Design of Fixed Wing Unmanned Aerial Vehicles
Dr. Subrahmanyan Sadrela
Department of Aerospace and Aeronautical Engineering
Indian Institute of Technology – Kanpur

Lecture - 18
Effect of Variation of CG Location and Static Stability

Dear friends, welcome back. So this is a wing so a symmetric wing we were discussing in the last lecture, right. Do you want to see how it is going to fly? So, does it fly? Is it going to fly or not if I through this as it? Right. So let us have some marking on this like, as I told you this is a 0.23 meter, I mean the chord is 0.23 and a 1-meter span, right. I request Himanshu my TA to help me with this. So sorry he is my Researcher.

So this is approximately 0.23 meters, you can see here this is 0.23 meters or 23 centimetres is a chord of this. So what will be the aerodynamic center for this configuration? 5.75. Yeah. So this is about; it is a rectangular wing, so the chord remains same, lambda is 0 in this case, so what you have is 2/3rd of; 0.25. Yeah, the c bar will be CR and the aerodynamic center will be 0.25 times of this? 5.75. So 23/4 is approximately 5.75 so this particular marking here is 5.75.

Say, this is my marking. What about this side? Can we mark it again? So, this again is 5.75 meters. So if I draw a line representing the mean aerodynamic chord of this. Can we do that? Can you hold this? You come this side. So this line which is connecting the aerodynamic center from tip to tip, right. So this line is going to represent the aerodynamic center of each and every aerofoil and the entire wing, right.

Now let us mark this as ac of wing, right which is at a distance of 5.75 meters – centimeters from the leading edge of this wing, okay. This is your aerodynamic center. Now, let us measure the weight, is approximately 150 grams, right. And let us throw this as it is, right. So, let us see what he is going to do. See, it just flipped. Shall we do it again? I am just throwing it. It is flipping. It is continuously flipping. Okay. Let me do a small magic or a trick say.

This is a molding clay which I borrowed it from UAV lab here. So attach this molding clay to this rod here which is connecting the wing, right. So now let us see how it is going to fly. Should

I throw it? So it is better right. So as soon as I add this molding clay to this it started gladding; it is not flipping the way it behaved in the first case, okay. Let us take another wing. So this is cambered aerofoil it is wing of 1-meter span so both of these are same planform geometry, if you can see.

So both the models are from the planform geometry but what is the difference between these two wings is a cross-sectional profile. You have used different aerofoil. So the one in my right hand with a symmetric aerofoil and the one here is with a cambered aerofoil, right. The one in my left hand, you can see it is a cambered aerofoil, right. Now I have already attached some clay here molding clay to this particular configuration and I am going to through this. And again both of them are which similar chord, let me mark down the aerodynamic center of this.

So it is again 23 centimeter chord, right. It is a rectangular wing. So the mean aerodynamic chord is again 23 centimeters, so 23/4 will be my aerodynamic center which is 5.75 approximately. So this is my, so aerodynamic center. So how about this side? So I have marked aerodynamic centers on the other side. So let us join these aerodynamic centers, okay.

This line represents your aerodynamic center. That is a cambered aerofoil, right. The wing is with the cambered aerofoil. See the gild; (()) (07:51) crucial role in this, yes. So what is the reason it started flying as soon as I attach this clay, right. What must be the reason? So we have not changed the wing, right. All we did is shifting the CG; that means say with this clay how to measure the CG in the first place? CG of this wing?

Either you need to have two balances like this, you place it on one balance here another; you have another balance at an offset, right; you take M1 X1+M2 X2/M1+M2 that is one way. Other simpler way here is our case, so above CG the model will exactly balance, right. Let us say if you take a horizontal rod and if you place this model on that horizontal rod, so above CG this model will be balanced exactly, right.

So that location you can consider it has CG location, right. So in this case about this location the model is exactly balanced, right in this case. This is the location about which the model is exactly balanced and this location is along the aerodynamic center of this wing. So if I am shifting the aerodynamic center towards the; I mean CG of the wing towards the aerodynamic center then it started gladding. So say, in this case without any clay, right.

