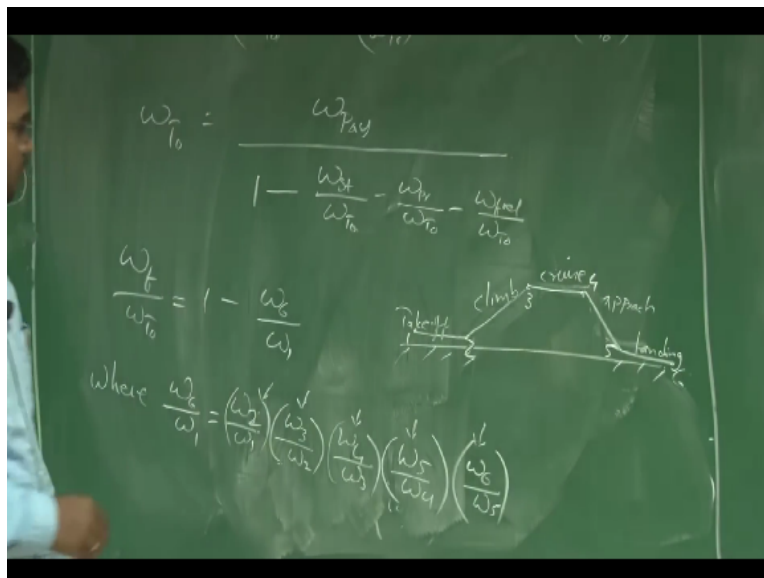


Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture – 15
Weight Estimation Contd., Electric Propulsion, Battery Sizing

Dear friends, welcome back. So in our previous lecture, we discussed about weight estimation where the total weight of this aircraft is summed up as the structural weight, as the propulsion weight, right.

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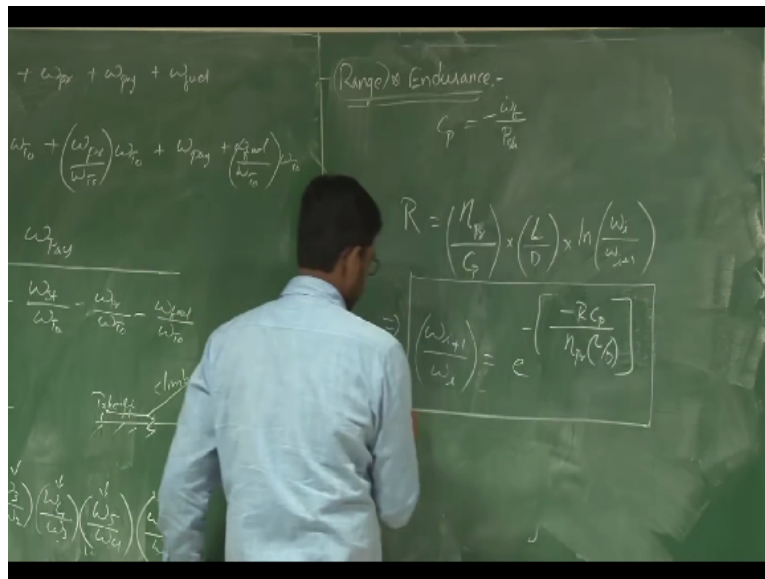


Total weight is summed up with structural weight, propulsion weight and payload as well as weight due to fuel. So we express this total weight or takeoff weight as the weight fraction, structural weight fraction+propulsion weight fraction, payload+fuel weight fraction, right. So where $W_{takeoff}$ is obtained with the help of, so $W_{takeoff}$ is obtained by rearranging this equation where in the denominator we have all the weight fractions.

And we also witness $W_f/W_{takeoff} = 1 - W_6/W_1$ where $W_6/W_1 = W_2/W_1 \cdot W_3/W_2 \cdot W_4/W_3 \cdot W_5/W_4 \cdot W_6/W_5$, right. So these are the weight fractions, weight ratios for various segments, right. So say typically we have segmented the entire mission as takeoff, climb, cruise, approach and landing. So we have segmented this entire mission based upon the dynamics during that particular phase, right.

So this weight fraction is the fuel weight that is consumed during these phases of this mission, right. Now out of which we also discussed that these were like more or less, I mean, for takeoff, climb, approach and landing, we have weight fraction from the historical database. So what they differs from UAV to UAV or the aircraft to aircraft is the cruise profile or the cruise fuel consumption which mainly depends upon the mission requirements, right or the task to perform.

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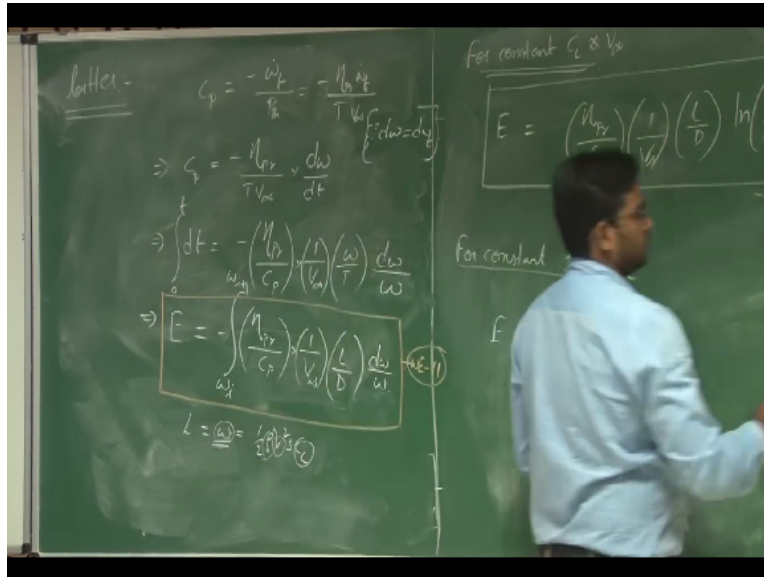
Now so in order to calculate this cruise performance, we studied about range and endurance, right. We started with range and endurance but we finished what is range. We have defined specific fuel consumption of this IC engine based systems. This is the amount of fuel consumed to produce unit power, right. So the power that we had discussed is the shaft, so this will be the shaft power.

