha of tail, so angle of attack is defined with respect to the reference line here, I am talking about tail, right, total angle of attack of tail is the angle of attack with respect to the flow + tail setting ahead.

So, this is alpha of T, right, so what this angle become, if this is alpha and this is epsilon, what this angle will be? Alpha – epsilon, right, this particular angle will be alpha – epsilon, so total angle of attack of the tail I of T = sorry alpha of T, total angle of the attack of the tail = tail setting; this particular angle is the summation of this angle and the corresponding angle marked in the; marked with the white chalk, right.

So, this particular angle is alpha – epsilon + tail setting angle I of T, right and we can further express alpha as epsilon as epsilon 0 + dou epsilon / dou alpha * alpha this we already discussed in our relay lectures, assuming this contribution is very small, you can say epsilon is dou epsilon/ dou alpha * alpha, right. So, what is alpha of tail is I of T + 1 – 2 epsilon/ dou alpha * alpha, this is the angle of attack at the tail, right in terms of angle of attack of the wing and downwash, right I of T, it is a function of all these 3 parameters.

Now, let us look at what is the total lift of this aircraft, since we are using a symmetric wing; symmetric air foil for tail, we are not considering the moment about aerodynamic centre in this case, right.

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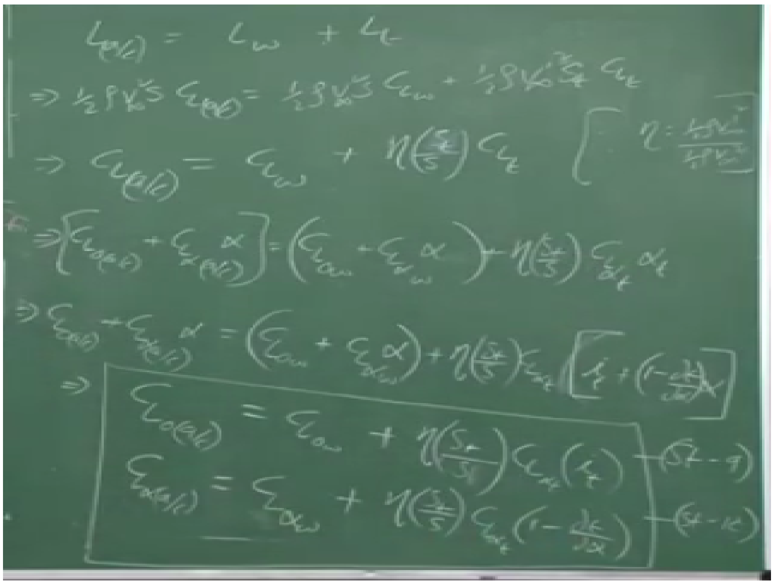

So the total lift of the aircraft =; so the overall force is acting lift, I mean upward forces are lift of wing + lift of tail, right, so if I am lifting a rigid body assuming the aircraft whatever we are considering is a rigid body here, so you are considering a rigid body, the total upward force that is acting on the; it is the summation of force acting at this point + the force due to my left hand, right, is the total force.

Similarly, the total force on this craft lift force acting on this aircraft is lift of wing + lift of tail, principle of super position, right. So, this is 1/2 rho V infinity square S * CL of the aircraft = 1/2 rho V infinity square S * CL of wing + 1/2 rho V infinity prime square rate, cannot you see this because the lift at the tail is because of the flow at the tail and the resultant flow here is V infinity prime which is modified due to the downwash, right.

And the corresponding surface area to generate lift at tail is S of T, where S of T represents the planform area of the tail, right, 1/2 rho V square ST * CL of tail, right, so if you want to see what is the non-dimensional lift coefficient for this entire aircraft is contribution from lift of wing + eta ST/S eta ST/S * CL of tail, right where, eta is 1/2 rho V infinity prime square tail efficiency factor, 1/2 rho V infinity square.

So, when this can be 1? When there is no interference, right when there is no downwash here that means, if the wing and tail are separated by enough distance, we can assume that eta = 1. So, this = CL0 of the total aircraft + CL alpha of the total aircraft * alpha = CL0 of the wing + CL alpha of the wing * alpha of wing + eta ST/S *; see since we are using a symmetric air foil for this, this CL0 will be; CL0 of the tail will be 0, because of the symmetric air foil.

So, what you have is CL alpha of tail * alpha of tail, right, this implies CL0 of the aircraft + CL alpha of the aircraft * alpha = CL0 of the wing + CL alpha of the wing * angle of attack at the wing * or say this alpha wing is nothing but the actual angle of attack here, right; * alpha + eta ST/ S * CL alpha of tail *; what is alpha of tail? We already derived it right, which is the summation of tail setting angle + the resultant angle with respect to V infinity prime, right, resultant angle when I say, with respect to fuselage reference line here.

So, this is I of T + 1 - dou epsilon/ dou alpha * alpha, right. So, now by comparing the constant and the coefficients of alpha, right, what you have is CL0 of this entire aircraft with the wing and tail combination = CL0 of the wing is contributed due to CL0 of the wing and tail setting angle, right, lift because of tail setting angle, see although, the I of the angle of attack is 0 here, let us say, if alpha is 0 for this entire aircraft with the wing and tail combination, there is always the lift from the tail which is contributing towards pitching moment right.

So, this lift from the tail is irrespective of whether alpha is; whether we are flying at positive alpha or not, right, you understand this point, so this I of T is the one which is contributing towards a lifted 0 angle of attack, right. So, CL0 of the aircraft at the lift at 0 angle of attack is contributed because of the CL0 of the wing. Say, if we also consider a symmetric air foil here for the wing in that case, CL0 of the wing is 0, right.

And when you do not consider any I of T that means, I of T is 0 that means for symmetric wing and symmetric tail, the CL0 of the aircraft is 0 and CL alpha of the aircraft = CL0 yeah, so, CL alpha of wing + eta ST/ S is an interference factor CL alpha of tail * 1 – dou epsilon multiplied by –dou epsilon/ dou alpha. Do you remember this numbers; equation numbers, ST 9 and 10, stability 9 and stability 10, right.