So the CG is somewhere here, right. Almost close to this point. This is at this particular location. The CG in this case at this particular location which is half the aerodynamic center here. That means; let us say if I make CG2 ahead of this aerodynamic center, right what; to do that what should I do? I mean if I want the CG ahead of this aerodynamic center I need to add more weight here, right.

Yeah, so there is some more molding clay. Okay. Now the CG is at this particular location which is ahead of this aerodynamic center, okay. During the; if I fly if I try to throw this wing with this particular CG setting let us see what happens. There is a nose down, immediate diving, there is no gild, started diving down, right. See, with this particular CG setting this model is started diving down, right.

Let us I reduce this clay again I am bringing the CG back, right. What I am trying to do now is I am bringing close to aerodynamic center here. Now it is behind the ac. So right now I am able to balance this model about the aerodynamic center. See this is my aerodynamic center. I am able to balance this model. Now let us see how it is going to behave. It is able to fly. So we repeat this, it is a gild, right. Now let me remove this clay again.

So let us see where is the CG location of the CG; it has definitely shifted backward. So this is the point about which I can balance this model, so this is your CG location, this is your CG location right now which approximately, this is your CG location and this is half this aerodynamic center. If I throw this; so it flipped, (()) (14:40) right. Let me do it again. I am not trying anything different compare to earlier cases.

It is automatically flipping. Let us see whether it repeats or not. Whatever the angle I am trying to do but it will flip. So it cannot be to ahead or behind. What do we infer from this? It should not be to ahead or too behind, that means you need to figure out what are the limits of this CG.

And right now we do not have a tail, right. What happens if you add a tail? How far this CG traverse can be allowed? We will discuss in this lecture.

Now to do this, when the CG is close to this aerodynamic center in this case for a wing along configuration so that too for a cambered aerofoil, so we see a smooth flight, smooth in the sense; it was actually the intended path right which I was trying to or say it should be the intended attitude of the flight so it should gild that is the whole idea why I am throwing. But when the CG is behind it started flipping. Okay.

So what is the difference in between these 2? In the first case it has something called, we call it as a stable flight, the second case which is unpredictable at the same time unstable here, right; started flipping. And in the other case where the CG is ahead again it is diving down. So which check the; in one of the flight where the CG is near to the aerodynamic center we witnessed it stable fly which let us say we consider it as a reference.

And in other case we witnessed an unstable or offset flight, flight conditions right. So what are the parameters that govern this stable and offset flight conditions, fine.

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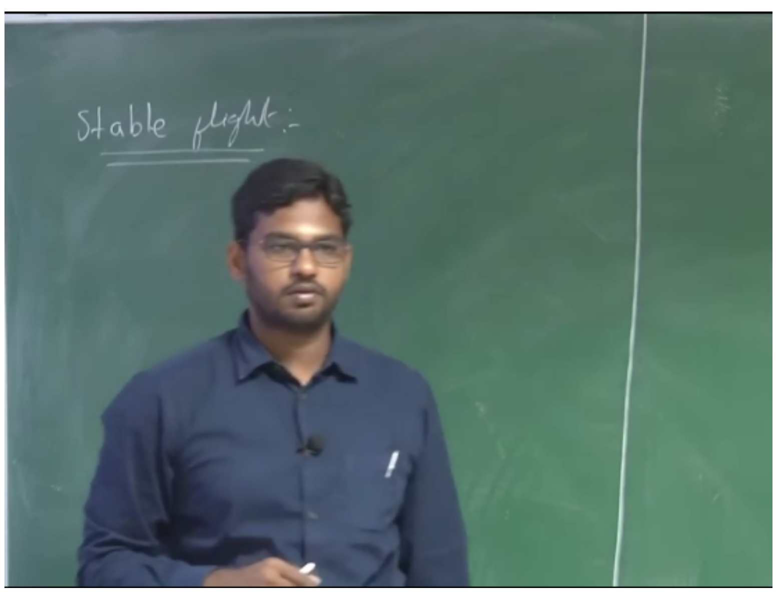

So we use something called stable, right. Stable flight. What is stability, right? So stability is a property of a system or the nature of the system to come back towards some reference that you consider when it encounters a small disturbance about the reference. So this reference in our case or in general is an equilibrium. So now we should talk about; when we say a stable flight first we need to understand what is stability of a system, right.

