Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture – 11
Thrust Required and Power Required

Good evening friends, welcome back, you know previous lecture we are discussing about thrust requirement during a level flight, the case of general flight; level fight is the case of general flight, where the equations of motion for level flight have been derived from the general equations of motion with some assumptions okay.

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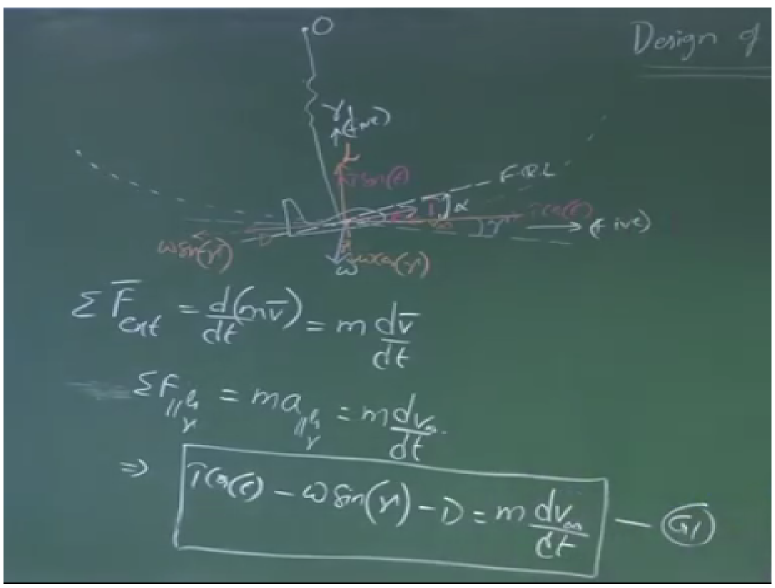

Now, let us go back to the general equations of motion, let us consider this is your UAV acted upon by external forces thrust, due to this thrust, the aircraft; and you have weight acting vertically down and due to this thrust, the aircraft will move at a certain velocity; V infinity, so this is your V infinity and once it start moving, there will be flow due to which there will be lift aerodynamic forces, external forces like lift and drag.

Lift is acting perpendicular to V infinity, drag along the direction of V infinity, now consider a fuselage reference line FRL of this aircraft; FRL, so the angle made by this V infinity with this FRL is alpha and in this particular case, we see the thrust is misaligned with the infinity or the direction of motion by an angle epsilon which is considered as thrust misalignment, yeah, now consider the local horizontal here.

Consider a local horizontal, so the angle made by this free stream with respect to this local horizontal is a flight path angle, gamma right. Now, w acts perpendicular to the local horizontal here, now let us write equations of motion along V infinity and perpendicular to V infinity, so this is your lift which is acting perpendicular to infinity, what we have here from here is; according to Newton's second law, the total external forces acting is; will result in the change of rate of change of linear momentum assuming the aircraft has a point mass.

And further these forces are acting at the CG of the system, so we can write the equations of motion, with these assumptions, we can write the equations of motion along and perpendicular to flight path angle. Let O be the point about which the aircraft is rotating in this vertical plane with the radius r1, right, so the force is; total force is acting parallel to gamma is equals to mass * acceleration, which is parallel to gamma, right.

So, parallel to gamma, the velocity is v infinity we can write, rate of change of velocity dv/dt is here and what are the forces acting parallel to this v infinity; we have thrust, component of thrust acting along v infinity, which is T cos epsilon and this is T sin epsilon and this weight have a component perpendicular to v infinity and this angle is gamma, this is w cos gamma and what we have; along v infinity is w sin gamma, right.

With this, we can write the equation here as; T cos epsilon assuming this is your positive direction of coordinate system here, or the axis system and this is your post 2 direction of this axis system, so the forces along this direction of positive, so T cos epsilon is one force acting along that direction; -w sin gamma - D = mdv/dt, this was a first equation, general equation 1.

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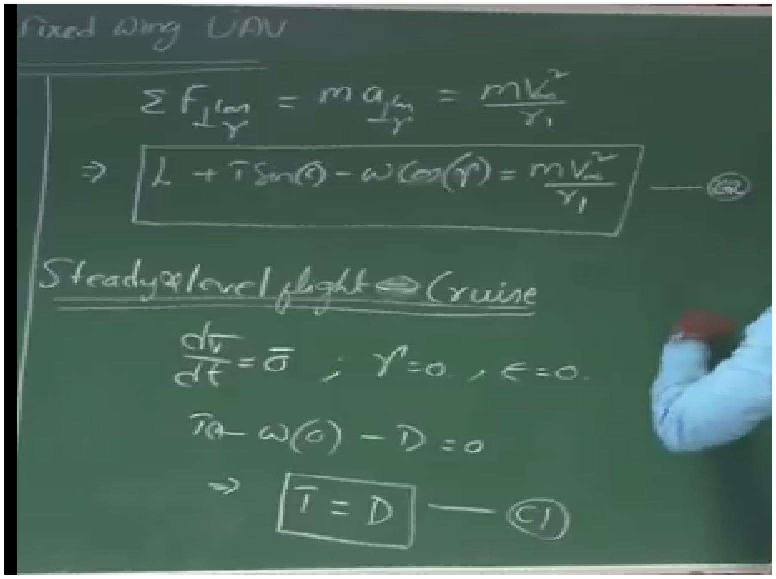

And what we have is the forces acting perpendicular to gamma which is equals to mass * acceleration which is perpendicular to flight path that is equals to m v infinity square/ r1, this implies the total force is acting perpendicular to flight path L and the component of the thrust - the rate contributes along the negative; negative axis of this lift cos gamma, sorry, please correct it, this is cos gamma = m v square/ r, so these are the 2 general equations.

And now, we considered a special case of this general flight known as steady and level flight which is also known as cruise, right, so what does it mean? Steady flight means the rate of change of velocity in this case is 0 or the net acceleration is 0 and level flight is gamma is 0; flight path is 0 and further we also assume that there is no thrust misalignment for this; during this particular case.

Now, with this assumptions, these equations G1 and G2 reduce to the following; T - w * 0; T * 1 – w0 – D = 0 that implies T = D is my first equation of level; of the level flight that is be cruise 1, first equation in cruise.

