## **Design of Fixed Wing Unmanned Aerial Vehicles Dr. Subrahmanyan Sadrela Department of Aerospace and Aeronautical Engineering Indian Institute of Technology - Kanpur**

## **Lecture – 02 Measurement of Flight Velocity and Standard Atmosphere**

Dear friends. Welcome back. Now in the previous lecture, we saw flight test of a fixed wing UAV. It was a flying wing which weighs about 1.5 kgs, 1.6 kgs approximately and with a span of 1.5 m, right. So what do you think is the velocity of that flight? Or in other words, how to measure the velocity of that flight? Right. What are the sensors that we require to measure the velocity and the same time, the altitude, right?

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So what are the various methods that we can measure this velocity? One simple thing is, install an onboard GPS, right. Second thing is can have an IMU, inertial measurement unit which measures the accelerations, say linear accelerations and integrate this linear accelerations to get the corresponding velocity. And say, if we have a radar where ground station, right, by means of radar.

If you have a radar at the ground station with which you can measure the position as well as the velocity. And then we also have some sensor called pitot and static tube, right. See we are not going to argue about how accurate these measurements are but these are the various sources from which we can measure the velocity, right. Now in this lecture, let us discuss about how this, how to use this pitot-static tube to measure the velocity, right. In the first place, what is this pitot tube and static tube?

Consider a hollow cylindrical tube, right with an open mouth at one end and the closed mouth at the other end, okay. Now drill a small hole here to tap the pressure reading and connect this to a pressure sensor. Say this is my pressure sensor, right. Okay. Now let us take this setup and place it in a wind tunnel. These 2 lines represent the boundaries of the wind tunnel test section, right. Now run this wind tunnel.

So wind tunnel is a device in which we can produce the required velocities, right. We can produce variable velocities of flight, right. Here the affect is similar, right. Either you are moving in a static air or you are holding a body and you are blowing the air, right. In both the cases, the affect is same. Now let us run this wind tunnel to produce a velocity V infinity. Let V infinity be the free stream velocity here.

Now let us consider point A, right. Initially, what happened as soon as you run this wind tunnel, the air will fill this tube, right, this entire apparatus. Now once the tube is completely filled, this pitot tube, right. I name it as pitot tube, okay. Let us assume a fluid particle which is coming from point A. So since it is in the free stream, the velocity of that fluid particle will be V infinity. V at A is V infinity.

Let us assume this fluid particle is brought to rest at point B isantropical, right, which is reversible adiabatic process. No addition of feed or losing the heat, right from this setup. During this process, there is no addition of heat or lose of heat. Now since we brought the molecule to rest at this particular, fluid element to rest at this point B, so the corresponding velocity here will be 0, right. Let P0 be the velocity, pressure measured at this point B, right.

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Now from the Bernoulli's theorem, we have  $P+1/2$ rho V square=constant, right which is derived from the law of conservation of energy. Since we are not removing or adding any energy here, we can apply this Bernoulli's principal across this points A and B, right. So the pressure measured at point A BPS, right, so PS+1/2rho V infinity square is the total pressure at point A=total pressure at point B where P0 be the corresponding pressure measured by this pitot tube at point B, right.

And what is the velocity? 0, right. So this P0=Ps+1/2rho V infinity square. So this particular pressure corresponds to static pressure and the pressure due to flow or velocity is known as dynamic pressure, right. Where rho is the density of fluid, right. Now this is the total pressure which is the summation of static pressure and the dynamic pressure. Here the pressure that you measure is ultimately the total pressure here, right.

Now if you want to find the velocity of the flight vehicle, we can rearrange this equation. Say this is my equation number 1. This is 2. Rearranging equation 2, you get, you can measure the velocity by measuring the differential pressure and the density, right. So from this equation, I will get the velocity in m/s, the units of rho are kg/m cube and P is in Newton/m square, right. Now what are the quantities that I need to know if I want to calculate V infinity, right? V0, PS and rho.

We witnessed that P0 is measured by this pitot tube. How about PS? What exactly is the static pressure? So what is air? It is a mixture of gasses, right. So we are in atmosphere which is a mixture of gasses and gasses are characterized by its, is a state of matter which is loosely packed and character is by its random motion, right. Now due to its random motion, these gasses will exert a normal pressure, right which is the rate of change of momentum which happens across the surface, right.