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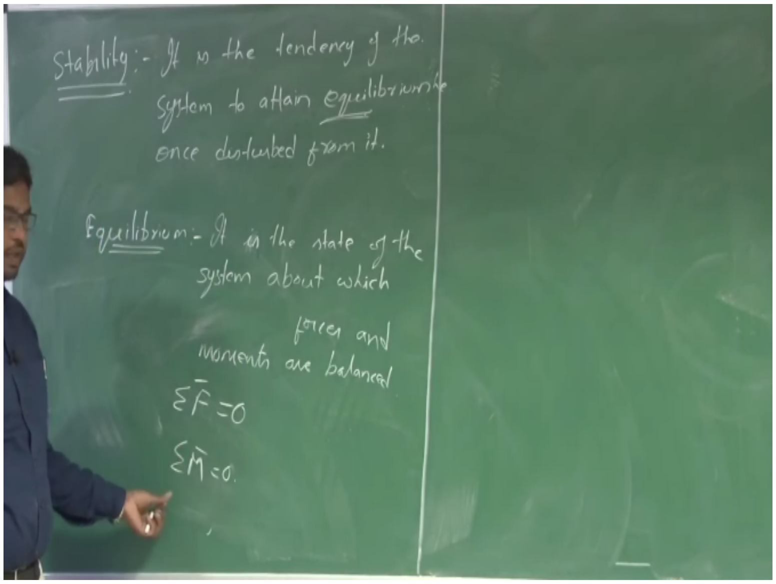

Let us talk about stability. Now with the help of the concept of stability let us see how do you design the tail and how do you find the limits of the CG traverse and what should be the size of the tail and the offset between the tail and the wing. So all these parameters come from the stability analysis of the system, okay. So what is stability? It is the tendency of the system to attain equilibrium once disturbed from it, right. What is this equilibrium again?

It is the state of the system about which resultant forces and moment are balanced, that means the forces and moments are balanced or there are no resultant forces and moment; forces and moments are balanced. That means Sigma F = 0; Sigma M = 0, right. So about equilibrium of a system the forces and moments are; resultant forces and moments are 0 there are no resultant forces and moment.

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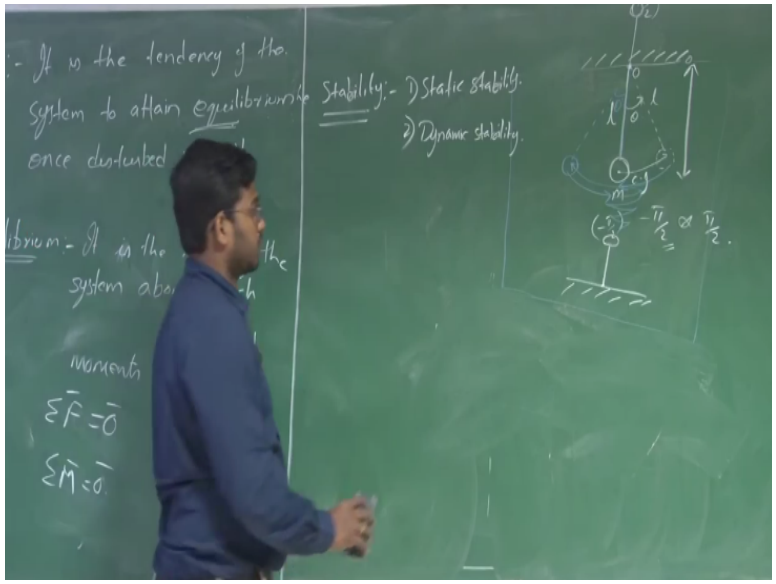

So there are many examples you might have studied in your earlier courses. You consider simple pendulum. What are the equilibrium points of it? That is -pi/2 and pi/2, right at theta = say this is your 0 inclination; this is your pi/2 and this is your –pi/2, right at this. So there are two equilibrium points of a simple pendulum, right.