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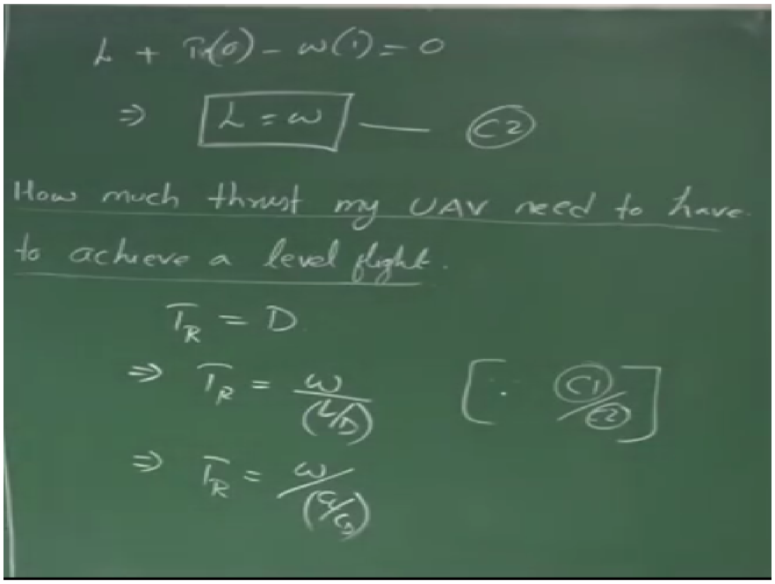

And now, substituting these conditions in G2, what we have is; L + T * 0 – w * 1 = 0 that implies lift = weight, so the weight is balanced by the lift and the drag is balanced by the thrust in a steady level flight. Now, what we understood is how much; how much thrust my UAV need to have to achieve a level flight, right, the answer is from this equation c1, so this drag is the systems output, right so we need to; which can be considered as a requirement of the system, drag is the negative force that we can see here which is the requirement of the system. So, what happens in the steady level flight?

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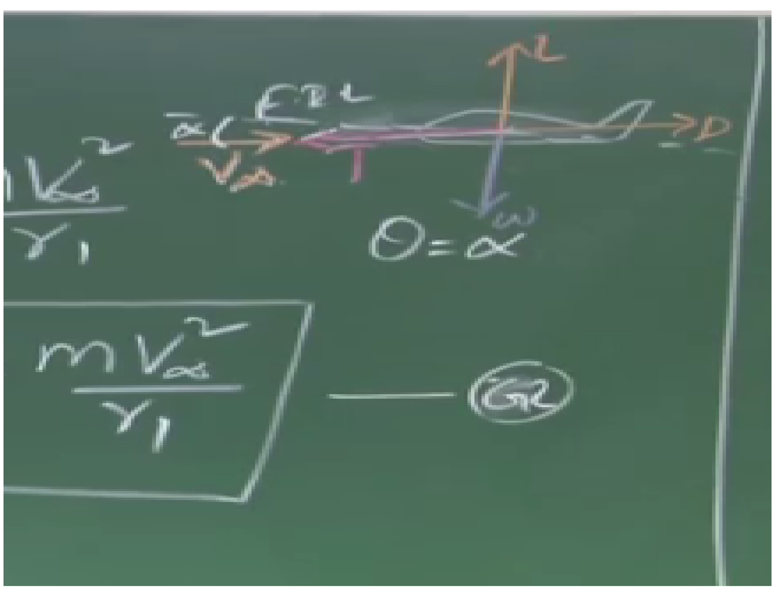

So, this is your reference axis and say this is your thrust, this is your V infinity, this is the reference axis, this is your thrust T and this is your v infinity, the lift acts perpendicular to v infinity and drag will be along this v infinity and you have weight acting perpendicular to local horizon. Now, the steady level flight does not mean that the orientation is 0; theta is not 0 in the steady level flight.

This is my alpha, it is still fly at a particular alpha, it is still fly, the reason orientation even in the steady level flight which is where theta = alpha, it does not mean that the aircraft has to be; the reference line has to be parallel to the local horizontal that is not the condition for level flight, velocity vector has to be; has to be parallel with respect to local horizontal, right, we need to fly at a particular angle of attack that is the whole idea, right to generate the lift.

So, the orientation; the aircraft still have an orientation during the level flight. Now, for this level flight we have thrust = drag that means, so the drag is the requirement; if drag is the requirement of the system and the corresponding thrust is the thrust required, so this much amount of thrust that the propulsion system has to generate to have a level flight, right, so we also witnessed thrust required = w/ L/D/ c2/c1 or c1/c2.

This = thrust required = w/cL/cD, right, now the minimum requirement of the thrust can be achieved when you have maximum L/D, so why we are mentioning this? We need to know which power plant that has to be used, right let us say, if you are working with the jet engine for this UAV, a miniature jet engine, then you need to know what should be the thrust boundaries, what are the boundaries of your thrust, right, so that you can select a particular power plant.

Now, what are the minimum requirement of this of a given UAV that means, your weight is constant of that UAV, relatively constant, right, so the minimum thrust requirement condition can be attained from maximum cL/cD.

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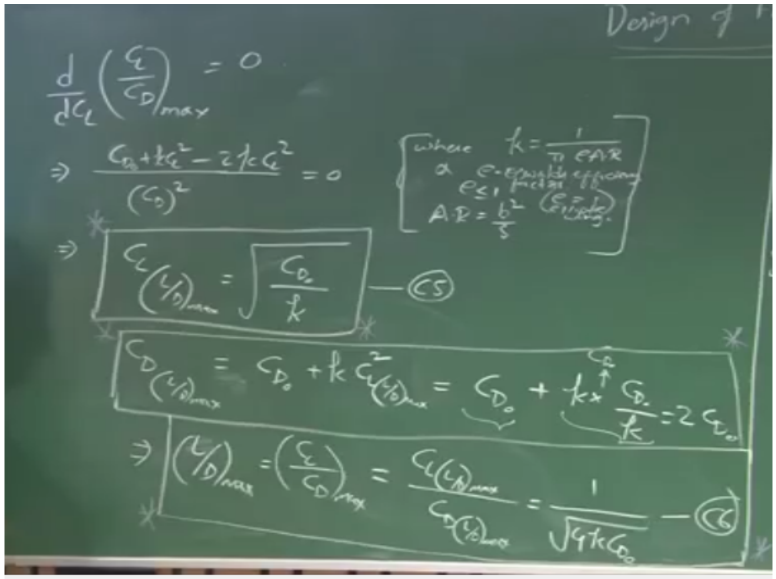

cL/cD max is attained by differentiating this with respect to cL, since cD is also a function of cL, right and equating it to 0, which implies thrust required minimum = L/D maximum, cD square * cD0 + KcL square – 2KL cL square = 0 which implies cL for L/D max = square root over cD0/K, where cD0 is the profile drag coefficient and K is the induced drag correction factor which is = 1/ pi; E is Oswald's efficiency factor whose limits are < or = 1, right.