So this pressure, static pressure if you want to tap, you need to measure this rate of change of momentum normal to the surface, right. Now for this case, see here at point A, this pressure is known as static pressure, right which we need to capture normal to the surface, right. Let us say if I take another cylindrical tube which is also hollow but with a closed mouth, okay.

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This is with the closed mouth. And again I make a small drill at this end and tap the corresponding pressure by means of this pressure sensor. Fine. Now if I have to capture this static pressure, what I need to do is to drill the holes along the surface or on the circumference of this tube, right. So I can tap static pressure by using this sensor, using this setup. This particular tube that helps you to capture this static pressure is a static tube.

A combination of this pitot and static tube can give you a differential pressure, right. So once you have a differential pressure and density, right. But we test that this UAV is flying a different altitudes, right. So we need to know whether this density is changing with altitude or not. It is important to know that, right. So we will look whether the density changes with altitude or not, right?

The answer is standard atmosphere will go into the topic. Yes, so now if I can make this pitot and static tube coaxial, right. So it will look something like this. If you attach this setup to your UAV, you will be able to measure the static and total pressure from which you will be able to find out the corresponding velocity of flight. Either you can use 2 separate sensors or you can have a single sensor with differential which can measure the differential pressure, right. Here what you get is PS, here P0. This is your V infinity. Now let us have a close look at this pitot-static tube. **(Refer Slide Time: 12:44)**



See this is one VTOL UAV which we are right now working on. This was built by our team members and I do not know whether you are able to see this pitot tube or not, but I will try to make it clear, right. So this is your pitot tube and there is a coaxial tube here where you have the holes on the surface with a closed mouth that is your static tube.

Here you can take the static output as well as the total output and you connect it to the corresponding sensor, right. So if you have to accurately calculate the velocity of flight which I can use it at a later stage to control the UAV, right. So I need to know what is the corresponding density at that particular altitude. Now let us look at how to find density at a particular altitude.

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So what is standard atmosphere? It is a mathematical model to estimate variation of pressure, density and temperature with altitude, right. Now we have defined something called altitude here. So there is a need to define it more precisely, right. So what we use as altitude in standard atmosphere is a geometric altitude. Let us consider an UAV, right which is flying at a height h, right which can be directly measured from the UAV to the surface by means of the tape, right.

So if you can able, the altitude that can be measured directly, right from the aircraft to the sea level, mean sea level or to the ground, right, surface. So that is the geometric altitude. So there is another altitude as well, right. If you want to, let us say if there is a satellite or another planet, right. If you want to find out the distance between the 2 planets, then we have to talk in terms of their center of masses, right.

Let us say there is a planet revolving around this, satellite revolving around this or a moon revolving around the earth, right. Now if you want to talk the distances between them, we have to talk the distances between their center of masses, right. In that case, what happens is, what we need to talk is with respect to their center of masses. Let R be the radius of earth, right and the distance between these center of masses is ha, right which is known as absolute altitude.

This ha, so ha=hG+R, right. And now, so researchers have performed sounding rocket as well as

hot air balloon experiments to measure the temperature by varying the altitude, right. **(Refer Slide Time: 18:26)**



So this typical plot represents how the temperature varies with altitude. It continues, right. So it is up to say 11 km, right. Initially the x axis here is temperature in Kelvin, right. Y axis here is altitude, h or hG you can say, right. So this is in km, okay. So initially it is observed that there is a change with respect, temperature change with respect to altitude. After a while there is a constant temperature regions and again there is a gradient regions, right.

So as you can see this is like change in temperature with respect to altitude here. Although the altitude is, in fact, we are varying the altitude here. It was supposed to be plotted on the x axis but altitude is, it is apt to plot along the z axis, right here, vertically. So that is the only reason why it has been plotted in this way, right. So the slope of this curve is defined as lapse rate, right. So which is again dT/dh, not dh/dT, right.