So, the CL alpha lift curve slope of the aircraft is the combination of the lift curve slope of the wing + the angle; it is due to the tail lift curve slope, right, these are the normalising factors for the; so the next step is to figure out what is the pitching moment, so you ultimately what we want to know is what should be the CG location for which this wing and tail combination becomes stable.

So, for a wing alone configuration, we figured out, so qualitatively we discuss may not be the exact with the exact number that will come up, right, once we discuss about static margin and all, then we will come up with what should be the exact value of CG; quantitative definition of CG, right or CG limits, so for each and every aircraft it changes and for a given aircraft at the given static margin, you can get it, get to know what is the corresponding location of the CG.

But qualitatively, let us first understand where the CG should be, right, either should be ahead of the aerodynamics centre or what should be the tentative location at which it has to be, so the system behaves stable, why we are doing this exercise is to figure out what is the CM0 because for a statically stable aircraft, we figured out CM0, in the longitudinal case CM0 has to be > 0 and CM alpha should be < 0, right.

So, in order to see what is the CM alpha and CL; what should be the CG location for which the CM0 of this aircraft; of the wing and tail combination has to be positive and at the same time with the same CG location, it has to give CM alpha for this wing and tail combination negative, right, so to look into that we have to first write down the equation; moment equation about the CG, right.

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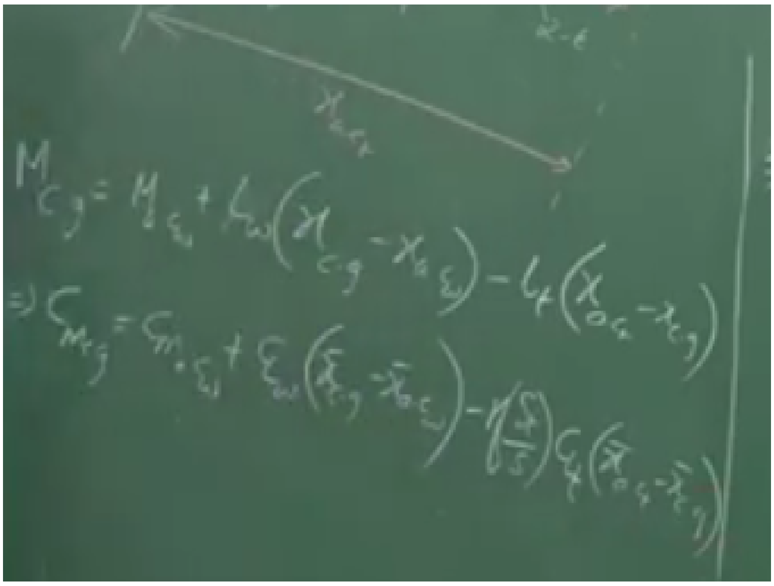

Moment about CG of this aircraft is; moment about aerodynamic centre of the wing + lift of wing, see moreover we have not assumed any offset of this CG with respect to fuselage reference line, so z CG is not considered, so since we are considering, say if this is your; this is your aircraft, this is your x, this is your c, this is your y, right, so for the longitudinal case, we are talking about aircraft; motion exert plane right.

Now, the z CG, x CG, we are talking about only x CG not this z CG here, we assumed that for this UAV's, the z CG is coinciding with this fuselage reference line, right, there is no offset with respect to; z off set with respect to fuselage reference line, so and yeah here, we are not considering the drag because anyways we are going to neglect them that we have witnessed in the first case where wing alone, while deriving the moment equation for wing alone configuration.

So, lift of wing * x bar x CG – x AC because the CG is behind the aerodynamic centre here the lift contributes towards positive moment right, pitch up moment, so lift of wing * x CG – x AC -; why – because the lift of the tail is behind the CG in this case, right, so the force * moment term here contributes towards the negative moment right, so lift of tail * multiplied by the corresponding moment term, which is x AC of tail – x CG, right.

These -, this -, these distance will get you with the moment term with respect to the CG right, so CM CG = CM AC of wing + CL of wing multiplied by x bar CG – x bar AC of wing – lift of tail is eta ST/S * CL of tail * x bar AC of tail – x bar CG.

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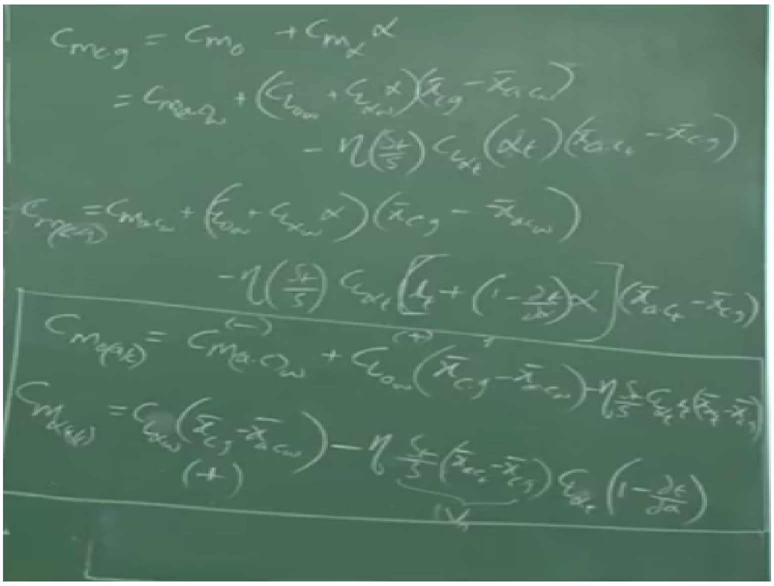

CM CG = CM0 of the aircraft + CM alpha of the aircraft * alpha which is = CM AC of the wing + CL0 of the wing + CL alpha of the wing * alpha and the corresponding moment associated, momentum associated with it, right – eta ST/S * CL alpha of tail * I of T x bar CG

–; x bar AC of tail – x bar CG; alpha of T okay. This implies okay this = that is CM AC of the wing – eta ST/S * CL alpha tail * I of T + 1 - dou epsilon/ dou alpha * alpha * the corresponding momentum; x AC of the tail – x bar CG, right.