And AR is the aspect ratio which is given by b square/ s, right, so for a elliptic wing, E = 1, right, = 1 for elliptic. Now, what is the corresponding drag coefficient for this L/D max condition, which is CD0 + kcL square from the drag polar, right, where the cL here has to be cL for L/D max condition or cL/cD max condition, this = cD0 + substituting this particular equation here, say here this is your equation sitting is yours this is your seat this is a safer substance if I in this equation c3, see this is your c3, this is your c4, right.

This is your c5, so substituting c5 in this equation, what you have is; k * cL square is cD0/ k, so this is = 2cD0, which means here, this turns out to be cD0, right, so here the induced drag; profile drag coefficient = induced drag coefficient, right, in that case, you can attain L/D maximum, which is; so, in this case, the drag coefficient is twice the profile drag coefficient, so the corresponding L/D max condition.

Or cL/ CD max condition is attained by substituting the cL for L/D max and cD for L/D max, which is cL for L/D max and cD for L/D max = 1/ root over 4k cD0, so L/D max condition is 1/ root over 4k cD max, so it is very important, you please, that is the reason why we are repeating this lecture, right this is an important part of aircraft design, where the wing sizing can directly done once you know the level flight requirements, right.

What you need to note down is; cL for L/D max and cD for L/D max which is 2cD0 and what is L/D max condition, which is 1/ root over 4k cD0, so these are noteworthy equations, right.

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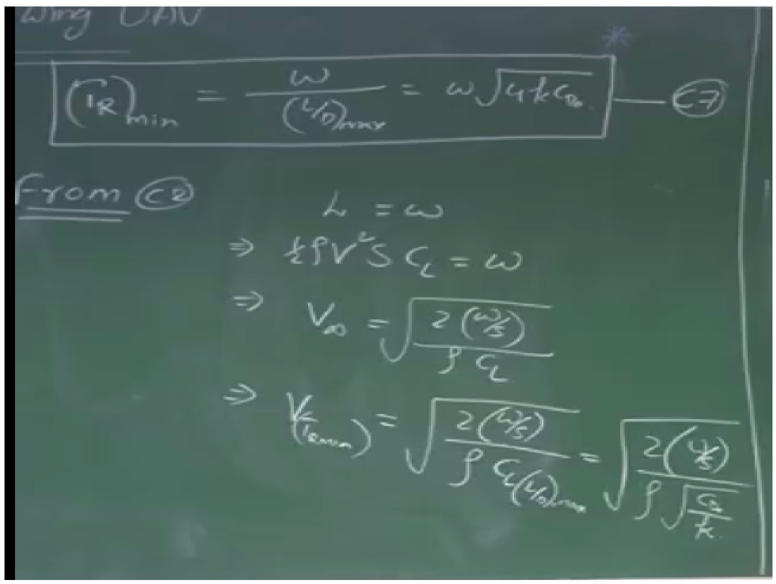

Once I have this L/D max, I can immediately find what is the thrust required minimum condition, right, this = w/; what is L/D max, L/D max which is L/cD max, this = w * root over 4k cD0, if you know the profile drag of your aircraft of your UAV and if you know what is the induced drag correction factor, you will be able to find out what is the minimum thrust requirement of that system for a given UAV, right of weight, w.

This is again, this is your equation c7, okay, and now, can we measure this thrust during the; during the flight, can we measure forces during the flight, (()) (19:26) is how to measure a force, how do you measure your weight; by means of weighing scale, right, what does it have? A load balance, right so, load balance needs a reaction force, right but aircraft in space you cannot get a reaction force there, right.

So, it is not possible to measure the force but you can estimate the force, right so why we are discussing this, you cannot ask the pilot or the controller to fly at this Tr minimum, right rather you should translate into a parameter which can be readily controllable or measurable; measured, right, so let us look at how to achieve this Tr minimum or what is a condition I need to maintain so that I will achieve this Tr minimum or L/D maximum, right, what is that condition?

Now, let us look at this equation c2; from c2 what we have is; L = w that means the weight is balanced by the lift of the aircraft, so what I can express; I can express this lift as 1/2 rho V square dynamic pressure times the reference planform area * lift coefficient = w, so the velocity; corresponding velocity of flight = root over 2w/s twice the wing loading/ rho * cL. Now, we earlier discussed during our initial lectures, right, how to measure the velocity of light.

And we are now confident enough that we can measure the velocity of light and can be controlled by varying the throttle; throttle setting right, by varying the thrust output, which means the throttle setting you can vary the velocity of flight, right, so that is a reason why we discussed how to measure the velocity, fine. Now, if I want to fly at thrust required minimum condition what I need to do is; I should fly at the velocity at which your systems thrust requirement is minimum, right.

This velocity I can easily measure by means of (()) (21:52) or say GPS with anyone of these, methods that we have discussed, right so in order to fly at the thrust required minimum condition, what I need to do is; I need to trim this aircraft at the cL that corresponds to minimum thrust requirement which is nothing but L/D maximum, cL for L/D maximum, so this cL for L/D maximum is root over cD0/k.

So, if we can substitute this here, so velocity for Tr minimum, thrust required minimum or L/D maximum = square root over 2 ws/ root over 2 ws/ rho * cL for L/D maximum or Tr minimum, right, so what is a corresponding value here? Root over twice the wing loading/ rho * square root over cD0/k.