About –pi/2 and pi/2. Now consider the case where it is at –pi/2. So this is your pivot point O; let m be the mass of the system. Now you displayed this mass by an angle theta where L is a length of this; this is your theta and you displayed this mass to an angle theta here. This will still remain L the length will remain constant here. So as soon as you leave it what happens this bob will try to come towards to the equilibrium this mass will try come towards equilibrium.

And because of its inertia it starts accelerating; yeah till here it will accelerate after that it will decelerate and attain an orientation theta prime where the velocity terminal velocity here is 0, right. And then it will start coming back, right and it will go forth and back and finally attains a equilibrium if there is enough friction in the hinge as well as there is a friction in the surrounding atmosphere, right. Air friction or a Fluid friction.

That means once you disturb this particular object it; from its equilibrium it will try to come back towards its equilibrium. See this particular equilibrium is known as stable equilibrium that means the system is finally attaining its equilibrium, right. So let us consider in case where; so there is no error resistance. And say this connecting rod have negligible mass and there is no friction. So you are performing this experiment in a vacuumed chamber.

So what happens is this will keep oscillator. We can say; see the system still trying to attain the equilibrium but it is it will not raise the equilibrium because there is no resistance there is no damping, right. So there is no damping in the system it is trying to oscillate about this equilibrium, right. So the system; although the system tries to come back towards its equilibrium but you cannot say the system is stable in that case.

Why? Because it is ultimately not coming back to that equilibrium, right. After a while it should come back towards that equilibrium. But it has a tendency towards to come back towards but it is not attaining the equilibrium in this case. Even in that condition we cannot say the system is stable, right. So when you talking about stability you are talking about two conditions here. One is the tendency to come back towards its equilibrium.

And finally over a time period you are looking at the behavior of the system and commenting whether system is coming back towards that equilibrium or not. So there are two phases here, I mean there are two cases for this stability. One is; so initial case is Static Stability. So at any instance you see the pendulum is here trying to come back towards its initial equilibrium position; that talks about your static in instance that it has this tendency or not. That is a static stability.

The study about such condition is a static stability. Whereas if you are looking at the time history of this motion and say how long it is going to take to come back to its equilibrium. Say if it has to come back towards its equilibrium what should I do? How should I increase the damping? Or what should be the time to half this oscillation? So the study, I mean the study related to this time history of this motion once disturbed from its equilibrium is a dynamic stability.

So dynamic stability talks about the time history of motion of the system once it disturbed from it equilibrium position. Now consider this; see we have another equilibrium point which is at pi/2. So assume that pendulum now is at the other equilibrium point which is pi/2, right. So this is similar to this case, right where you are holding a stick right. So at some time, yeah this is stable here when you make it; when you place it on the table vertically it is stable.

When you place this pen the marker vertically on this table, right; so this is also a equilibrium point because if there is an resultant force there will be a motion, right. And moments it will try to rotate resultant moment, but here the resultant force and moments are 0 that is why it is, it is adheres right now. But whether this position is stable or not? Say, if there is a small disturbance it has attain the new equilibrium.

It has not come back to its previous equilibrium, right. Let us say in the other case if I hold at the top of this marker, say if I give some external force, right it will start oscillating and coming back towards its equilibrium, so this particular equilibrium is a stable equilibrium. So the one at pi/2 is an unstable equilibrium. Because it is not coming back to its initial equilibrium once disturbed from it, the same thing here.

So this; so you see right now since I am folding it, now I can vary the friction here, frictional force so, so say if I disturb this it may go down, say if it is a proper hinge it will definitely; any small disturbance will definitely take it away from this particular equilibrium which is an unstable equilibrium here. Now what are the conditions here for which; so in this case; in case of a pendulum see if the CG is beneath the hinge point.

So this is here cg right, if your cg is beneath the hinge point it is a stable equilibrium. The equilibrium in which the cg is beneath the hinge point is a stable equilibrium. See if the equilibrium in which CG is above the hinge point is an unstable equilibrium, right. So similarly, do you have something in the aircraft? Right? But in this case the cg and the hinge point in the vertical axis, so what happen to the aircraft what is.

So we have lift from the wing; lift from the tail, right and cg acting at some other location. The hinge point and cg in the straight line here in both the equilibrium right in the vertical axis. So but in that case there is an offset where the cg as well as the aerodynamic forces acts, right. So we need to see where should I locate this cg so that the system is automatically stable, right. See here you are not controlling the system.