Now, by comparing these 2 equations, this is also a moment about CG of the system, this is also the moment about the CG of the system, so now comparing these 2 equations, right what you have is; CM0 of the aircraft A/C represents here the aircraft right = CM about aerodynamic centre of the wing + CL0 of the wing multiplied by the corresponding momentum – eta ST/S CL alpha of tail * I of T x bar AC of tail – x bar CG.

And CM alpha of the aircraft = CL alpha of the wing, so this is the thing, right, the coefficients of alpha will be CL alpha * by the corresponding momenta, in the same time this tail CL alpha * corresponding CL alpha of wing * x bar CG – x bar AC of wing – eta ST/S, so this particular quantity ST/S multiplied by x bar AC of tail – x bar CG is considered as tail volume ratio, right.

This particular quantity is known as Vh tail volume ratio; tail volume ratio of horizontal tail, right ST/S * x bar AC – x bar CG * CL alpha of tail 1- dou epsilon / dou alpha, this is your yeah, pitching moment like these parameters are important to analyse the stability of the system, right, static stability of this system for the longitudinal case, so for a system to be statically stable say wing and tail, if I have a combination of wing and tail, if it has to be statically stable, then CM0 has to be positive, right.

So, CM AC in general we considered is the cambered air foil is negative, right and say here in this case, this CG is behind the aerodynamics centre, let us say that means this contributes towards positive, right, so this is positive, so this is positive right and – eta ST/S CL alpha of tail * I of T, if I of T is negative, this will; this particular term also will contribute towards positive moment if I of T is 0, you do not have any tail contribution in the moment coefficient at 0 angle of attack, right.

So, if I of T is 0, you do not have tail contribution in pitching moment, so all you need to play around is this, if the CG is behind the aerodynamic centre, right, so the advantage let us see what is the advantage of this tail right, let us say in the first case, I of T is 0, there is no I of T, now what happens in that case? This I of T is 0 that means the chord line of this tail coincides with the fuselage reference line here, right.

Now, this particular term is 0 and we assume that the CG is behind the aerodynamics centre, so this is positive, so this particular term we need to ensure that we have considered enough distance between these 2 and we have chosen a proper aerofoil to make sure that it all comes the corresponding moment about aerodynamic centre of the wing, right this particular term turns out to be positive, right.

So, this is a first case and say now, by doing that what we are fixing is this CG should be behind the AC, now if you have a CG behind the AC, this is positive; CL alpha is positive, so this entire contribution from this term is positive, in that case. Now, if CL, if CM alpha has to be negative, you have to consider the enough tail volume ratio and CL alpha to make sure that this wing and tail combination will satisfies this necessary condition of longitudinal static stability, right.

So, this particular term should be; so if this has to be negative, this particular term should be greater than this particular term, first term, right, this is the moment contribution by the tail and this is the moment contribution because of the wing, right, so this CL alpha and this particular term tail volume ratio plays a major role in making sure in ensuring the stable flight of this configuration, wing and tail combination.

Now, let us consider the second case, where I of T is non 0 right, so what happens if I of T is positive, so this whole contribution from the tail becomes positive for CL0, negative for CL0 right, so if you have I of T positive; positive means the chord line of this is inclined above the fuselage reference line, right that is I of T is positive. Now, say if the CG, this particular term the distance between CG and AC are very close, right.

Otherwise, the CG of the wing and AC are the same location, this contribution is 0, right, what you have is CM about aerodynamic centre of the wing, right, what we are doing right now is we are thinking that the CG of this coincides with this point, right that means, this becomes your x CG again effectively, so x CG and x CG are same, you do not have a momentum there to account for CL0 of the wing or the pitching moment contribution from the CL0 of the wing.

So, this particular term is 0 that means, what you have here; so if you are left with CM AC of the wing – eta ST * CL alpha of tail + I of T * the momentum, right here, so now if CM0 has to be positive, so I of T has to be negative and this particular value should yeah, should be more than this value, so if I of T is negative, this trail contribution is positive and CM0 becomes positive if it is more than this CM AC of the wing, right.

So, what happens in the second case, if the CG is exactly at the AC, so if it is at the AC, this becomes 0, CM alpha is the negative term, eta ST/S, this particular term is positive because this CG is ahead of the aerodynamic centre of the tail, right, so this is positive and CL alpha of the tail is positive, 1 – dou epsilon/ dou alpha is positive, so this is a positive term that contributes towards the negative pitching moment, right.

So, this is one case where you still have the; even if your CG is exactly at the AC of the wing, you can still have a flight, right because it is satisfying the condition of CM alpha negative and CM0 positive right.

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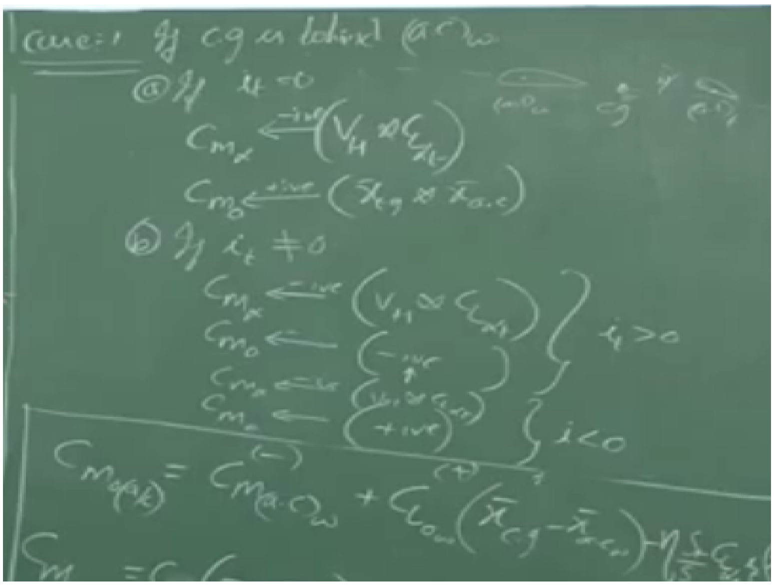

So, case 1, if CG is behind AC of the wing right, let us say this is your case 1 and we have witnessed that for so in this again, there are 2 cases, where if the CG is behind AC that means, you have a combination like this, wing and tail combination, this is your I of T, right and say this is your CG, this is your aerodynamic centre of tail and this is your aerodynamic centre of wing, right.