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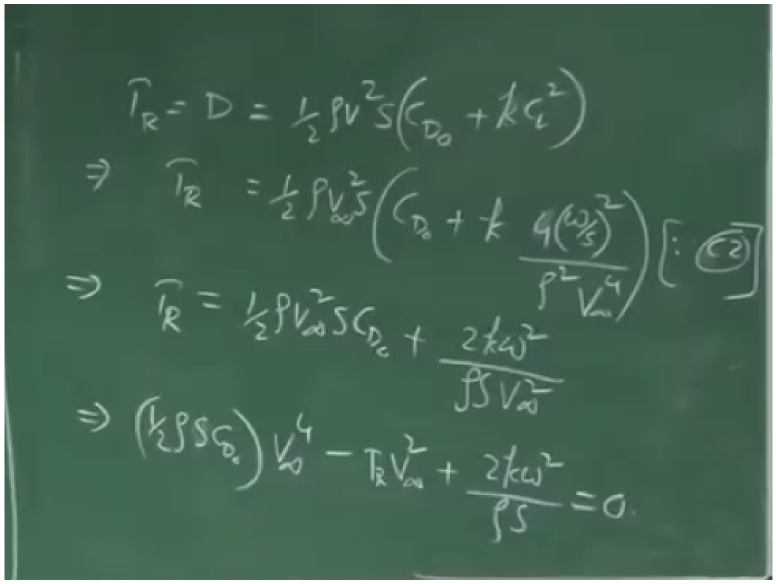

So, velocity for Tr minimum, which is = velocity for L/D maximum = root over twice the wing loading/ corresponding density at that altitude * k/ cD0 raised to the power of 1/4 or 0.25, this is a corresponding velocity that you have to fly, so you can ask the controller to trim your aircraft to this particular velocity or to adjust your throttle setting to this particular velocity to achieve the minimum thrust required for a level flight.

So, if you; in cruise, say which is one of the major; major phase of a particular; any mission of UAV or in general any aircraft, so if you; this is an amount of thrust that you need to generate, right that means, the power plant minimum capability should be during the cruise, it should be Tr minimum, right, so the available power should be equal to this particular power, right, then you can fly at this particular velocity.

We have also solved for, what is a velocity that are possible to achieve the level flight at a given thrust required condition, right how we have done that? So, we have T = D, thrust required = drag which is = 1/2 rho v square, s * cD which is cD0 + kcL square = 1/2 rho v square s * cD0 + k *; cL is again a function of velocity, right, cL square turns out to be twice w/s or 4w/s square/ rho square * V infinity for raised to the power of 4, right.

Since, during level flight L = w or since from equation 2; c2, right, this is your thrust required, yeah, now this =; Tr = 1/2 rho V infinity square SCD0 + 2kw square/ rho SV infinity square, right, this = 1/2 rho V infinity; 1/2 rho SCD0 * V infinity raised to the power 4 – Tr * V infinity square + 2kw square/ rho S = 0.

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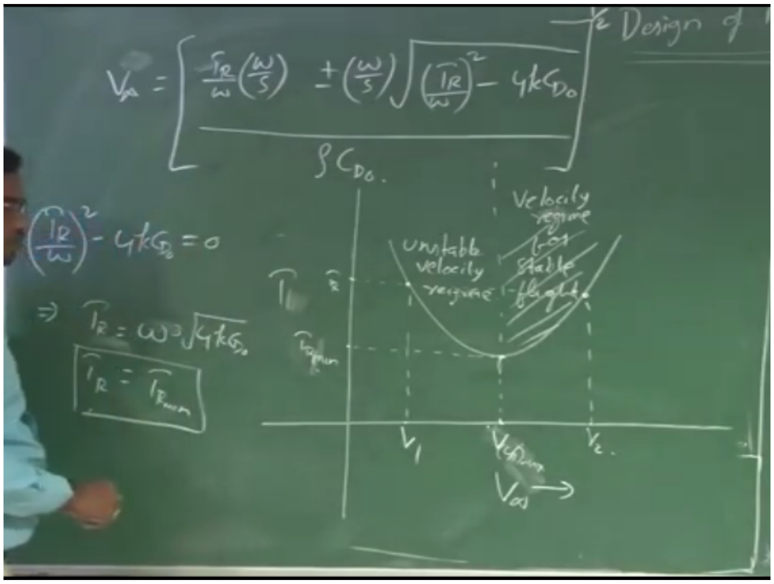

So, V infinity square = Tr/w * w/S + or – w/S * square root over Tr/ w whole square – 4kCD0/ rho CD0, so V infinity = this whole thing raised to the power of 1/2, right. So, if you plot this Tr verses V infinity, example; for a given UAV, so this is a thrust required which is drag verses V infinity, you will have a curve like this, right and particular point corresponds to thrust required minimum condition which is thrust required minimum is h1/ L/DS maximum.

So, this particular point is V for L/D max, this is your V infinity, this is the corresponding velocity for L/D maximum. Now, the CL trim at this point is root over CD0/k, right. Now, this particular case is when you have a new solution, right, here for a given thrust required you have only one velocity which is a velocity for minimum thrust requirement that can be achieved from here when this b square – 4AC = 0 that is thrust required by w whole square- kCD0 =0.

This implies thrust required minimum, this = w/; w * root over 4k CD0 which is a condition for minimum thrust required, right. So, the thrust required during this condition is a minimum thrust required and the corresponding velocity is Tr * w/S/ rho CD0. Now, we also witnessed, we also discussed, so this particular region which is to the right of this; this velocity region which is to the right of this vertical line, right is a velocity for stable, velocity regime for stable flight, this is the unstable velocity regime.

Why? Because for a given thrust requirement, you have 2 solutions for; of velocity you can either fly at this velocity let us say, this is your thrust required condition you can either fly at velocity V1 or velocity V2, right. So, which velocity you need to choose? When you fly at both the conditions, your thrust requirement will be same but which velocity you need to choose, right so that we have discussed earlier, you can refer our earlier lecture, right.