It is in terms of Kelvin/km or m, right. So there are some regions where the temperature is changing with altitude, right. Those regions are known as gradient layers. The regions where the temperature remained constant with altitude are known as isothermal layers. So this goes up to, first gradient layer is up to 11 km and the first isothermal layer is up to 25 km and then up to 47 km to 55 km and so on and so forth, right.

So we are not interested in this. Why? Because we are talking about atmosphere for a flight vehicle and we know even the commercial aircraft will fly up to 11 km altitude, right. So what we are more interested is in this region. So this slope is defined as lapse rate, is also known as, a is also known as lapse rate which is dT/dh and the units were Kelvin/km here, right.

So at sea level, it is 288.16 Kelvin. This is 216.66, this corresponds to 216.66 Kelvin, right. So with this information, can we reconstruct or can we estimate density and pressure variation with respect to this altitude. Can we estimate? That is the idea of the standard atmosphere, right. So you will be able to estimate. Now the basis of the standard atmosphere is a hydrostatic equation, right.

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Consider an infinitesimal fluid element with rectangular cross sections, right, rectangular faces in unit cross sectional area. Each face is having unit cross sectional. Let this face be, let us label this as A and the top face be B, right. Say these 2 faces, it has a unit cross sectional area and these 2 faces are separated by distance dh, right. So further since it is a fluid element, there will be forces acting on it, right.

So pressure, because of the other fluid elements present nearby. So the force here is due to other fluid elements is P\*. **"Professor - student conversation starts"** (()) (23:54) Yes. **"Professor student conversation ends."** Let us say on bottom face it is P\*cross-sectional area is 1, P\*1, right. What about this? Let this, let the force acting on the face B is P+dp, right. Further assume that this fluid element is at rest, okay.

So when we say it is at rest, it is at equilibrium, say the forces, this is your z axis and this is your x axis. So the force is acting along (()) (24:35) to z, I mean along this z are positive and acting opposite to this, I mean, yes. So what is this Fz here?  $=0$  that implies  $-P+dp$ , right+P+ there is a weight of this fluid element, right which is acting downwards. So that weight is mass\*acceleration due to gravity and since we are talking about a fluid element, it is apt to talk in terms of density and volume, right.

Let rho be the density and what is the volume of this fluid element? **"Professor - student conversation starts"** (()) (25:24). Yes. **"Professor - student conversation ends."** Volume of this fluid element is, dV is 1\*1\*dhG, right. This is the volume of this fluid. Rho\*dhG, -g\*rho\*dhG=0. This implies dp=-rho gdhG. Say this is my equation 1. But if you observe here, what we are interested in? Like coming back to the question like why we are doing this?

We want to know how the density is varying with altitude, right. So if I integrate this equation, I will get to know what is the variation of pressure with respect to altitude given how rho is a function of altitude. But we should not miss that g is also a function of altitude, right. Yes. So what we need to for the sake, in order to ease this integration. Otherwise, let us take one more step, right.

We have equation of state, right. What is equation of state, which is valid everywhere. P=rho RT. Divide these 2 equations. So dp/P=-gdhG/RT, right. Now if I integrate this, I will get variation of pressure with respect to altitude but the issue is, there is a change, variation of temperature with respect to altitude. Now that we can figure it out from this gradient layer and isothermal layer. But how about these 2 parameters, is g is also a function of altitude here, right.

Now we have to define new altitude called geopotential altitude. In order to ease this integration, let us now define a new fictitious altitude called geopotential altitude, right. So what is that? So dp, the same equation here, equation 1, can also be written as -rho g0dh. Say this is my equation number 4, if I name it as 3, right. So this also represents the same pressure, right at that point. So equation 1 and 4 represents the pressure at the same point.

But what is the difference here? In order to ease the integration, we have defined a new altitude called geopotential altitude dh where h is the geopotential altitude which physically does not exists. It is a fictitious altitude, right. Since 1 and 4 represents the pressure at the same point, we can equate these 2, right. To figure out the relation between real altitude and the fictitious altitude, let us equate 1 and 4, right. From equation 1 and 4, we have gdhG=g0dh. This implies  $dh = g/g0dhG$ , correct. Now what is  $g/g0$ ? I am erasing this part.