Now, in this case of the CG is behind this AC; AC of the wing right, so there are 2 cases, if I of T = 0 and this again case a, if I of T = 0, so this particular term x CG – x AC right, so for CM alpha to be negative is that going to affect, so the condition what we need is CM alpha to be negative, if I of T is 0 in the CG is behind the AC, so this term is positive and you have to choose this tail volume ratio and CL alpha plays a major role here, right.

So, for CM alpha to be negative what you need is; tail volume ratio VH and CL alpha of the tail plays a major role, right, so these 2 are crucial in deciding CM alpha negative, for it to be negative, we need to choose a proper values of this. So, when you are choosing this tail volume ratio, which means you are actually sizing the tail in terms of both distance between CG and AC at the same time, area; area of the tail, right.

So, you know how to select the wing area here, we are trying to select the tail area in the corresponding distance between, right and the second condition is CM0 should be positive how it can be weight positive, in this case if; if CG is behind the AC and there is no I of T; I of T is 0 that means, the tail contribution is 0 here, so this is positive right, CG is behind the AC, this is positive right.

So, this is negative and you have to choose this CG -; this distance between the; because CL0 of the wing you are deriving it from the design requirements, right or say mission requirements as per the design CL, we already did that exercise right, where you figure out what is the CL0 of that, so you do not have much control at this particular stage on the CL0, right.

So, what you can do is; like choose the proper distance between CG and AC, so x bar CG and x bar AC; distance between them plays a major role in this particular I of T = 0. Now, in the same case, where CG is behind AC if I of T is non 0, so when you say non 0, it can be either positive or negative, let us say if we consider negative I of T, so I of T is not going to affect your CM alpha right.

So, this again depends upon the same conditions for the CM alpha to be negative, so it is not going to change, it has to be negative, then VH and CL alpha of the tail plays a major role, right and what about CM0, so say if I of T is positive that means, this particular chord line is above the fuselage reference line or oriented above the inclination is above the fuselage reference line, right.

So, in this case, I of T is positive, x AC – this distance is positive and the rest of the terms are also positive, so this contributes towards negative moment, so this distance has to increase a lot, if that is happening, if this distance is increasing because these are the only positive term, this is negative say if it is a cambered air foil and this is negative, right, so what happens in that case, CL alpha of the wing if you consider a large distance between these CG and AC, this will also become large.

So, you have to keep increasing your tail volume ratio that means, the size of the tail increases and the wing; the distance between them should also increase, right, so if we have negative, this is the problem, if you want to give a positive, of course it will be helpful in generating lift; overall lift because the total lift is lift of wing + lift of tail that we have derived.

And we witnessed at 0 angle of attack, the lift coefficient of the total aircraft is the summation of lift coefficient of the wing + contribution from the tail because of the I of T, right, so positive I of T will contributes towards positive lift that means, lift will increase but it will not contribute towards your moment, this necessary condition for stability. When IT is > 0 here, so for CM0 to be positive, the CL0; the contribution from this wing CL0, and the corresponding momentum should be higher right.

This particular term has to be positive because all these becomes negative, so CM0 to be positive, this distance has to increase, if you have a greater momenta with respect to say aerodynamic centre of the wing, then this particular tem will also be positive right, is the strong contender to this negative right, so ultimately what we want here is CM alpha to be negative that is first or the necessary condition here, right.

So, we cannot, what you call that we need to increase the tail volume either by increasing the area of the wing or by distance between the AC and CG, right at the same time, we have to choose a CL alpha, so the advantage for having I of T positive is that you have contribution in the lift but in case of pitching moment, it is the disadvantage. So, CM0 when I of T is; I of T is > 0, what happens is CM0 will be; will become negative, right.

Let us say, if you fix the CG and AC, the case where I of T is 0, okay. Now, say you have I of T positive that means, this is becoming negative, right, this initially you have fixed to certain CM0 that CM0 decreases, right that is the; if you have I of T positive in that case, so compared to that you need to increase the CG and AC distance. At the same time, will be negative more negative compared to initial case.

This negative when I say is compared to a small compared to the initial case, where I of T is 0, right and say if I of T is non 0 and it is negative, if I of T is < 0 in this case, what happens is; this contributes I of T is negative, then the tail contribution is positive towards CM0 and yeah, if the CG is behind the AC here as this case, this particular term is positive, so CM0 will be automatically become positive.

So, positive for CM and in CM alpha, it is not going to affect ever, CM alpha again depends upon tail volume ratio and CL alpha of tail.

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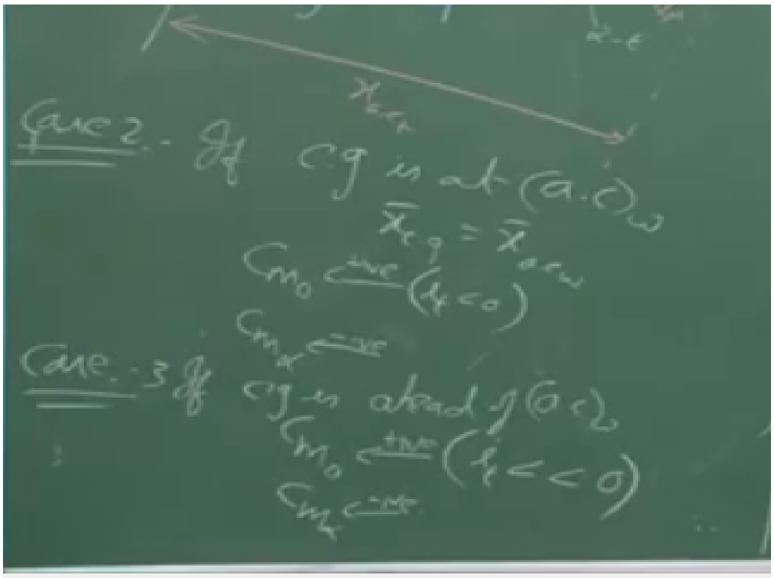