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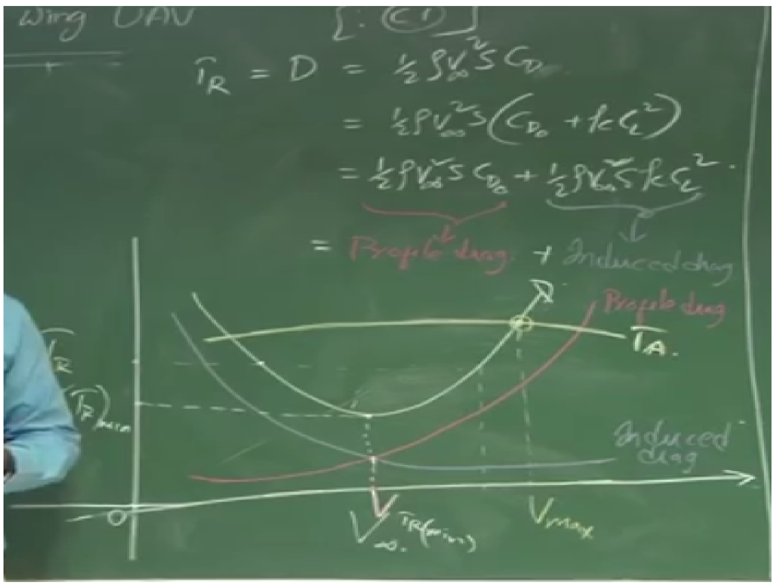

Right now, let us concentrate why this curve look like this, right why there exist 2 solutions, what is thrust required, from C1 thrust required is drag for a level flight, thrust required is drag which is expressed as 1/2 rho V square S * CD that is = 1/2 rho V square S * CD0 + kCL square, please make a correction here, it is CD, not CD0, right, this = 1/2 rho V infinity square S * CD0 + 1/2 rho V infinity square S * kCL square, so we call this as profile drag and this part as induced drag.

Profile drag is the summation of skin friction drag and pressure drag due to flow separation and as a velocity increases, there will be another component called wave drag that that that appears at high mach numbers, right, so we are not going to discuss about that. Now, so we witnessed thrust required with velocity where is something like this, right, according to this curve, so this is your

0. Now, say at minimum; at lower velocities, right, so if you want to sustain the weight, we have another equation right from C1, C2 what we have?

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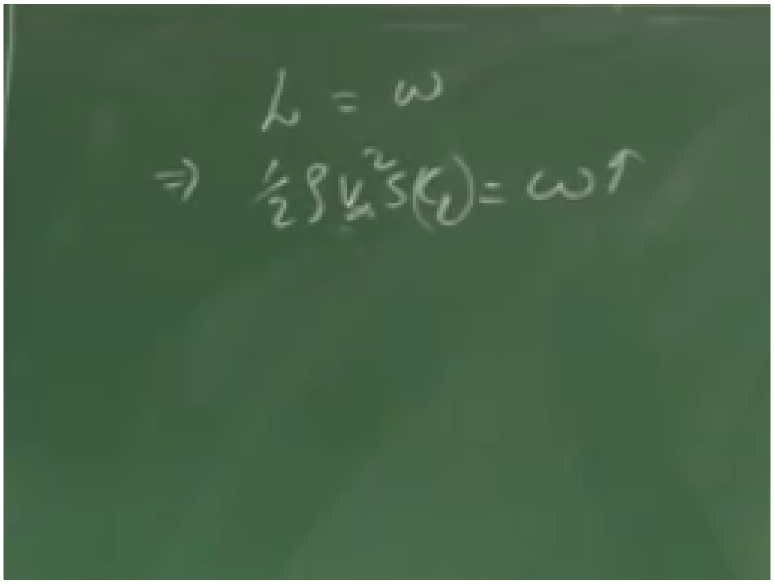

If we want to lift this weight, if we want to lift or lift this weight or hold this weight at particular altitude, you need to have a combination of V infinity and CL right, for a given UAV, if you want to fly at a required altitude, you need to have a combination of CL and V infinity, right, so at lower velocities, to sustain the same weight, I need higher CL, how can I achieve higher CL? By trimming it at higher angle of attack, right.

So, at this higher angle of attack, there is more induced drag because the lift dependent drag is increased, right that means at lower velocities, this particular curve represents the variation of induced drag with respect to velocity. At lower velocities, we have higher value as it the velocity increases, your CL requirement decreases that means you will be trimming at lower angles of attack.

Higher CL means higher angle of attack, right, that means higher the pressure difference on the upper and lower surface, greater the spanwise flow and greater is your induced drag at the local sections, spanwise; spanwise sections, right, so as your velocity increases, your CL requirement decreases, so this plot represents how the induced drag varies with velocity, so this talks about induced drag.

And how about profile drag; so, profile drag is proportional to velocity here because CD0 almost remains constant, so at lower velocities, it has minimum and as the velocity increases, your profile drag requirement increases, right, otherwise the profile drag increases here, this is the profile drag curve variation with velocity, this is the induced drag curve variation with velocity, so ultimately the point at which; what is this point, can you notice this like minimum thrust requirement, we have induced drag = profile drag, right.

At this point we have profile drag = induced drag and this particular point corresponds to velocity for Tr minimum, this particular point corresponds to velocity for Tr minimum, so what is this Tr curve; white curve? At each and every point of V, it is a summation of profile drag + induced drag, so sum of these 2 will be this point, here sum of these 2 will be this point and this is a corresponding thrust required for V, yeah thrust required minimum condition.

The respective velocity is Vr, V for thrust required minimum or L/D maximum, depreciate this, so this plot is nothing but overlap, summation of these 2 curves. Now, this thrust requirement curve indicates like the systems requirement to fly at a particular velocity, right this much is the systems requirement fly at different velocities. Now say, you have selected a power plant, you have a power plant.

Now, what should be, with that power plant there is a maximum limit of the power that or the thrust that the power plant can generate, right, so what will be the velocity or the maximum velocity you can fly with a given power plant.