So C_p , since it is, W_f is negative, so C_p is a figure of merit, so we will define C_p as $-W_f/P_{sh}$, right, shaft power. And we witnessed that the range or range for this propeller driven UAV is given by $\eta_{propeller}/C_p * L/D * \ln$ of W_i/W_{i+1} . So if you want to get, if you want to know the weight fraction, fuel weight fraction, then what you have is $W_{i+1}/W_i = e$ raise to the power of $-R * C_p / \eta_{propeller} * L/D$.

It is the same equation what we discussed in your previous lecture. Now let us discuss about fuel

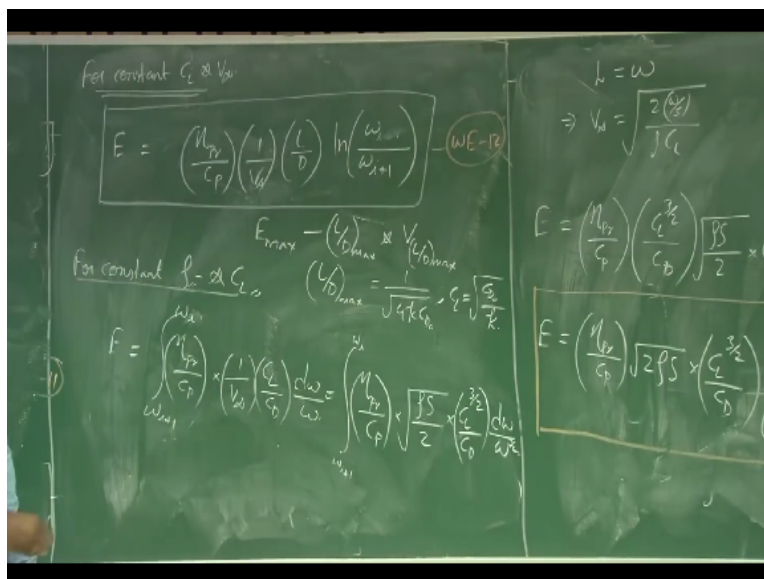
weight fraction for loitering.

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So what is the specific fuel consumption by definition. $C_p = -\dot{w} / P_{sh}$, right. This is equals to $-\eta$ of the propeller $\cdot \dot{W} / T \cdot V_{\infty}$, right. This implies $C_p = -\eta$ of the propeller $/ T \cdot V_{\infty} \cdot dW/dt$, we know that, right. So $dW = dW_f$, since $dW = dW_f$, you can write $\eta \cdot dW/dt$. So $dt = -\eta$ of the propeller $\cdot C_p \cdot 1/V_{\infty} \cdot W/T \cdot dW/W$. This implies, so in order to get the endurance, I have to integrate it over the time, 0 to t, right. So the endurance or the time of flight $= -\int_{W_1}^{W_2} \eta$ of propeller $/ C_p \cdot 1/V_{\infty} \cdot L/D \cdot dW/W$, right.

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So endurance $=$, there are 2 ways to integrate this. One like constant velocity and constant CL

climb, CL cruise, like both the lift coefficient and velocity remains constant but as the fuel is consumed, since you are having same lift and same velocity, we know $L=W$, right in cruise, which is $W=1/2\rho V^2 S*CL$, right. So one such condition in which cruise can be written is constant CL and constant V, right.

And in that case, this W is continuously changing because the fuel is getting consumed, right. The only variable that has to change is density. That means you will end up gaining altitude which is known as cruise climb, right. Just like a balloon. You do not actually climb but you keep gaining the altitude because there is an additional lift acting on the system which is greater than the weight, right.

Why? Because there is fuel consumption. We call it as cruise climb. So if you consider say assuming that density, variation in the density is allowed, right and say V and CL is constant. So these particular terms remains constant. So in that case what will be the equation. So there are 2 cases, right. Let us do, I mean, let us solve this for this particular case where density is allowed to change.

That means the aircraft can gain altitude as it cruise, right. So in that case, for constant CL and V infinity, so for that case what happens is the endurance= η of the propeller/ C_p and velocity remains same, constant, $1/V$ infinity, $*L/D$ remains constant because CL remains constant here. When CL remains constant, C_D will also remain constant. So the integral will be \ln of W_i/W_{i+1} . So this is the endurance of a UAV when, of a propeller driven UAV when you have constant velocity and CL, right.

And for constant density and CL, that means you do not want to vary the altitude. You want to have your cruise at a particular altitude and then CL remains constant. Which means the pilot has to continuously decrease the velocity as the fuel load is decreased. It is more a (()) (11:58) study, right. So what happens in that case, what will be the endurance? =integral over, so this is the equation of endurance in general, say W estimation 11. So endurance is W_i, W_{i+1}, η of propeller C of $p*1/V$ infinity $CL/CD*dW/W$. Now from, since we are talking about cruise condition, we know $L=W$.

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The chalkboard contains the following derivations:

Left side (WE-13):

$$\left(\frac{M_{pr}}{C_p}\right) \left(\frac{1}{V}\right) \left(\frac{L}{D}\right) \ln\left(\frac{\omega_{i+1}}{\omega_i}\right) \quad \text{--- (WE-13)}$$

$$E_{max} = \left(\frac{L}{D}\right)_{max} \times \left(\frac{V}{\omega}\right)_{max} \times \left(\frac{1}{\omega}\right)_{max}$$

$$\left(\frac{M_{pr}}{C_p}\right) \times \left(\frac{1}{V}\right) \times \left(\frac{L}{D}\right) \frac{d\omega}{\omega} = \left(\frac{M_{pr}}{C_p}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{L}{D}\right) \frac{d\omega}{\omega^2}$$

Right side (VE-13):

$$L = W \Rightarrow V_d = \sqrt{\frac{2W}{\rho C_l}}$$

$$E = \left(\frac{M_{pr}}{C_p}\right) \left(\frac{C_l^2}{C_d}\right) \frac{1}{2} \times (-2) \left[\frac{1}{\sqrt{\omega_{i+1}}} - \frac{1}{\sqrt{\omega_i}}\right]$$

$$E_{max} \rightarrow C_l = \frac{2W}{\rho V^2}; C_d = C_l C_{pr} \quad \text{--- (VE-13)}$$

So for this, lift=weight which in this case the corresponding velocity of glide twice the wing loading/ $\rho \cdot CL$, substitute that here. So what you have is integral ω_{i+1} to ω_i η of the propeller/ $C_p \cdot \text{square root over } \rho \cdot S/2 \cdot CL \text{ power } 3/2 / C_D$, right, and $dW/W \text{ power } 3/2$, right because the rest root over W here and $1/\text{root } CL$, so it becomes $CL \cdot CL \text{ power } 1/2$ which is $CL \text{ power } 3/2 / C_D$ and $W \cdot W \text{ raise to the power of } 1/2$ which is $W \text{ power } 3/2$, okay.