Case 2; if CG is at AC of wing that means, x CG = x AC of wing, right. What happens in that case, this particular contribution is 0, right, what happens to CM0 of the aircraft, so if CM0 is; this contribution is 0 because CG = AC, the difference is 0 here, there is no momenta and say in this case, if I of T is positive, if I of T is 0, let us say to start with if I of T is 0, then CM0 of the aircraft will become negative automatically, unless use a reflex air foil here, right.

So, for a non 0 I of T, see if I of T is positive that means, CM0 again becomes negative, so the only condition where CM0 becomes positive is I of T is negative, so CM0 becomes positive, when I of T is negative and CM alpha in this case, will be always negative as long as you have a tail, right automatically, because this difference is 0, so this becomes, this is the positive quantity which is automatically becomes CM alpha, automatically becomes negative, okay.

And let us consider another case where the CG is the ahead of the aerodynamics centre of the wing, so what happens in that case? This particular term becomes negative right, so CL0 becomes negative; this particular; I mean the entire contribution from CL0 of the wing towards pitching moment will become negative because this x CG is ahead, right and CM is negative, this is the more, higher negative term.

And say now, you have to have a bigger I of T, right, there is a negative; higher negative tail setting angle that means, the tail should be at a negative angle of attack; continuous negative angle of attack, so there is a disadvantage right, your lift is effective, lift is decreasing at the same time, there is a drag because we are using symmetric air foil, on either side in the drag polar, you have a positive drag other than 0 alpha, right.

A positive drag in the sense, higher drag here okay, please make a correction, there is a higher drag, so you have to continuously overcome that additional drag, so that is the reason why if you place it, if you place the CG ahead of this, this become, you need a higher I of T, negative I of T that adds to the drag as well, right, so this can become positive by case 3, if CG is ahead, AC of the wing; if CG is ahead of the AC of the wing, what happens; this particular term becomes negative.

So, for CL0 to be positive, IT should be very negative, right and if this is ahead, then CM alpha is negative and this is already negative, so this becomes negative but there is again a disadvantage like there is a too much CM alpha right, the slope is very stable even for a; if you want to trim, if you want to change the attitude of the aircraft, you need a larger deflection, larger elevator force.

So, there is a disadvantage that we will discuss later. So, CM alpha will be negative, the necessary condition is already satisfied here, now this is the qualitative discussion, now let us look into the quantitative discussion of this CG; CG traverse, limits of the CG traverse. See, there is the same case right, if we can see for a symmetric aerofoil, we have 2 flights last time; one with 2 wings right, made out of Styrofoam.

One is the cambered wing alone and the other one is the symmetric wing, it is the rectangular planform with 1 litre span approximately and 22 centimetres chord right. Now, in that case what happen, why initially it did not fly? When there is no dead weight, the CG is behind the AC, right, if the CG is behind the AC for a wing alone configuration, this becomes; CM alpha becomes positive, right.

If there is no tail, that means that this contribution from this particular quantity is not here, right, not present during for that particular flight, so if the CG is behind the AC, this CM alpha is positive that means, system becomes unstable that is why it is flipping, right, so for a wing alone, so by placing the C; dead weight, what I did is; I brought CG to the AC almost right, in that case, I do not have CM alpha.

So that is the reason why, whenever at that particular location, so if it encounters any disturbance, it will try to achieve a new equilibrium, which we do not want in general, right, you want the aircraft to fly at a particular CL; the design CL, so the CL is controlled by alpha, we want to have it a particular alpha, we do not want whenever there is a disturbance the aircraft should adjust to the new equilibrium.

So, if that is the neutral stability, right, so the system becomes neutrally stable, when I bring this CG close to this aerodynamic centre for this wing alone configuration, right, you understand, this particular point is like neutral stable; neutrally stable position where it achieves new equilibrium, whenever there is some disturbance, right. Now, say will there be any such neutral point or a point, which is neutrally stable for a wing and tail combination, if so what is that?

Right, even for wing and tail, we should have the same thing, right, like there should be some point in this case of wing alone, we say there is a neutral point where if the CG of the aircraft = the aerodynamic centre of the wing, then your aircraft is neutrally stable at that point. After that if I place the CG to ahead, then it is also diving down, right, so it is stable but it is also diving down, it is not performing the intended task.

So, let us see what is the neutral location or the location at which the system is neutrally stable right, for a wing and tail combination, let us derive that first.

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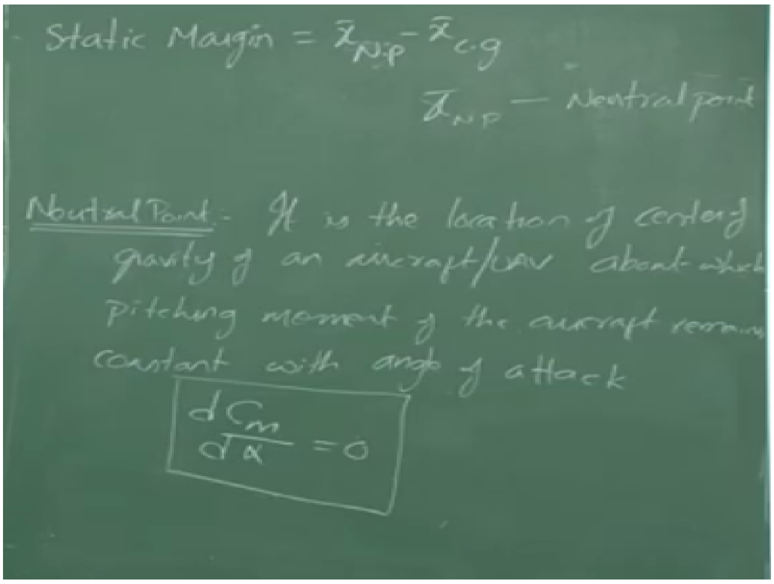

So, let us define something called static margin; static margin is a distance between x bar CG and or x bar NP - x bar CG right, where x bar NP is the neutral point location or location of neutral point with respect to in this case, is the location of neutral point with respect to a reference, right. In our case, we always consider the leading edge of the root chord is the reference, so this is your neutral point x NP bar right.