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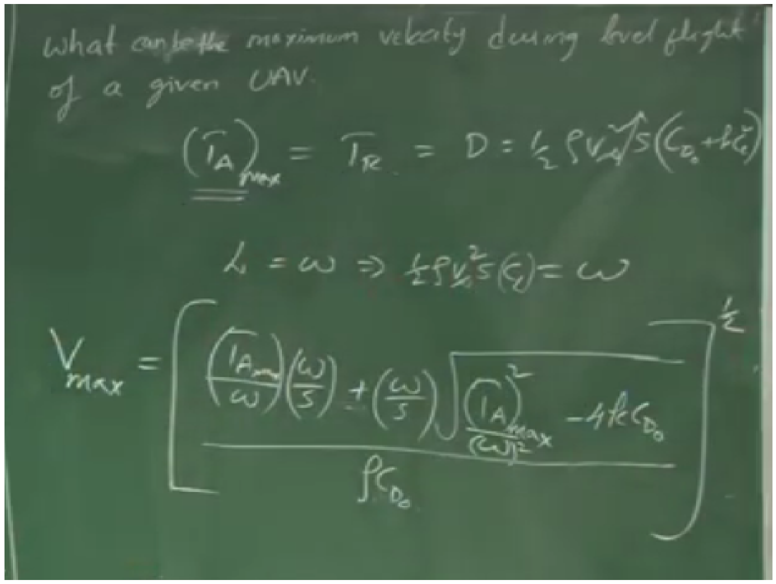

What can be the maximum velocity during level flight of a given UAV when you say given UAV, you have weight fixed planform area fixed, right and the power plant is fixed which means the maximum and minimum thrust that it can generate is fixed right, so in this case the thrust available is the thrust generated by the engine, so this condition is possible when thrust available = thrust required.

So, this condition can be attained when thrust available = thrust required that means you need to; you will be, so this is the maximum thrust that the system can generate, some thrust available max = thrust required that means you are utilising the entire thrust that is produced by the power plant, so when you have such a requirement, this thrust required = drag which is 1/2 rho V infinity square S * CD0 + kCL square, right.

So, if this is the maximum available thing, you will end up flying at this maximum velocity right because at higher velocities, your CL requirement is very less from L = w, you know 1/2 rho V square SCL, which we just discuss. So, at higher velocities, you need very less CL to achieve a level flight, so that means if you utilising the maximum output of the power plant that means you are actually travelling at the maximum possible velocity for a level flight for that particular (()) (39:14) right.

So, how the curve looks like, so this A, this is the thrust available by the power plant say, jet engine, when we used to talk about thrust, it is like we are discussing about jet aircrafts, jet powered aircraft, right. So, the thrust available at a given altitude more or less remains constant with velocity. If this thrust available = thrust required, this is the point where the thrust required curve and thrust available curve coincides, right.

This is your thrust available and this is your thrust required, so this point coincides, so the corresponding velocity of flight will be the maximum; V max. So, can we find that point from this equation, what you need to do? You need to replace Tr with T available and take the positive sign of this, you will reach this particular point right, so this sign, this point will be negative sign of that right.

Yes, so replace the Tr by Ta to get maximum velocity level flight is thrust available/ weight * wing loading + or -; + w/S * root over thrust available square max, see this thrust available is thrust available maximum by w square, right, -4 kCD0, which are constant/ rho CD0, so at a given altitude, if you are utilising the maximum available thrust, then you are flying at the maximum velocity raised to the power of 1/2, thank you, right.

So, this is a condition for maximum velocity that you can attain, so what about the minimum velocity? So, we have discussed this is our minimum velocity of flight right but let us see when can we attain this minimum velocity, let us look at this minimum velocity further close.

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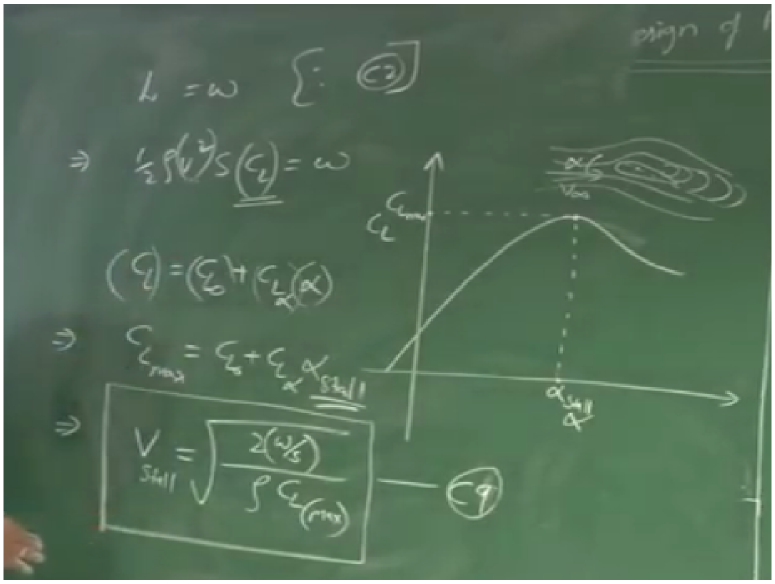

We have L = w for level flight from c2, this = 1/2 rho V square S * CL =w, the controller should give an achievable figures as an output, right, it is not that it is not demand from the system which is out of its limits. Why we are doing this exercise? We want to figure out what are the maximum and minimum limits of this velocity, so that the controller can play in between that numbers, right.

So, it can demand the system only within that range which is feasible, right. Now, from the level flight condition, we have 1/2 rho V square SCL = w, right, so if you want to see the minimum velocity then you have to fly at the maximum possible CL, right, so what is the maximum possible CL; CL versus alpha, how can we achieve CL; different CL, we know CL = CL0 + CL alpha * alpha in the linear regime.

So, maximum alpha, you will get maximum CL because CL alpha is constant for a given wing or once you have airfoil, and you have made the wing out of that airfoil, then CL alpha remains constant and CL0 is constant again, so alpha is the variable here, you can trim a different angle of attack and achieve difference here, so what is the maximum alpha that you can fly? The answer is; stall angle of attack, right.

So, during our introductory lectures, so we discussed about a point called CL max where which is attain at an angle of attack called alpha stall, what happens beyond this stall is, what is angle of attack here like, so the orientation of this with respect to flow right, so during this condition as the alpha increases, the flow tries to separate, so the pressure distribution whatever which was responsible for generating lift vanishes, right.

So, if the flow separates what happens is the lift drastically decreases, beyond this point the lift drastically decreases and drag increases at the same time, right so this particular point corresponds to CL max and the corresponding angle is known as alpha stall, so we cannot; the one condition is that your CL should not cross this CL max, right, for a during a level flight, so the minimum velocity is constrained by CL max as well, higher the value of CL, minimum this VL, velocity required.