Once you disturb from it equilibrium there is no need for any external control, it will automatically come back towards its equilibrium. Once you; once the disturb; once; it will automatically come back towards its equilibrium once the perturbation is done; once the external force is removed from it. So here the system will come back towards its equilibrium automatically once there is no external force acting on it, right.

So another example that you may consider is, take a bowl; these examples you might have studied already, so you take a bowl stick it with the; at this location at the bottom location of this bowl, it is a conquer bowl, right. Now you displace this to a particular location here either this side or this side, it will start moving towards the initial equilibrium. So in this case the resultant forces and moments are 0;

So once you displace and leave it, it will start oscillating about this particular equilibrium, right. So it will start oscillating about and finally attains equilibrium which we can say it is a stable equilibrium, okay. So if you take a convex bowl you should place a ball here, what happens is as soon as you perturb this it will either go this side or this side, right. It will not come back towards its equilibrium. So here in a first case we are not controlling any; we are not imparting any external forces here.

It is automatically coming back towards it equilibrium. So similarly, the UAV should have the initial tendency to come back towards its equilibrium then we can say it is a stable UAV, right. So while varying the CG what we witnessed is the stability of the system is changing, right. So let us see how this stability changes as a CG varies.

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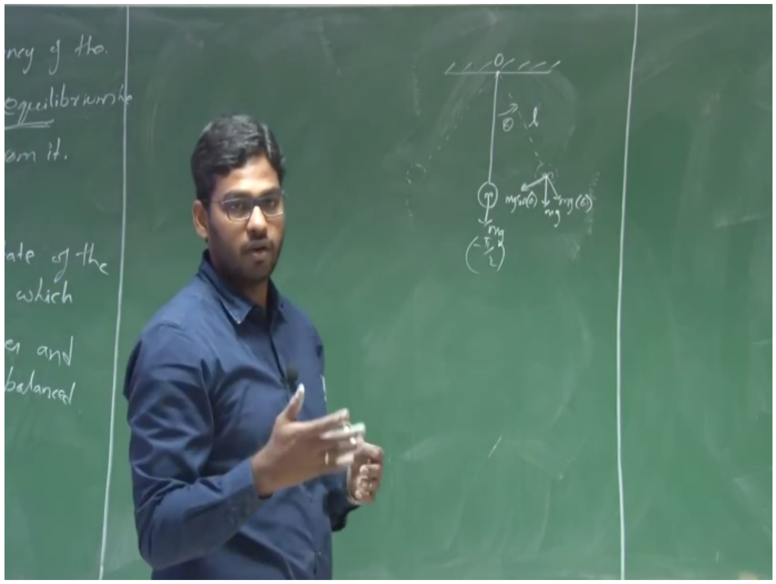

So again in this pendulum case; in this particular equilibrium location where it is –pi/2, right. So at this equilibrium point we say this is a stable equilibrium, right. If there is a friction, enough friction that helps this pendulum to come back towards its initial equilibrium. So in this particular case, we considered it as a stable equilibrium. Why? Because it is attaining its actual equilibrium. But how? What is helping? Why not in the other case? Yes, okay mg even gravity is there in the other side.

If there is a component of this mg, right; mg sin theta and mg cos theta, so this mg sin theta is what driving it towards its equilibrium right? But this force about this particular point hinge point is creating a moment, right. This is into L mg sin theta * L is a moment that is created about this hinge point; that means there is a moment that is restoring its equilibrium, that moment is known as restoring moment, right.

Now to have this equilibrium; I mean if you need to; say if a system have to be stable then there should be a restoring moment.