Now, let us define what is this neutral point? okay. It is the location of centre of gravity of an aircraft or a UAV about which; about which pitching moment of the aircraft remains constant with angle of attack, so it is a location of the system right, it is the location of the CG of the system that means, the neutral point is the location of centre of gravity for this particular system about which the pitching moment of this aircraft remains constant with angle of attack, okay.

So, do not you see it is similar to that of aerodynamics centre of a wing, the pitching moment remains constant with angle of attack about the aerodynamic centre for the wing for the entire aircraft about neutral point, this pitching moment remains constant, so say this is my NP, neutral point, now say this distance is considered as x NP, right, so about neutral point, the CM variation with angle of attack.

The change in CM. with change in angle of attack is 0, this is for the entire aircraft, so this is the definition of neutral point so this is for the entire aircraft, is it clear? So, the pitching moment or the change in the pitching moment with respect to change in the angle of attack; Why because? As the angle of attack change the lift of the wing and the lift of the changes and so, the pitching moment of the aircraft.

So, the change in the pitching moment of the aircraft with respect to angle of attack is 0 or the pitching moment is independent of angle of attack about this neutral point that means, there is a constant pitching moment but which is unaffected with change in angle of attack, that is like similar to the concept of aerodynamic centre of wing, so neutral point is like an aerodynamic centre for the entire aircraft, right.

So that means, the CM is constant or the pitching moment is constant. Let us see what is that constant value and where this MP located for a wing and tail combination, centre of mass in neutral point, you see it itself is a CG, centre of gravity, it is a location of centre of gravity about which pitching moment is independent of angle of attack but see how; why does this point exist in the first place, why there should be a neutral point, right?

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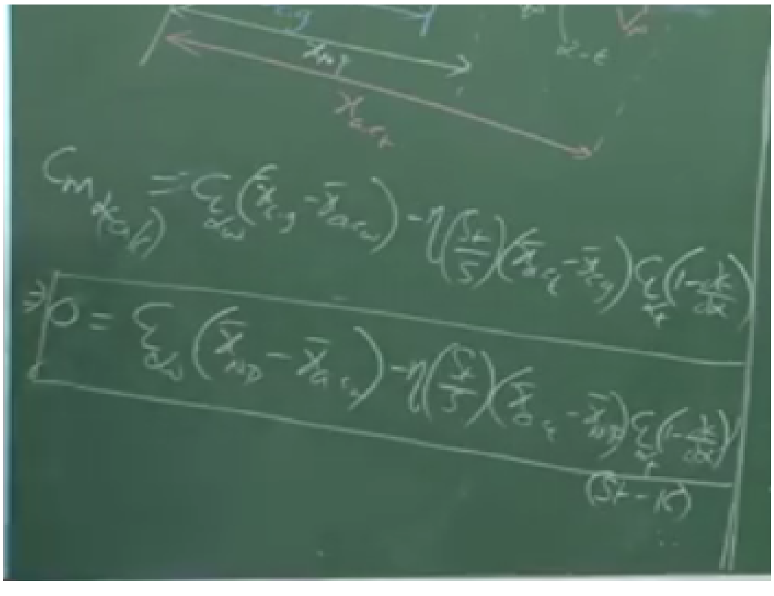

Now, we will address that so, what; so, let us say this is the CM alpha is 0 that means, let us consider this CM alpha of this wing and tail combination here, what is that? CM alpha of the aircraft which means, which implies, CM alpha of the entire aircraft is 0, about this CG location, right, so that particular CG location is known as neutral point, so what is the CM alpha of this wing and tail combination? We derived it just now.

CL alpha wing * X bar CG – x bar AC of the wing – eta ST/S * x bar AC of tail – x bar CG of wing; x bar CG * CL alpha of tail * 1 – dou epsilon/ dou alpha, right. So, from the definition of neutral point, if I want to find out what is the neutral point for this combination, what I need to do is; the CG here should be the NP and the corresponding CM alpha should be 0 that is what the definition says; it is the location of CG right.

Now, say if I am at the neutral point, CM alpha has to be 0, right, so by the definition of neutral point, I substitute x CG as x NP x bar NP and the corresponding CM alpha is 0, x bar NP, right, so consider this definition as ST, ST 15, right, so this equation is 16; equation and stability, right. Now, solve this ST 16 for neutral point.

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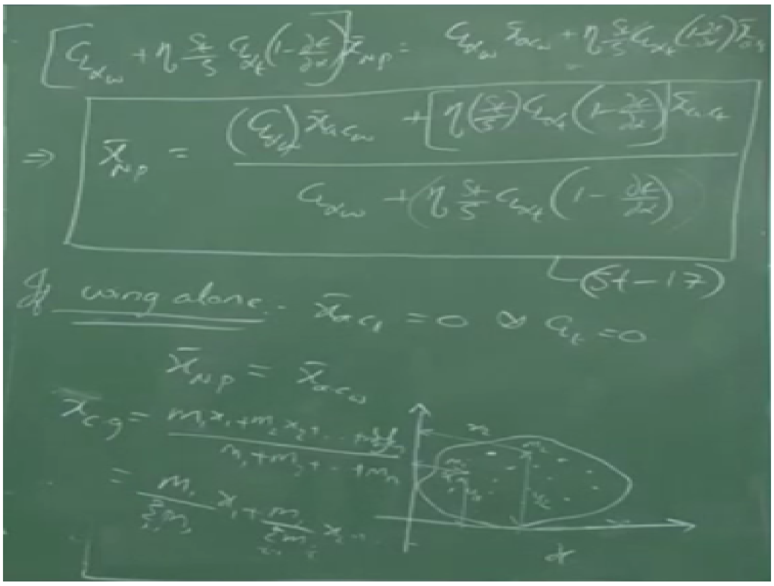