Now, the maximum or the upper limit of the CL is the CL max here, which is obtained at alpha stall, so that CL max, I can write it assuming a linearity here, linear relationship although it is not but for the timing let us assume it is a linear thing like CL alpha * alpha stall, although it is not linear we still let us assume it for the time being, right. So, what you have here is the corresponding velocity at which the CL is max is known as V stall.

V = square root over twice w/S/ rho * CL, now if you say this is your maximum CL, which is obtained by alpha stall and the corresponding velocity of flight is known as stall velocity, right, this equation is c8, if I am not wrong, this one is c8, okay let us name it as c8 and let this be c9, this is the cruise, equation 9 in cruise for this lecture. The question is which one is the minimum velocity?

Is V minimum, V for Tr minimum is the minimum velocity or V stall is the minimum velocity, this is the unstable region for flight, right, this is the velocity region for unstable flight but we want a stable flight, so this is a minimum velocity from this curve but we have V stall which is obtained from the aerodynamics, right, when we trim the aircrafts at CL max you will get the corresponding stall velocity.

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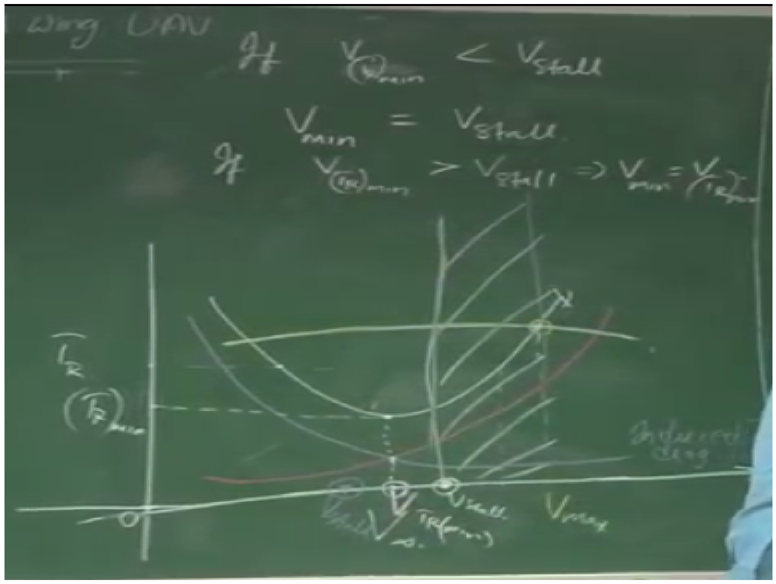

Now, which one we need to consider as the limit; lower limit, there is an ambiguity right, now if V max or V for thrust required minimum that is nothing but the minimum velocity for level flight right, which is = V minimum is < V stall, right, otherwise, this V; velocity for thrust required minimum is < V stall, < V stall, then minimum velocity during cruise = V stall right. What does it mean? So, we have a stall velocity here right, so this is your stall velocity, V stall in the first case.

So, V Tr minimum is < V stall, this is the point okay, this is higher than this V stall, right, sorry, V stall is higher than V Tr minimum. Now, you should consider the minimum velocity of flight from here, so this is the regime that which in which you can fly why; because if you fly at this velocity, you cannot attain the required CL because it will stall beyond this angle of attack, now this particular regime corresponds to stable region of flight which you can give as a limits for velocity for your controller.

Now, say if V stall is here, somewhere here, say this is your V stall, right, so which one you need to choose? If V Tr minimum is > V stall, right, what will be the minimum velocity? V minimum is V Tr thrust required, why because so, this falls to the; this V stall velocity lies in the unstable region of flight, this is the velocity for unstable region of flight, so we cannot fly at this particular velocity, so that is why we can consider this as the minimum velocity in that case.

So, these are the conditions for minimum and maximum velocity for a jet aircraft if you are planning to build a UAV by using a mini jet engine but most often, we see a UAV is with propeller driven aircraft, right, so now we need to understand the requirements of a propeller driven aircraft.

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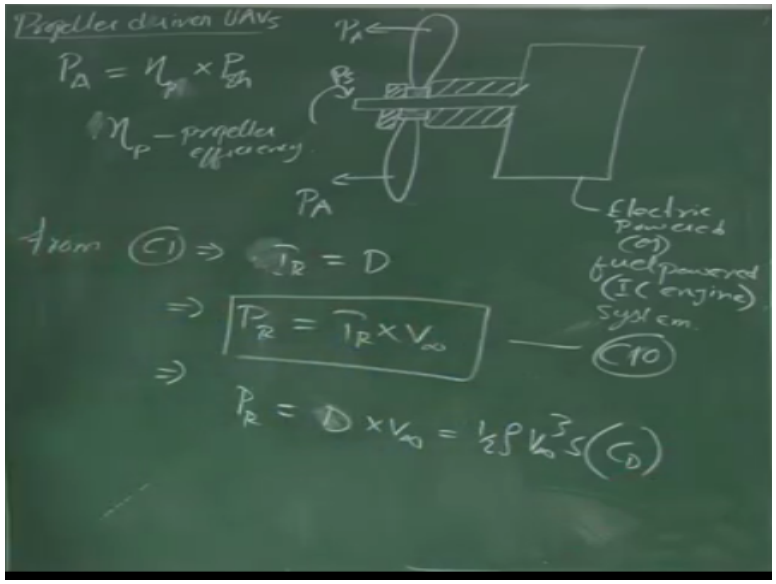

So, when you have a propeller driven aircraft, which is for a propeller driven aircraft, this is a typical propeller driven aircraft, where this propulsion system may can be either electric power or a fuel powered system or IC engine, IC engine yeah, system right. So, in either the case you input energy and the output in this case is mechanical power called shaft power here, PS, you will get mechanical power, right. Now, you need to convert this mechanical power to the useful power, for your case, to propel the aircraft forward, right.

So, how you are doing it? You are trying to convert this shaft power to available power; power available, right by means of this propeller, so this power available will helps you to achieve different velocities when you want to use a propeller driven aircraft or when you want to build a propeller driven aircraft UAV, so this power available = shaft power times, sorry shaft power times the propeller efficiency, eta P stands for propeller efficiency, eta P stands for propeller efficiency, right.