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What is  $g/g0$ ? So let us go back to this definition of absolute altitude, right. Say we have our moon here, moon, okay. This is our earth and moon or you can also assume a satellite. There is a satellite revolving around this earth, right. Now we have defined absolute altitude which is a sum of geometric altitude and the radius, hG, right. So according to Newton universal law of gravitation, the force of attraction between these 2 bodies is directly proportional to the product of their masses where m is the mass of the satellite and capital M be the mass of this earth, right.

And inversely proportional to the square of the distance between them, right. That is ha square which is hG+R square, right. So this equals to GmH/hG+r whole square, right. Am I correct? **"Professor - student conversation starts"** Yes. **"Professor - student conversation ends."** Now let us bring the satellite on to the surface of this earth, right. What it becomes? Mg=GmH/R square directly. Let this be g0. This equals to,  $g0 = GM/R$  square, right and  $g = GM/R + hG$  whole square. So we have equation number 6 and 7, right.





So if you divide this equation 6 and 7, what you have is, yes, this implies like, 7/6 implies g/g0=R square/R+hG whole square. Now substitute this g/g0 in this equation. What you have is dh=R square where R+hG whole square\*dhG. Now integrate between your altitudes of interest. Say from 0 to h, and the corresponding limit for right hand side is 0 to hG.

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So by integrating that, what I have is h=R square\*-1/R+hG from 0 to hG, right. So this equals to

R square/-1/R+hG-1/R. Am I correct? So h=, sorry +.  $R/R + hG * hG$ . So this is the relation between fictitious altitude and real altitude. This is my equation number 8. Now why we are deriving this? Since we want to ease this integration, we have converted the actual hydrostatic equation which is in terms of geometric altitude to a geopotential altitude, right, to an equation with geopotential altitude.

Using this step 4 in equation number 3, what we have  $dp/P = g0*dh/RT$ ,  $-g0$ , right. Now g0 is constant, right. It is on the surface of the earth which is 9.81 m/s with the average value of g0, right. Now if I integrate this equation 9, so I will be able to figure out how the pressure varies with altitude and then we will see how the density varies with altitude, right. Now let us consider this layer, gradient layer, first layer, right.

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So for a gradient layer, right. So dp/P, integrating it from P1 to P2=-h1 to h2 g0/RT\*dh, right. Now from the gradient layer, we have lapse rate which is defined as dT/dh, right, okay. So what is dh?  $dT/a$ . Using this, substituting this here, what I have is h1 to h2 g0/aR\*dT.

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So corresponding integration limits will also change to T1 to T2, where T1 be the temperature at altitude 1, T2 be the temperature measured at altitude 2, right. Now in this equation if you see a is constant for a particular gradient layer, right. R is constant which is, R=287 J/kgK, right and a for this first layer is about -6.5, the average value is -6.5 K/k, this is the change in temperature. Once you move, if you change 1 km altitude, like if you move up, if you travel 1 km vertically up, then there is a drop of 6.5 K.

So now let us integrate this equation. What you have is ln of  $P2/P1 = -g0/aR^*$ ... What we have done is RT, RT right. This is R\*T. So please make a correction. Here it is R, aR\*T, right. So what we have here is g0/aR\*ln of T2/T1. This implies P2/P1=T2/T1 raise to the power of -g0/aR, right. Now how to find out, so now we got the variation of pressure, right, with altitude. So if you know the lapse rate, you will be able to find out what is T2 and if you know P1, that is let us say if you are taking sea level condition, you know sea level condition, values at sea level, right. P is 1 atmosphere.

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56 k

Here what is at STP, standard temperature and pressure is 1 atmosphere 1. **"Professor - student conversation starts"** 1.01325. **"Professor - student conversation ends."** 1325\*10 to power of 5 N/m square or Pascal. Rho=1.225 kg/m cube and temperature is 288.16 K, okay.

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We know the equation of state, P1=rho 1 RT1 at an altitude h1. We can also relate this, right and we also have equation of stated altitude h2, right. Now dividing these 2 equations, may be 10, 11. **"Professor - student conversation starts"** It was 10. **"Professor - student conversation ends."** 11 and 12. Dividing these 2 equations, you have P2/P1=rho 2 T2/rho 1T1, okay. So what is rho 2/rho 1, is P2/P1\*T2/T1=T2/T1.