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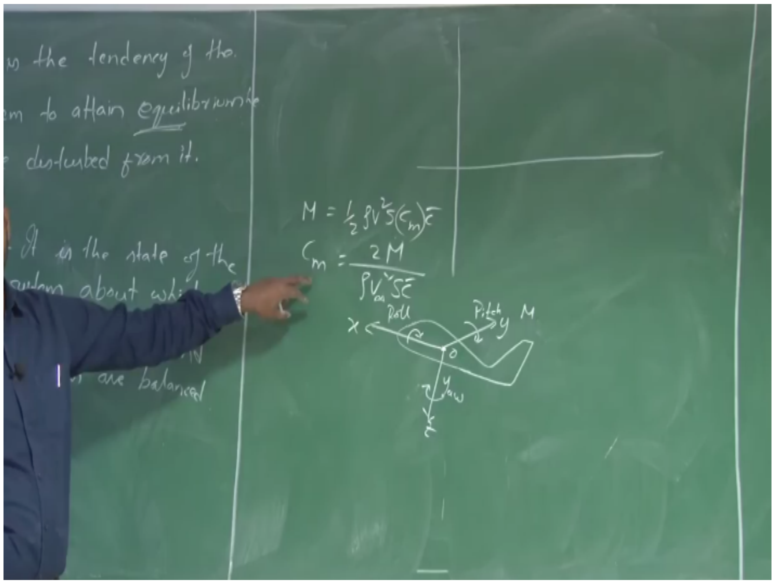

So we know aircraft, aircraft have 6 degrees of freedom here, right. We can translate as well rotate along the three access here of this body frame. So it can translate along xyz and rotate along these three access, so the rotation along the x body x-axis is known as Roll and Pitch and Yaw, right. You have Roll, Pitch and Yaw of the system.

Okay, so the pitching moment is about y-axis. Now say, let M be the moment here which is given as 1/2 rho v square S Cm*c bar; 1/2 rho v square S is a force; dimensions of the force and c bar is the dimensions of the length so force*length is a moment; it is a moment term you can say c bar and Cm is a non-dimensional pitching moment coefficient. Cm is a non-dimensional pitching moment coefficient. So Cm is twice the pitching moment/rho v square S*c bar. Okay.

Now this is the moment that we are talking about. Even in the pendulum there is a moment, right because of the force. So this is the moment because of the aerodynamics. So there in the pendulum case we have that moment due to gravity. Here we have a moment this Cm is a moment that is due to the aerodynamics here.

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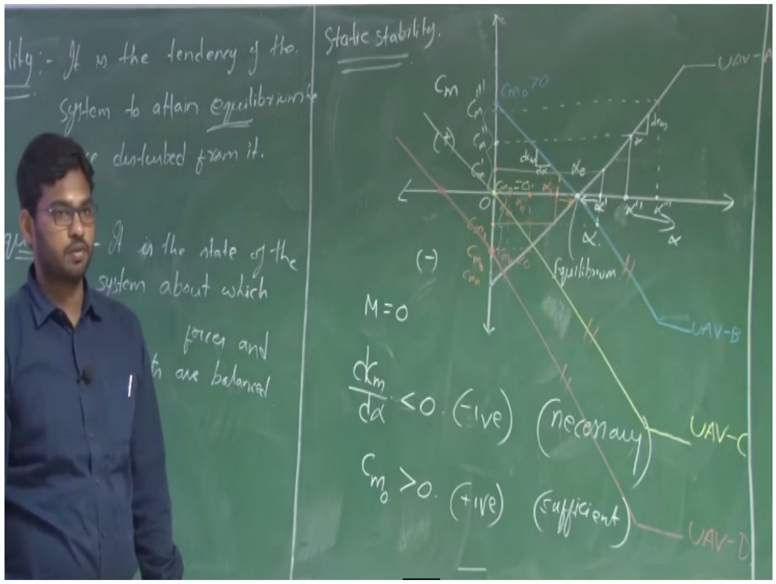

Consider the Cm variation with angle of attack; since we mentioned it is an aerodynamic moment so these aerodynamic force and moment depends upon the angle of attack, right. Let us consider this variation of Cm with angle of attack. So let us say this is for the entire aircraft or a UAV entire UAV. So say in the first case you have a UAV which is, so let us say this is you UAV-A. This is your UAV-B, right. So what is this particular point?

So for this UAV-B you have Cm variation; this blue line represents a pitching moment coefficient variation with angle of attack; so the UAV-A, this pink line represents the pitching moment coefficient variation with angle of attack. So what is this particular condition? This is a increasing alpha, okay. So this is your positive and this is your negative. What is positive pitching moment? So we discussed, right?