Now here we have, density is constant. We assume density as constant in this case and CL is constant, right. ρ is constant and CL is constant. If you say CL is constant, C_D also remains constant. So other than this particular term, everything else is constant in this case. So what you have here. So the endurance for this case= η of the propeller/ $C_p \cdot CL \text{ power } 3/2 / C_D \cdot \text{under root } \rho \cdot S/2 \cdot 2 \cdot 1/\text{root over } W$ from ω_i to ω_{i+1} , right.

So this will turn out to be, so the ratio, propeller efficiency $\cdot C_p$ specific fuel consumption $\cdot \text{root over } 2 \cdot \rho \cdot S$, density at that particular altitude $\cdot CL \text{ power } 3/2 / C_D \cdot 1/\text{root over } \omega_{i+1} - 1/\text{root } \omega_i$, right. This is our equation WE-13, weight estimation 13, right. So ω_{i+1} is smaller than ω_i . So this quantity is larger than this quantity, right. So this is a positive value. So everything else is positive here, right.

Now from here we generally consider for initial weight estimation. That is why we get an

estimate out of it with the assumptions with, right. We are having certain assumptions here. With that assumptions, we will get an estimate of the fuel load. So which may be off by at least 20% of the actual value, right. So to start with, we can go ahead with this assumptions and say if we want to find out W_i/W_{i+1} , all you have to do is multiply the given endurance by C_p and V infinity.

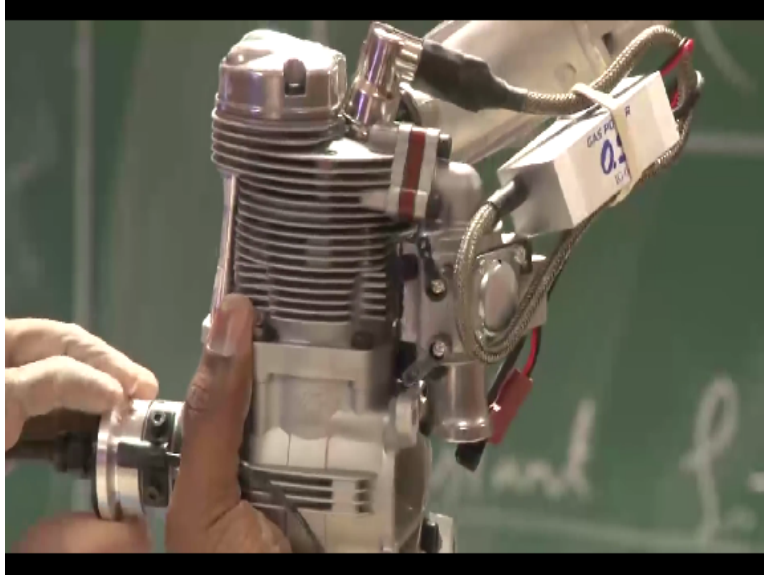
V infinity is the corresponding velocity of flight and you should know what is the corresponding L/D , right and propeller efficiency as well, right. Once you know this, then you will be able to find out what is the weight fraction. So we will do a small example here once we discuss this weight estimation as well as wing sizing, right. So when this endurance can be maximum? In this case, you have to fly at L/D maximum.

So the corresponding velocity for L/D max is V for L/D max, right. You should fly at the corresponding velocity where L/D is maximum, right. In this case, when E can be maximum when CL power $3/2/CD$ has to be maximum here, right. You fly at sea level, this particular term will be higher, right. Assuming these parameters are not going to be affected with either velocity of altitude, right.

And you have, if you want this E to be maximum, ρ has to be at sea level and CL power $3/2/CD$ has to be maximum and you should carry more fuel, right. This difference has to be more, okay. That means there should be lot of reduction in weight, right. When there is a lot of reduction in weight, then this quantity will be higher compared to this. So the difference will be higher there where you end up having high endurance.

So for this case, so E max can be attained at L/D max and V for L/D max. So what is L/D max? $1/\sqrt{4K CD_0}$, where $CL = \sqrt{CD_0/K}$ and the corresponding velocity you can find out. Similarly for this case, CL should be $\sqrt{3 CD_0/K}$ for E max, in this case corresponds to CL for CL power $3/2/CD$ max and $CD = 4CD_0$. So the induced drag is 3 times the profile drag in this case. Induced drag coefficient is thrice the profile drag coefficient, right. Okay till now we are talking about fuel based system, right.