CL alpha of wing + eta ST/S * CL alpha of tail * 1 – dou epsilon/ dou alpha * x bar NP = CL alpha of wing * x bar AC of wing + eta ST/S and x bar or say CL alpha of tail * 1 – dou epsilon/ dou alpha * x bar AC of tail, right. So, this implies x bar neutral point = CL alpha of wing * x bar AC of wing + eta ST/S * CL alpha of tail 1 – dou epsilon/dou alpha * x bar AC of tail/ CL alpha of wing + eta ST/S * CL alpha of tail * 1 – dou epsilon/ dou alpha, right.

So, this is the corresponding neutral point for wing and tail combination, this is ST 17 stability equation, so this is the neutral point corresponding neutral point, right. So, if there is no tail, let us say if there is no tail what happens? x AC of tail is 0 and there is no lift from the tail, so if wing alone, right, which is wing alone right that means, the tail is not x AC of the tail is 0 and CL from tail is 0.

So, by substituting that you figure out the neutral point = x AC of the wing, as we discussed the neutral point of the aircraft is similar to that of aerodynamic centre of the wing. In case of; if there is no tail in case of wing alone configuration, neutral point is nothing but the aerodynamic centre of the wing, right that is the reason why we are trying to shift CG of the wing by adding the dead weight to the aerodynamic centre, so that to make it neutrally stable, right that was the reason, this is fine.

But what is this neutral pint, let us say if I have a mass or if I have an object, how do you find the CG of this mass, I take a reference right, say I take a coordinate system tangential to any one of this, right, so if I want to figure out the CG of the system, what I have is; this is say y axis, this is x axis, I have mass m1, which is at a distance of x1 and from here it is at a distance of y1.

Similarly, if I have mass m2 located at a distance x2 and y2, right so on and so forth, how I can find x bar CG =; or x CG = m1 x1 + m2 x2 + so on, mn xn/ mn sigma m, right, similarly y CG, fine. So, what are these m1 or say if I divide this, m1/ sigma m * x1 + m2/sigma m * where I starts from mi; I starts from 1 to n, x2 + so on, so is that not a weight, is it not a coefficient that we are weight to this particular x1, right.

We are assigning some weight to this particular quantity, which depends upon this m1, > 1 greater is the weight, weight in the sense here weightage; weightage of this x1 increases as the m1 increases. So, cannot you see a similar arrangement here; CL alpha of wing * x bar AC of wing and this is what; what is this? This is (()) (01:19:41) see, you can see a similar weightage here, right this CL alpha wing * x AC of wing + this is nothing but CL alpha of tail, but it is a modified, normalised one, right * x AC of tail/ CL alpha + CL alpha of tail that is sigma CL alpha of the total aircraft, which is CL alpha of wing + CL alpha of the tail.

But CL alpha of tail is the modified CL alpha that is the normalised CL alpha, so it is similar to that of; see it is the weighted average of CL alpha, so neutral point is the weighted average of CL alpha and CM alpha, sorry CL alpha of wing and CL alpha of tail. Now, let us say if the tail and wing are separated by infinite distance; infinite distance that means, epsilon is 0, what this becomes, eta becomes 1, because the flow at tail and wing will be same.

Because they are separated by infinite distance, right, epsilon is here, so this becomes like ST/ S * CL alpha of tail, if wing and tail are same, let us say if I am using same wings, that means, this particular ratio becomes 1, all I have is CL alpha of tail say that depends upon the aerofoil of CL alpha of the tail, if I take the same area wing, this become CL alpha of tail * x AC of tail like m 2 * x2 and this one is m1 * x1 kind of; CL alpha of wing * x AC of wing.

So, divided by CL alpha of wing + CL alpha of tail, if you are considering equal area of tails; tail and wing, it is the weighted average of CL alpha, the moment you decide the distance between wing and tail, and the corresponding aerofoil of the wing in the planform geometry, you have fixed your neutral point, now you have to make sure that your CG is ahead of this neutral point.

Why ahead of this neutral point, we will discuss, do you appreciate this fact, see here this is like a weighted average of CL alpha of wing and tail, lifting characteristics of wing and tail, it is the weighted average of it.

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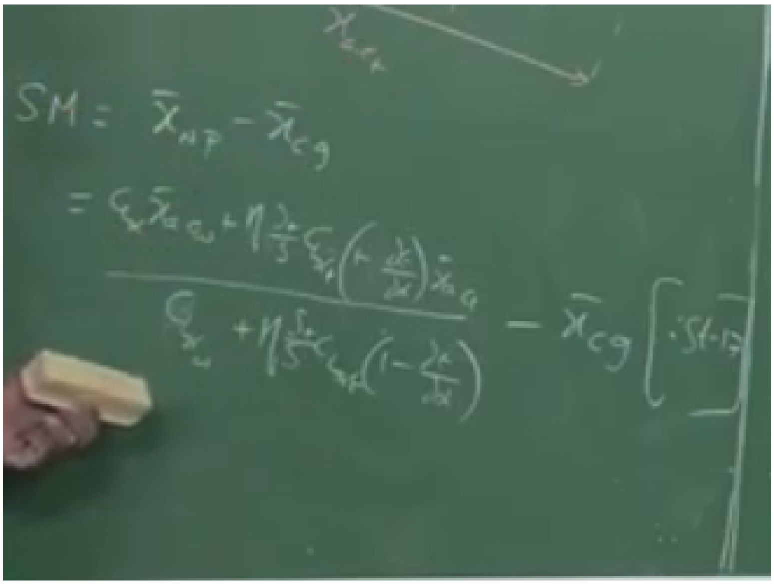

So, what is static margin? It is the x bar NP – x bar CG, what is x bar NP, okay CL alpha of wing * x bar AC of wing + CL alpha of eta ST/S CL alpha of tail 1 – dou epsilon/ dou alpha * x AC of tail / CL alpha of wing + eta ST/S CL alpha of tail * 1 - dou epsilon/ dou alpha – x bar CG, this is from neutral point, ST 17.