Right hand thumb rule, stretch your thumb along the positive axis right, positive axis in the corresponding curl, I mean the curl of your fingers will give you the corresponding positive moment. So for; this Cm is about pitching moment is about y-axis, so stretch your thumb along the positive y-axis of this UAV-And the pitch up is considered as positive and pitch down is considered as negative, right. Now what is this alpha? What is this point?

So you can see at this particular alpha you say the moment Cm is 0, that means there is no pitching moment for this particular angle of attack, right. Do you accept this? So which means the moments are 0 or a pitching moment is 0. Cm is 0 means of course M is 0. That means it is a equilibrium point. If there is imbalance in the force that creates a moment but when the resultant forces are equilibrium that has no moment here. So above this point there is not moment, right. Okay. We can consider this as equilibrium. So this is your equilibrium.

Now let us say there is a sudden increase in the gust or the wind which has increased the angle of attack. So this particular point alpha e equilibrium is perturbed by alpha prime. The new angle of attack is perturbed by an angle of attack, perturbed by wind. In the resultant angle of attack at that instance is alpha prime. So this alpha prime if I consider the UAV-A what happens because of this change, there is a pitching moment; it is a positive pitching moment.

That means the aircraft will have a nose up. So this is your positive y-axis, say this is your positive y; this your positive x and this is your positive z, so stretch your thumb along the positive y and the curl of your fingers will give you the corresponding positive rotation here. So this pitch up, pitch up motion is considered; pitching up is considered as positive; pitching down is considered as a negative.

So what; so with the increase in the angle of attack there is an increase in the moment. Initially, at this particular angle of attack where the resultant forces and moments are 0, right you have 0 moment definitely because that is the definition of equilibrium here. So as a angle of attack increase there is a change in the moment and the change is; in the resultant moment is positive which means there is a pitch up moment, right.

So initially say, see this is your angle of attack this marker represents the angle of attack, right. Sorry, V infinity and this is your angle of attack the angle between them. So because of a small change so this is a trim condition let us assume because this is your alpha e. Because of a small change there is an increase in the angle of attack, small disturbance external disturbance. So this angle of attack increases the angle of attack for an aircraft with that pitching moment curve;

Let us say that is the UAV-A, right. So this curve represents the pitching moment variation with angle of attack for UAV-A. So for such UAV what happens, so this increase an angle of attack is increasing the pitching moment that is no sub-moment; it further increases. So this further increase; due to this no sub it further increases the angle of attack, right. So that means, let us say this is your alpha double prime.

Now the increase in angle of attack due to pitch up is alpha double prime, so for this UAV you have Cm prime, Cm double prime, okay. That means there is a further increase in the pitching moment which further creates a no sub and further increases the angle of attack. So alpha double prime or triple prime will result in a Cm triple prime. This is alpha triple prime. That means you are moving away from this particular equilibrium point.

See here this is your equilibrium point you are moving away from it. Now in the other case say for the same UAV, UAV-A if there is a disturbance that has reduced the angle of attack. So initially, this is your angle of attack say at equilibrium, now this disturbance has reduced the angle of attack here. So this reduction in angle of attack for this particular UAV with this pink curve right, so will result in a;

So say this is your alpha, alpha prime in the subscript, right. So this results in a Cm prime in the subscript, okay. Now, which means your pitching moment is negative here, right. There is a negative pitching moment which is a nose down. That means this will further decrease the angle of attack. If you; the aircraft will do a nose down automatically and which eventually result in a reduction in angle of attack further.

So alpha double prime; so this will again, right decrease the pitching moment or gives a negative pitching moment, okay. So that negative pitching moment further will reduce the angle of attack, that means it will go to alpha triple prime in the subscript, so this will further create a pitching down moment. So in either case if you follow a UAV with this pink curve you will end up moving away from the equilibrium once disturbed from it.