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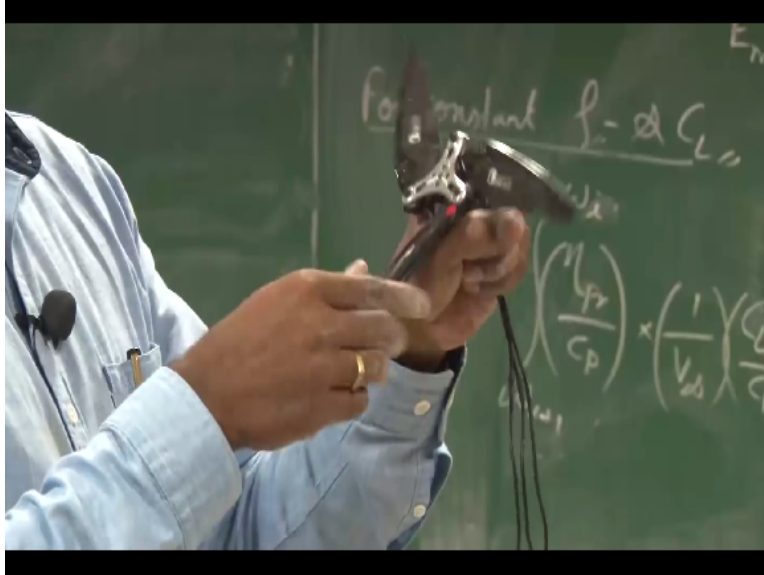


So in the previous lecture, we were not able to, I mean, show you a proper engine, right, proper IC engine. So this is a bigger version of it. So I have the specifications of this as well. So it is GF400S engine, right and it has a 39.96 cc and for displacement and a bore of 40 mm diameter bore and a stroke length of 32 mm approximately and feasible RPM is about from 1800 to 9000, right.

And the output from this is 3.5 HP at 8600 RPM. So it weighs about say 1.7 kgs approximately. So you can see this is a spark plug. There is an igniter for it, right. You can connect it here and this will be connected to this particular wire which actually generates power with the crank rotation, right. This particular valve that you can see here, see, it will automatically come back to 0. You can control the fuel-air mixture, right.

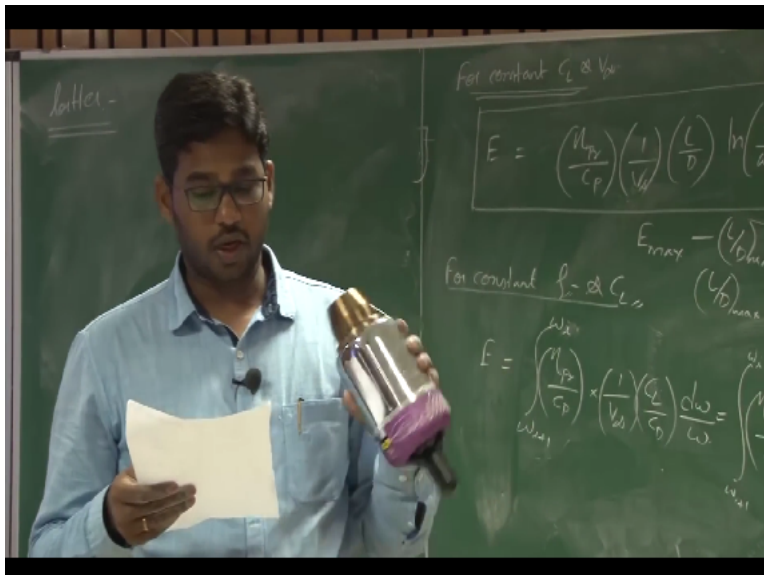
This is like a lid for this fuel-air mixture, right. It acts like a lid to this mouth and you can see there is a valve that will be rotating as I rotate this lever, right. So it actually controls the fuel-air mixture and so the RPM of this shaft, right. So once you mount the propeller here, you will be able to convert this shaft power, available mechanical shaft power to the useful power, right.

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So I thought of showing you some like brushless motors as well. So it is a carbon fibre propeller, 3 carbon fibre propeller, 3 blades mounted on this hub, right which is attached to the outrunner. It is a 515 kv outrunner and this hub is a foldable hub for this propeller.

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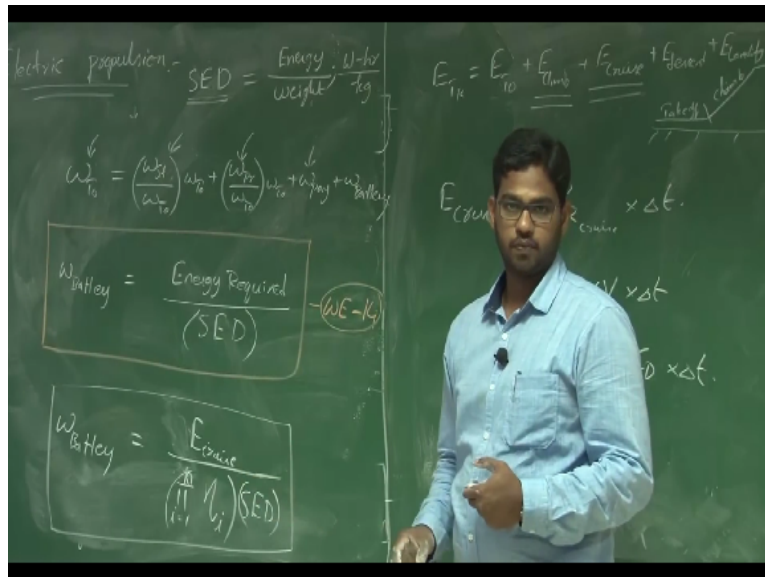


There is a jet cat, jet engine. You can see this. It is a small jet engine, mini jet engine which can produce a 22,000, 220 Newtons force which is approximately 22 kgs force, right. It can produce a 22 kg thrust. If you can see, you can see a straighter there in the front. It weighs about 1.8 kgs and yes, some of the specifications I have here. Maximum thrust is about 220 Newtons at 1 lac 17,000 RPM.

So this RPM has a range of 35,000 to 1 lac 17,000 RPM. So it weighs about 1.8 kgs and ideal thrust of 9 Newtons which is approximately 1 kg of thrust, right. So 30 cm and diameter of 11 cm, right. So this is the jet engine. So if time permits, I will try to install this jet engine in a delta wing configuration and we will fly. Otherwise, we will fly 1 or 2 delta wings with normal propeller driven electric based, I mean, propeller driven engines which are powered by battery, okay.

So let us talk about how to estimate the battery weight. Say if you are going for a brushless motor similar to one this. So if you want to power this brushless motor, you need to carry a battery. Say if you install this motor on a UAV, right. So you need to know how much battery I need to carry, say if I want to have a certain mission requirements? Say if 10 km range or say if I have, if I want to have an endurance of 1 hour, so how the battery weight changes, right as the mission requirement changes? So let us look at how to estimate the battery weight, right.