So with the help of this equation, so if 10, right. T2/T1 raise to the power of -g0/aR+1, right, okay. So rho 2/rho 1=T2/T1 raise to the power of -g0aR-1. This is your gradient layer relationship for density variation with altitude, right. So this is your pressure variation with altitude and density, this equation number 13 talks about density variation with altitude, right.





Now for isothermal layer. So coming back to this equation dp/P=-g0/RT\*dh, right. Coming back to this equation. In isothermal layer, we observe that temperature remains constant with altitude, right. So what we can do? We can consider it as a constant. T here is no more a variable. We can treat it as a constant and integrate this to get ln of P2/P1 from P1 to P2, from h1 to h2,  $=$ g0/RT\*delta h. Where delta h=h2-h1. So P2/P1=e raise to the power of -g0/RT\*delta h.

So by using this relationship again here, what we can see  $P2/P1=r$ ho  $2/r$ ho  $1*T2/T1$  where T2=T1 in this case, that means P2/P1=rho 2/rho 1 which is equal to rho 2/rho 1 for isothermal layer. So if you know the initial conditions and the change in the altitude and the corresponding temperature of the isothermal layer, you will be able to find out what is the pressure here, right. But we are interested in finding, so mostly our flight is constraint to, we are looking for flight in the gradient layer, right.

So let us solve few problems related to like figuring out the velocity as well as, yes, as well as pressure for this gradient layer, right, okay.

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Let us rewrite these equations for gradient layer and isothermal layer. For gradient layer, we have P2/P1=T2/T1 raise to the power of  $-g0/aR$ , right and rho 2/rho 1=T2/T1 raise to the power of  $-g0/aR+1$ , okay. These 2 belongs to gradient layer. And for isothermal layer, we have P2/P1=e raise to the power of -g0/RT\*delta h, right. So the same equation stands for rho 2/rho 1, right. So where delta h=h2-h1.

So what are these T1, T2 and h1, h2 for gradient layer and isothermal layer? So the sea level conditions are P1, T1 and rho 1 for gradient layer. Whereas for each isothermal layer or each gradient layer, the end point of this first gradient layer will be the starting point of the isothermal layer. So you have to consider P1, rho 1, T1 for isothermal layer at 11 km, right. If you want to calculate these thermodynamic properties at say 20 km altitude, then you have to consider P1, rho 1, T1 at 11 km only for isothermal layer.

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Now let us quickly solve 1 or 2 examples. Example 1, consider the fixed wing UAV flying at an altitude of 5 km, right with a velocity say, velocity of 30 m/s, okay. Find the corresponding total pressure measured by pitot tube, right? So here there is an UAV, fixed wing UAV flying at 5 km altitude and cruising at a speed of 30 m/s.

Now we have to find out the corresponding total pressure measured by this pitot. So how do we do this? So as we know 5 km corresponds to gradient layer here, right. Now what is the total pressure, =static pressure at 5 km+dynamic pressure with density at 5 km altitude and the velocity of 30 m/s. Given V=30 m/s, right, h=5 km. So here which in this case is delta h, hG we have to note it down, right.

Now first we need to find out what is the static pressure at 5 km and density at 5 km altitude. Now by using this gradient layer equations, before using that, what we need to do? We have to convert this delta hG to delta h, right. How we are going to do that? We have delta h=R\*hG/R+hG or h=R\*hG/R+hG because h1 is 0 in this case, right. So R here is approximately 6400 km.

Now at 5 km altitude, what is delta h? How to find out geopotential altitude for the corresponding? \*hG is, hG is what? 5 km.  $5*10$  power  $3/6400+5*10$  to power 3, right. What is delta h? Or h is equals to how much? h is approximately 4.9961 km, right. So what is the

difference between this hG and h at 5 km altitude? Hardly.

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What is the difference between hG and h, right? 5 km. 5000 m-4996.1, right, which is 3.9 m, right. So the difference is not much. It is just 4 m of difference, right. And so the result will also be not much affected by whether you convert this to delta h or delta hG, right. Now P2= or P at 5 km=P at sea level or say P at sea level, yes. PS is P at sea level\*T2/T1 raise to the power of  $-g0/a*R$ , right.