Similarly, to that of a pendulum case where you have an equilibrium but it is at –pi/2, pi/2 sorry. If the CG is above the pivot point then the equilibrium is not stable the system is not stable about that particular equilibrium, right. So on the other hand if you look at this blue curve, so look at this blue curve that represents the variation of pitching moment with angle of attack for UAV-B. So this alpha prime;

When there is an increase in the angle of attack there is a Cm prime in the subscript that is a negative pitching moment, right. So let us say, there is a disturbance that has increased; this is my equilibrium alpha, right so there is a disturbance which has increased this alpha but with the UAV with this particular blue curve will this change in the alpha will induce a nose down moment here, Cm is negative.

So this nose down moment will decrease the angle of attack, right. That means you will start moving towards this equilibrium point. It will start decreasing the angle of attack, right. So similarly, if the angle of attack is decreased; say this is your trim angle of attack or the equilibrium angle of attack, right. If the angle of attack is decreased due to the disturbance what happens. So this reduced angle of attack will induce a nose up moment.

From this curve you can see this reduced angle of attack will induce a positive moment which is nose up that means the aircraft will pitch up and go back towards its, that means pitching up means it is increasing the angle of attack that means you are moving from here to here, right moving towards your equilibrium. So what do we infer from this discussion; that means UAV to be statically stable, right.

So this Cm versus alpha has to be negative. The slope has to be negative. Here the change in Cm dCm with d alpha is negative where as the dCm/ d alpha in this case is positive, so for unstable aircraft this change is positive; for a stable aircraft the change in pitching moment with alpha is negative, right. So dCm/ d alpha = should be < 0 or negative. And by the way what is this moment about? This moment we are talking is about the CG but due to aerodynamic forces.

So we are starting with Static Stability. So we discussed that there will be two phases of this stability static and dynamic. Initially we are arriving at the mathematical conditions for static stability. If the UAV has to be statically stable the variation in pitching moment with angle of attack has to be negative, right. The slope of this Cm alpha should be < 0, okay. So this is a necessary, necessary condition. But is it the sufficient condition?

Let us say you have the Cm variation with alpha the blue one negative. Let me draw a parallel curve for this, right with a negative slope but passing through origin here, okay. This is your Cm versus alpha. So this UAV-C, right. This is a third UAV, okay. So still, even in this case with the change in angle of attack, say this is your equilibrium here that means you are trimming your aircraft at alpha=0; 0 angle of attack.

So at this equilibrium; about this equilibrium when there is a change in angle of attack; increase in angle of attack there is a nose down moment that will take you back towards your equilibrium and vice verse. But what is the trim for this UAV? It is a 0 angle of attack. Because you cannot trim at positive angle of attack, you will get negative moment. So let us; what you call? Let us I have curve which is offset to this.

So this, these three are parallel. So this is your UAV-D. So in this case where is the trim? Is at negative angle; this is your negative alpha, right; alpha negative in this direction. So the trim is negative here; alpha is negative. That means; but we witnessed CL=CL alpha CL0+CL alpha; when you have negative angle of attack you are actually reducing the lift CL, that means you will not be able to sustain the weight at this particular angle of attack.

So it is an inefficient flight if you have it. The whole idea is to generate lift at minimum velocity and the minimum conditions, power requirement condition or the minimum what you call minimum fuel consumption, right. So although you have Cm alpha curve; Cm alpha negative even for this UAV-D but you cannot trim this at a positive angle of attack. So the Cm0; what is Cm0 here? Here it is 0; here it is positive.

What is Cm0? Cm at which alpha=0, right. So here it is positive; this is for blue curve, this is your Cm0. So for yellow curve this is your Cm0 is 0 and for this brown curve is your Cm0 which is negative. This is > 0. This is equivalent to 0. Okay. So for this; to trim a positive angle of attack I need to create an offset here, right. to create that offset for a flat plate this is how it will be; we will witness for flat plate what is going to be the Cm, right.

So in order to trim at positive angle of attack you need to have; so this is your trim at positive angle of attack. Trim is an equilibrium, right. So in order to trim at positive angle of attack you need to have positive Cm0, right. This is your negative Cm0; this is your positive Cm0; this positive Cm0 will help you to trim at positive angle of attack. So Cm0 has to be > 0; Cm alpha should be < 0 or it should be positive for sufficient. So necessary and the sufficient condition for statically stable UAV.