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For an electric powered propulsion system or a battery powered propulsion system, what we need to define is specific energy density, specific energy density of the battery which talks about the amount of energy stored per unit weight, weight of this battery which is watt hour/kg. The units of this specific energy density are watt hour/kg. So each and every battery that we use to power this motors, they have certain energy density, right.

So based upon the available energy density you will be able to estimate the battery weight once you know what is the corresponding energy that is required to perform that particular mission, right, okay. So let us go back to this equation. So weight, overall takeoff weight = structural weight ratio * takeoff weight and propulsion weight ratio * the takeoff weight + payload + battery, right.

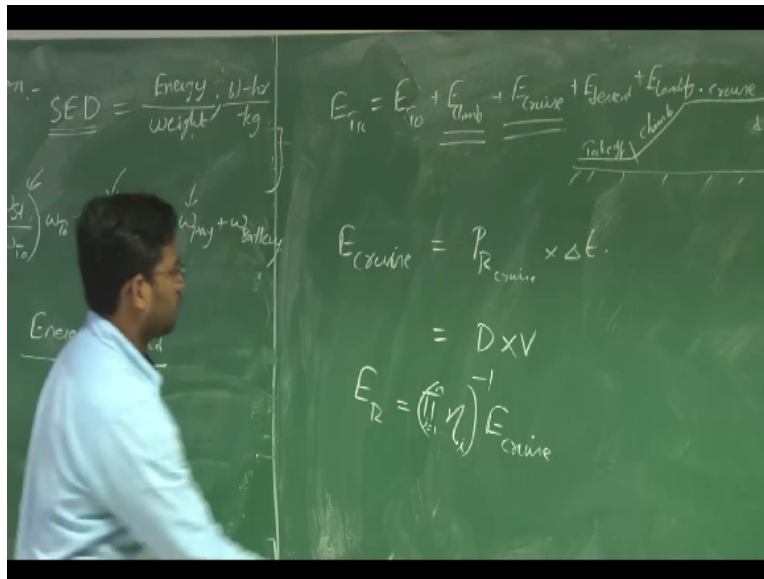
So it is bit different compared to that of fuel based system, okay. So what we go ahead with this is, so for this particular mission requirement, how do we estimate the battery weight and the overall takeoff weight? So similar to that of the rear case, we consider this structural weight ratio and the propulsion weight ratio from the historical database, right and we also consider what is the overall takeoff weight here.

How? We take the nearby flight vehicle which is performing a similar mission, right. So we will take the weight initially from the historical database, even the overall takeoff weight. At the same time, we will take the structural weight and the propulsion weight, right and the payload is given for you. So let us say these things are known for the time being. So it is an iterative process but in this, we will try to address if this is overall takeoff weight, what is the percentage of battery weight for that overall takeoff weight?

Let us see if we want to build a UAV of 5 kg. Now you should know what should be the fraction of this overall weight of 5 kg, should be the battery weight, right. So let us solve that problem first and then we will see how this iterative process works to actually find the weight estimation, right, weight estimation of the new design, okay. So how to go ahead with this? Say we know specific energy density.

Say this is known, specific energy density is known, you need to know what is the energy so that you will get to know what is the weight, right. That means weight of battery = energy required or energy to be supplied to the system / SED, specific energy density, right. So energy required, energy is watt hour or joules, right watt second in fact or watt hour/kg, okay. So this is, energy required is watt hour and this is watt hour/kg, so what you are going to get is the weight of the battery in kg, right. And now how to find out the energy required?

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So for any mission profile say, so again going back to this mission profile, typical mission profile where it involves takeoff, climb, cruise, approach or also descent for glide, landing. So the total energy required =, E_T is the total energy required, =energy for takeoff+energy for climb. It is a summation of energy for this entire phase, right. Energy for climb+energy for cruise+energy for descent and energy for landing.

So if you see here, so most of the UAVs, the major part of energy is consumed during cruise and to some extent, very little extent can be climb. Say if you know the cruise energy and descent is most likely a glide in case of UAVs unlike to this, unlike commercial aircrafts, the descent is most likely a glide here and landing of course, it hardly requires energy. So say if I estimate the cruise energy and also the climb energy required, then I will take some factor of safety as to account for this descent, landing and takeoff.

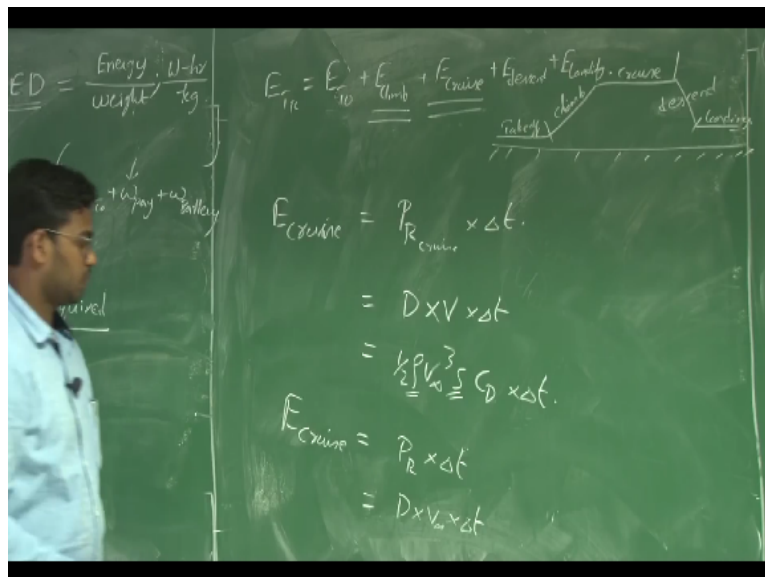
So this is my initial estimate, right. Now how do I find, how should I find this cruise energy? So one way to that is energy for cruise = power required during cruise * total time of flight. Fine, now the total time of flight during cruise, I will know what is the corresponding, and if I can estimate the power required during cruise, I will be able to estimate the corresponding energy required during this cruise.