Since ST 17, substituting x NP with ST 17, equation number 17 first stability part, right, is it correct.

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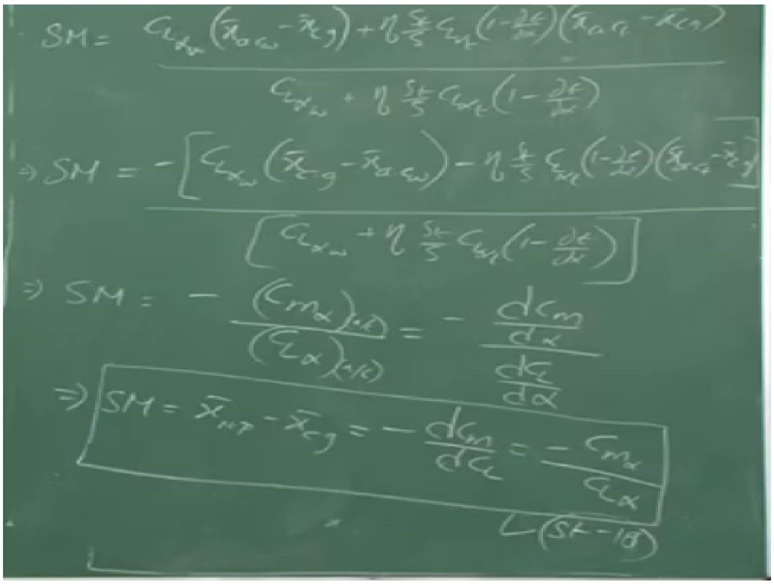

Now, this static margin = CL alpha of wing * x bar AC of wing – x bar CG + eta ST/S * CL alpha of tail * 1 – dou epsilon/ dou alpha x AC of tail – x CG / CL alpha of wing + eta ST/S * CL alpha of tail * 1- dou epsilon/ dou alpha, right, this implies static margin = okay, can I take minus common, minus out from this equation, this is x bar CG – x bar AC of wing, right + eta ST/S CL alpha of tail 1 – dou epsilon/ dou alpha x bar AC of tail – x bar CG / CL alpha of wing + eta ST/S * CL alpha of tail 1 – dou epsilon/ dou alpha, am I correct.

So, I we can refer back, this particular quantity or the numerator is CM alpha; CM alpha of the entire aircraft, so this static margin = - CM alpha / this is the total lift CL alpha, lift curve slope of the aircraft, right, CM alpha/ CL alpha for this entire aircraft, this is for the entire aircraft, this = - dCM/d alpha/ d CL/ d alpha, this implies static margin, which is x bar NP – x bar CG = - dCM/dCL, okay, which is –CM alpha/ CL alpha of the entire aircraft.

Now, for CM alpha to be negative, for example, we need CM alpha negative, right, let us say this is your stability 19, am I correct, 18, stability 18.

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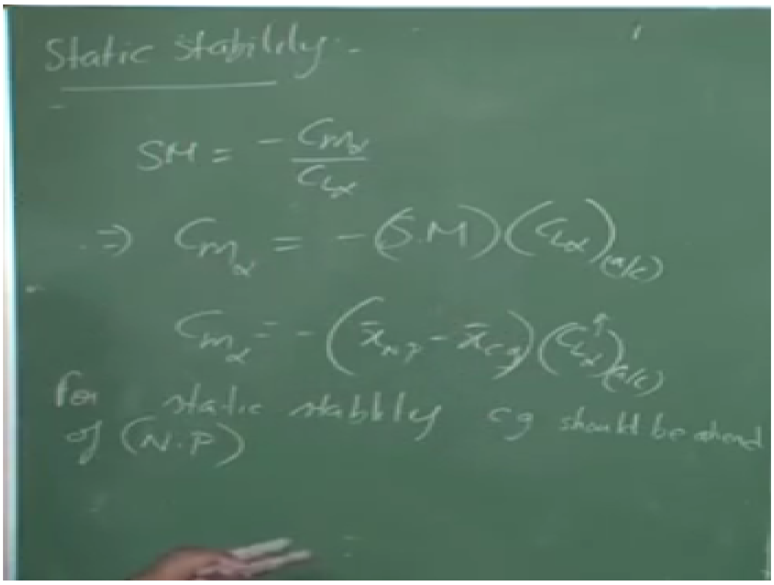

So, using this, the static margin = - CM alpha/ CL alpha, this implies CM alpha of the aircraft should be – the static margin * CL alpha of the entire aircraft, right. So for CL; for statically stable aircraft, we need CM alpha to be negative, right, so if this quantity has to be negative that means the static margin should be positive and CL alpha of the aircraft of course, is positive, right, what is static margin?

X bar NP – x bar CG * CL alpha of the entire aircraft, so this is the positive term, so for this CM alpha to be negative, the x Np – x CG should be positive that means, x NP should be larger than x CG which means, the neutral point should be behind the centre of gravity or say the CG should be always ahead of the neutral point, so that is the reason why, so for a positive static margin, you have CG is ahead of neutral point, right.

CG should be ahead of NP even for wing alone, the NP is nothing but aerodynamic centre of the wing that is why we need CG ahead of the aerodynamic centre of the wing. The questions; the question final exam paper is limited to the topics that we have discussed till date, so that second part of this course will be offered in the next semester that talks about trim and then modelling to simulation of fixed wing UAV's.

Due to the time constraint, we are not able to complete the internal task, so let us span it next series of lectures, so I will be offering them in the next semester, coming semester, that talks about trim and say modelling and simulation yeah and analysis as well, at the same time we will solve few example problems there that covers the entire design process of UAV, so meet you soon.