What is P at sea level? 288.16 K, sorry. So the pressure at sea level is 1 atmosphere 1.01325\*10 power 5 Pascal\* what is T2? Yes, how to find T2? We have lapse rate for this first layer which is  $-6.5*10$  power 3 K/m=dh/dT, right, dT/dh=T2-T1/h2-h1. This implies T2=T1+a\*delta h. So what is T2 in this case. T1 is  $288.16+$ , a here is  $-6.5*10$  to the power of  $-3$ .

Please correct here it is 10 to the power of -3 K/km\*, delta h is 5\*10 power 3. So this turns out to be... **"Professor - student conversation starts"** 288.1275. 288.1275. Kelvin. Kelvin. **"Professor - student conversation ends."** So now substitute this value here, 288.1275/288.16 raise to the power of -9.81/-6.5\*10 power -3\*287 J/kgK, right.

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So if you solve this equation, what you get is static pressure at 5 km which is equals to 53.75 KPa and the corresponding density at this altitude is, what you can do is simply by, you know equation of state which is P=rho RT. P at 5 km=rho at 5 km R $*T$  at 5 km. Now you can directly solve this since you have P5 and T5. Otherwise you can also use this equation like rho 2, rho at 5 km/rho at sea level=T at 5 km/T at sea level, raise to the power of -9.81/-6.5\*10 to power of  $-3*287+1$ , right.

You can solve from either of this equations to get density at 5 km which is equals to 0.7324 kg/m cube. Now that you have the density as well as the static pressure at 5 km altitude, so by substituting these 2 values in this equation and for a given velocity, you can find out the corresponding total pressure measured by the pitot tube.

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So what do we get, P0=P at 5 km+1/2 rho at 5 km\*, what is V? V infinity square which is, yes, V 30 m/s, right. So what is P at, is 53.75 KPa+0.5\*0.7324\*900. So this is equals 54.08 KPa. So this is the pressure measured by the pitot tube which is installed in this UAV, right, which is flying at 5 km altitude at a speed of 30 m/s. Now take another example. Example 2. Determine the cruise velocity of a wing alone UAV flying at an altitude of 10 km, right.

The pitot tube measures total pressure of 26.37 KPa. So in the first question it was like we need to find out the total pressure for the corresponding velocity, right. Here so if we have the total pressure, what will be the velocity of flight, right? So you can easily do it, right. Is it? But the difference is, difference here is, we are flying at 10 km and still we are in gradient layer, right. **(Refer Slide Time: 01:02:13)**



So here V at, V velocity=square root of twice the differential pressure, right at 10 km, density at 10 km. Now again consider the gradient layer equations where pressure at, static pressure at 10 km, = static pressure at sea level, right, \*T2/T1 raise to the power of -g0/aR. So the T2 here is T1+a\*delta h which is T1 is 288.16 K+, what is a -6.5\*10 power of -3\*, delta h is  $10*10$  power 3 km, okay. T2 is 223.26 K, right.

So static pressure at 10 km is 1.01325\*10 power 5\*223.26/288.16 raise to -9.81/-6.5\*10 power of -3\*287. This equals to 26.19 KPa. Okay. So we got the pressure, static pressure at 10 km. Now let us find out the density at this altitude, right.

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 $26.19k$ 

So we have from the equation of state, pressure at 10 km=density at 10 km  $R^*T$  at 10 km. So we can easily find out what is the density at 10 km from here. P at 10/R\*T at 10=0.4087 kg/m cube. Now substitute these 2 values here, like we have, P0 is given to us and we have P at, pressure, static pressure at 10 km and we have density of air at 10 km altitude.

Now if we can substitute in this equation, twice 26.37-26.19\*10 power 3/0.4087. This turns out to be 30 m/s approximately, right. So that is the velocity at which this UAV is flying, if the pitot is measuring 26.37 KPa, right and it is mentioned that the UAV is flying at 10 km, so the velocity, the cruise velocity of this UAV is 30 m/s.