So how should I calculate this power required during cruise? Power required is drag*velocity, right. Of course, this energy required during cruise, you need to have a factor of safety, right to account it for the other cases. At the same time, the efficiency factors, right. You can multiply all the efficiency factors that is $\pi \cdot \eta$, whatever the final or required = $\pi \cdot \eta$, yes, so say $\pi \cdot \eta \cdot \text{energy cruise}$, right.

Where π $i=1$ to n , η I, right. So where η talks about factor of safety, efficiency and all these things. So you multiply all those and find out what is the corresponding energy required. You should divide this in fact, right. Because this is the output that you, this is the input that you have to give, right. From the battery you need to supply this much of energy. So this is like inverse of it, right.

So if you do this, then you will be able to estimate what is the actual energy required. So this may also include the, what do you call, battery cannot discharge 100%, right. There is a limit for discharge. So that limit can also be included as well as there can be losses when you connect this battery to a particular power plant, right. Those losses can also be included here, right.

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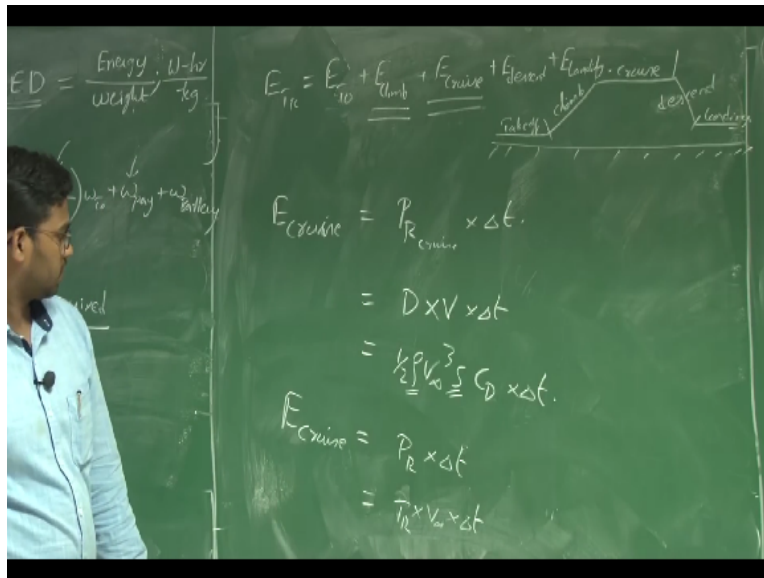


Let us go back to this energy for cruise. So this is drag*velocity*delta t. What is drag? $1/2 \rho V^3 S \cdot C_D \cdot \Delta t$, right. So velocity for this particular cruise mission will be given and you will be asked like how much time, I mean, as a part of mission requirements you will get this

delta t. You need to know what is this CD in order to figure out what is the cruise, energy for cruise, right.

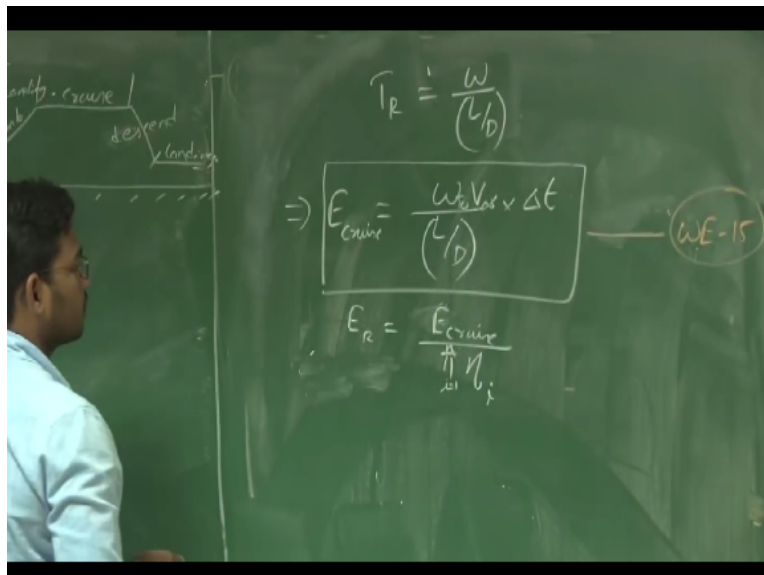
Instead of, there are 2 ways, right. So you are actually given the information about areas, reference area and CD. Let us say if you are not, if you do not know this reference area and CD, what happens, right? What I can do? So energy for cruise = power required * delta t. The power required is drag * velocity * delta t, right.

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Or power required is thrust required * V infinity. So what is thrust required?

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So thrust required is $W/L/D$. So if you can refer our previous lectures, you will be able to find. Because $T=D$ and $L=W$ for cruise, right. So thrust required is $W/L/D$. Now if you substitute that in that energy required equation, this is like $W/L/D * V_{\infty} * \Delta t$. So all you need to know is what should be the L/D of flight. So usually it will be, it will range from 10 to 15, right for a normal UAV.

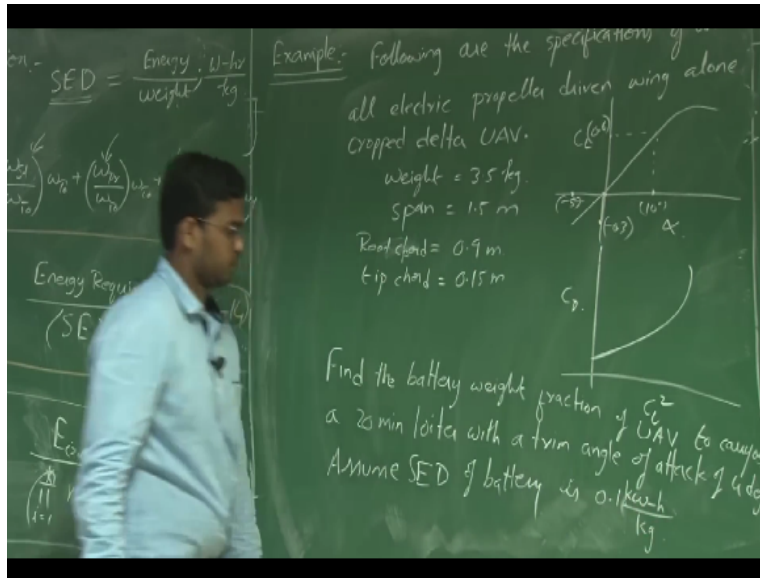
So V_{∞} will be given, you know, you have initially considered some W takeoff which is that same W here, right. This is takeoff, $*V_{\infty} * \Delta t$, right. Now you know what is the energy required here. Now the weight of battery =, or this is energy for cruise. So energy required = energy for cruise / efficiencies, η_I , whatever the efficiencies that you want to consider here.

So this is our WE, what is the number? **“Professor - student conversation starts”** 14. 14 **“Professor - student conversation ends.”** WE-14 and WE-15. So say if the cruise is at a constant velocity, then if we want to have maximum cruise or minimum energy, then you have to, for the same time of flight at the same velocity or say for the same time of flight if we want to spend minimum energy, all you need to do is to fly at maximum L/D where the corresponding V_{∞} will be, right.

You have to fly at the corresponding V_{∞} as well, V_{∞} at L/D max, right. So weight of battery = energy required by SED. Energy required we got it from the ER which is E_{cruise} . So this is for initial estimate, right. $\sum_{i=1}^n \eta_i$, right, *specific energy density. So this is the weight of the battery and the percentage, the ratio of the, what do you call, battery fraction/total takeoff weight, right.

Then you will get to know, say if you want to build a 5 kg UAV, how much energy density, how much battery you need to carry, right. So let us solve a small example.

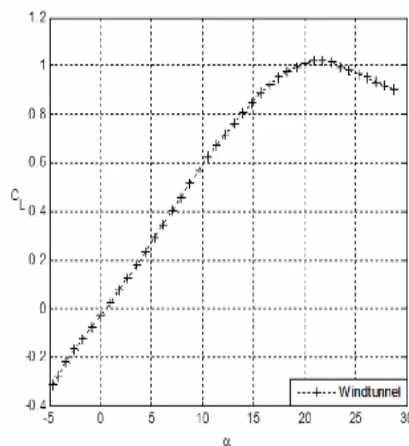
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So we are slowly started design in the design of UAV, right. Because we initially figured out how to select a power plant or the engine sizing and then we are trying to figure out how to estimate the total weight, right. So let us take 1 small example. We will also come back to that iterative process once we solve this example, right. Example here, following are the specifications of an all electric propeller driven wing alone cropped delta UAV.

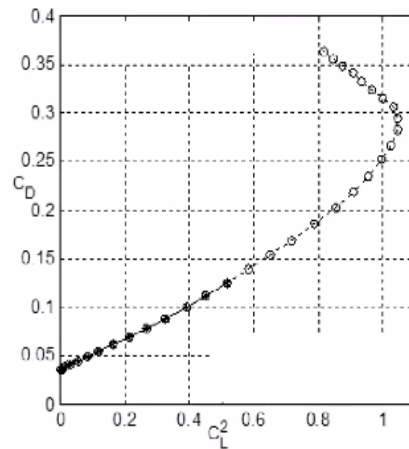
So what is given is weight, weight is 3.5 kg, span 1.5 m, root chord 0.9 m and tip chord 0.15 m.

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And also that plots of C_L versus alpha of this entire UAV, full scale UAV were given here.

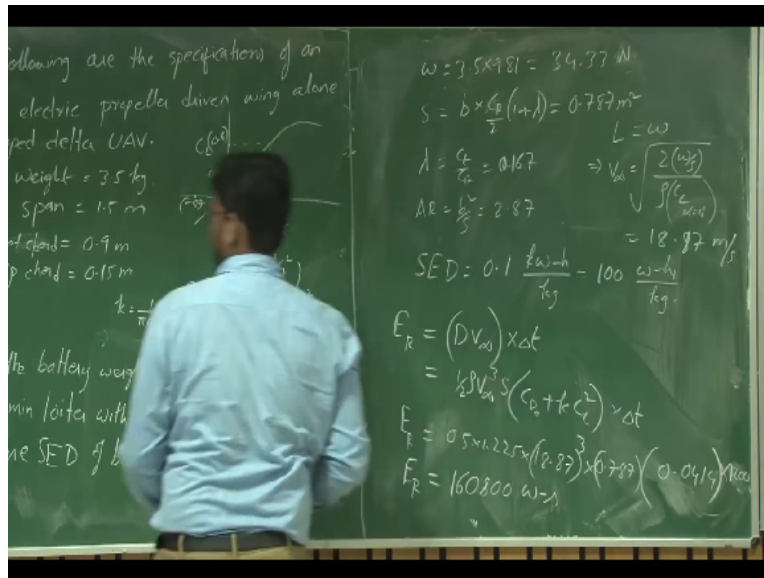
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And there is a plot that represents C_L square versus C_D for this full scale UAV is also given and then coming back to the question, find the battery weight fraction of UAV to carry out a 20 minute loiter with a trim angle of attack of 4 degrees. What should be the additional battery weight required to perform the same task at 3 km altitude, right. So first let us solve this question at first place.

Also assume energy density of the battery is 0.15, assume specific energy density of battery is 0.1 kilowatt hour/kg. Also find the Oswald's efficiency factor E . See I am using, if you observe I am using the same configuration in most of the examples. So it is worth you get used to this numbers, at least the aspect ratio, weight and all these things so that it will be helpful for you during the end semester examination, end examination, right.

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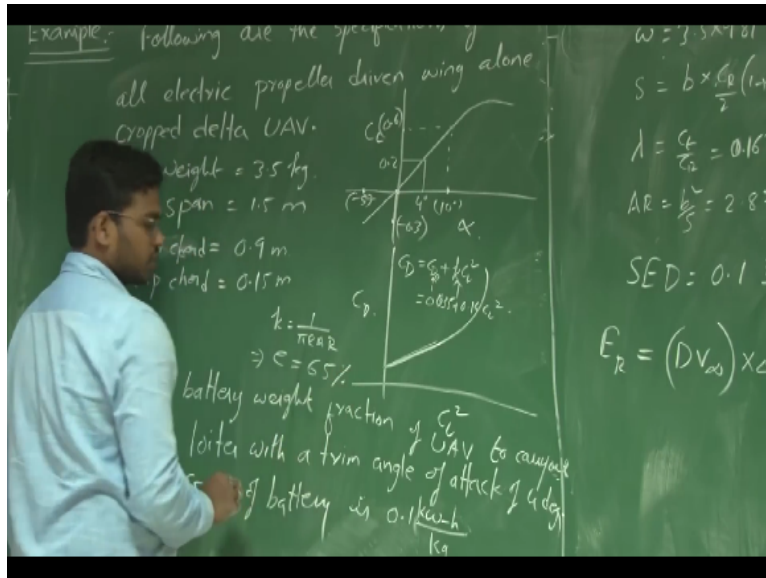


So you can refer our previous lectures. So for this configuration, the weight is given as 3.5×9.81 which is 34.33 kgs approximately, Newtons sorry. Weight is given. So S is again $b \times C_r / 2 \times (1 + \lambda) = 0.787 \text{ m}^2$ and then λ is C_t / C_r is 0.167 and AR is $b^2 / S = 2.87$, right. It is a crop delta wing UAV. Fine. Now what we need to do here? We need to find the battery weight fraction of this UAV to carry out a 20 minute loiter with a trim angle of attack of 4 degrees.

SED is given as 0.1 kilowatt hour/kg, right. So $SED = 0.1 \text{ kilowatt hour/kg} = 100 \text{ watt hour/kg}$, right. So let us figure out what is the battery weight that we need to carry in order to complete this task for this particular UAV. So in order to find the battery weight, what I need is the energy required for this system, right. How do you find energy required here? So energy required =, assuming it is 100% efficient in all the cases in this case, right.

I mean, say there are various losses that happens, right. So assuming there is no loss throughout the process, right. So energy required = drag * velocity is the power required, * delta t, right. So why we considered drag here? Because here we can also consider the other case like what is the L/D for this particular configuration? Or say if you know what is the C_D here, since you know $\alpha = 4$ degrees, you will know the corresponding C_L here.

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That $\alpha=4$, the corresponding CL is 0.2 approximately, right. And from here you will be able to find out, see this is CD/CL square, right. Which means in the (α) (45:25), this is like $y=mx+c$. It is a straight line, right where y is $CD=CD_0+\text{slope}$ here will be $K*CL$ square. So this straight line will give you the drag polar when you plot CD versus CL square, right. CD_0 you can find out and K you can find out.

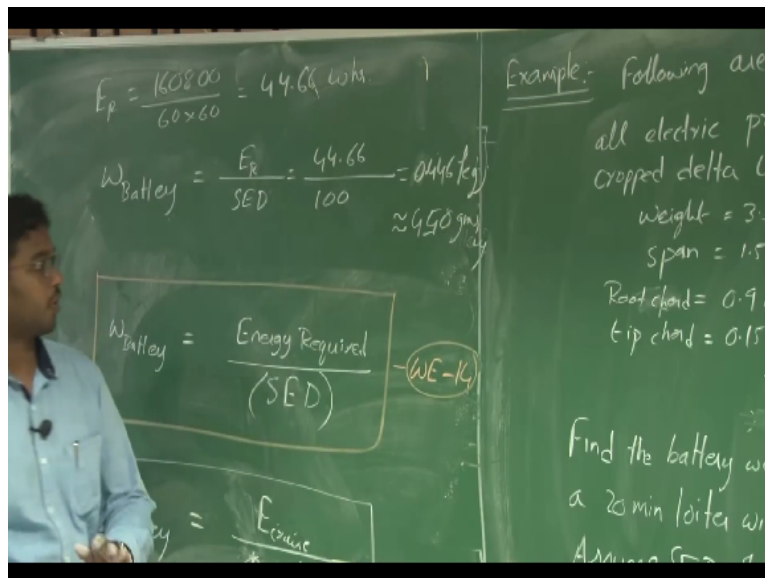
So for this particular case, it is 0.035 and K value is $0.16*CL$ square, right. Once you know K , $K=1/\pi eAR$, you know the aspect ratio, you can find out the corresponding Oswald's efficiency factor which is approximately 65%, right. You can calculate that here. So $E_R=\text{drag}*\text{velocity}*\text{the time of flight}$. Here it is 20 minutes of flight. Now what is drag here? $1/2 \rho, V \text{ square}*V \text{ infinity is } V \text{ cube}, S*CD_0+K*CL \text{ square}*\Delta t$.

So at $\alpha=4$ degrees, you will be able to find out what is the corresponding CD because you know CL and you know the CL squared value. You can find out the CD or you can simply, from this, you can substitute in this equation once you have this, right, from this 4 degrees, you know what is the CL value, you can substitute this. So $0.5*1.225*$. What is the velocity of flight here then?

We know $L=W$. For the level flight condition, we have $L=W$. So the corresponding velocity is square root over $2W/S/\rho*CL$, corresponding CL , CL at $\alpha=4$ degrees, right. So what is the

value here? It is 18.1 m/s approximately, 18.87 m/s. Now substitute that velocity here, $18.87^3 \times 0.787$. What is $C_D + KCL$ square or the corresponding value here? You can figure it out which is approximately 0.0414. So this is C_D is 0.787 and the total time of, 20×60 because it is in minutes right. So 20×60 is like 1200 seconds, right. So the total energy required is approximately 160,800 watt second. So if you convert that too.

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So let us convert that to watt hour, Energy required $160800/60 \times 60$ which becomes 44.66 watt hour, right. Now you know energy required in watt hour. You know SED, that is 0.1 kilowatt hour which is 100 watt hour/kg. Now divide this, so the total battery weight should be equal to energy required by SED which is 44.66 watt hour/100 watt hour/kg which is 0.446 kg, right which is approximately 440 or 450 g. So for this 3.5 kgs UAV, so 1.2 kg is the battery weight. You need to consider 1.2 kg as the battery weight for this UAV, right.