So, how to select a particular power plant for a UAV? We need to understand the power requirement for various phases, right then we will select okay what will be the output of this particular thing and multiplied by this efficiency will give the corresponding available power, right. So, if this available power falls, within the limits or the falls within the requirement of your system that is the power required, then you will be I mean you will; you can achieve the required flight condition using that particular propulsion system.

Now, what will be the; see again you cannot measure power on board, right, you can estimate but you cannot measure, so again we need to talk about what will be the velocity of flight in order to achieve the minimum power or what is the minimum power that is required to have a level flight for a given UAV, right. Now, let us look at that case, so from c1, from c1, what we have is power required or thrust required = drag, this implies power required = thrust required * velocity, thrust * force * velocity

This implies, so this is your equation c10, so power required = drag * thrust required is drag from this equation which is c1 * velocity, so power required is drag * velocity, so this is the requirement of the system, this = 1/2 rho V infinity cube S * CD.

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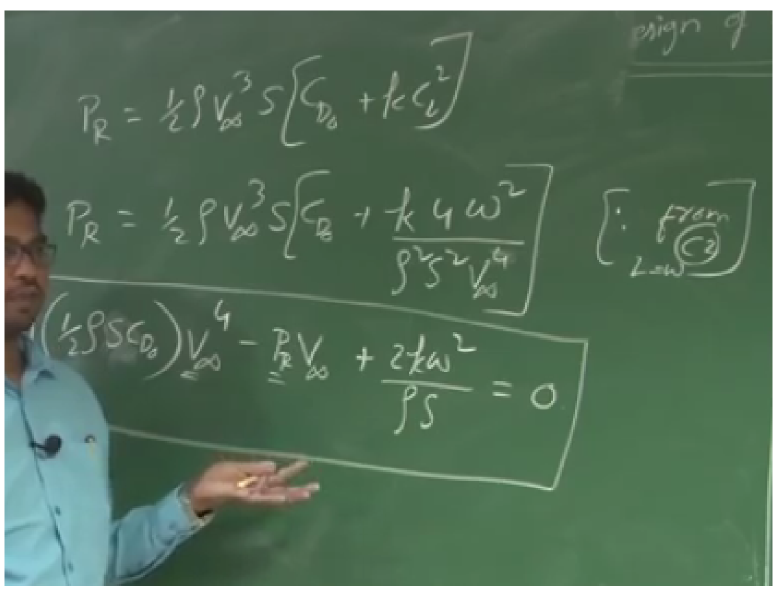

So, power required = 1/2 rho V infinity cube S *; CD is CD0 + kCL square and this implies power required of this UAV is 1/2 rho V infinity cube S * CD0 + k *; what is CL? 4w square/rho square S square V infinity square, V infinity raised to the power of 4, since from c2, L = w, right, c2 is L = w, this implies 1/2 rho; 1/2 rho S CD0 * V infinity raised to the power of 4 – PR * V infinity, right; PR * V infinity + 2k w square /rho S = 0.

This equation represents the relationship between velocity of the flight and the corresponding power requirement, so this does not have a closed form solution; you cannot obtain the solution analytically, so the general procedure that is adapted is by using graphical approach.

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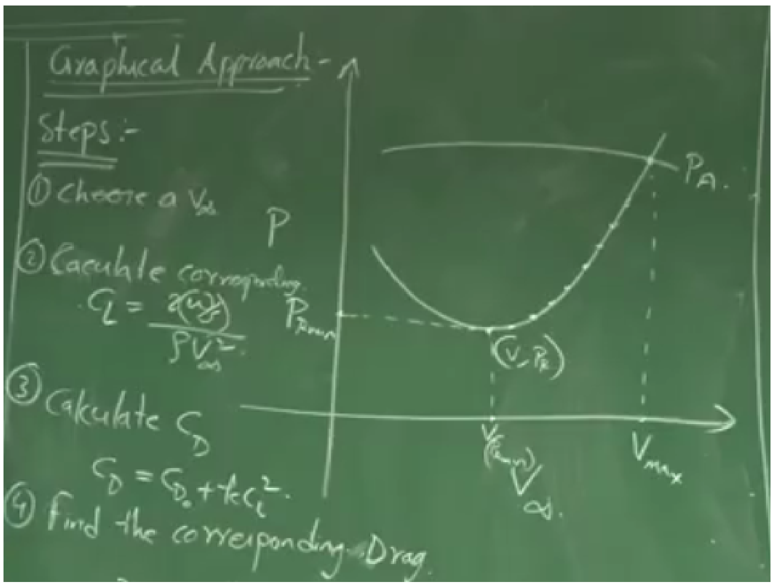

Power with velocity, so for a propeller driven aircraft aircraft, power required varies with the velocity, right, this is how it typically looks like and power available from the power plant almost remains constant, right, so this particular intersection point will help you to figure out what is the corresponding maximum velocity that you can fly during level flight, right and again the minimum velocity that we have already discussed.

But can we obtain this minimum velocity? The minimum velocity for a propeller driven aircraft aircraft will also follow the similar concept of jet aircraft, right, so this is the power required minimum, then the corresponding velocity is V for PR minimum okay, so how to obtain this plot? We need to follow certain steps here, so steps for this graphical approach, first one is to choose V infinity of interest, right, choose a particular V infinity of interest.

And now, calculate corresponding CL of this flight by CL = 2w/ S/rho V square, you know the altitude of flight and for a given UAV at the required velocity, you can find out the corresponding CL, this is from c1; equation c1. Now, calculate CD is the drag coefficient, how do you calculate CD = CD0 + k CL square, right, find the corresponding drag associated with the UAV at this particular velocity; corresponding drag, D = 1/2 rho V square S * CD, right.

Because for a given here, CD0 and k are constant, this CL is a variable which you can obtain once you know the velocity for the level flight, right. Once you know CL, you can substitute in this drag polar to find out what is CD, once you have CD, you can find out the corresponding drag, now once you have the drag, you can find out what is the corresponding power required. Power required = drag * velocity, so for each and every iteration by changing different velocity, you will end up having different PR.

That means, every iteration will get you different points, right. So these points are the coordinates of this curve, each point corresponds to V, PR, corresponding PR, right once you have this, you can plot this curve. Can we figure out this minimum power requirement analytically?

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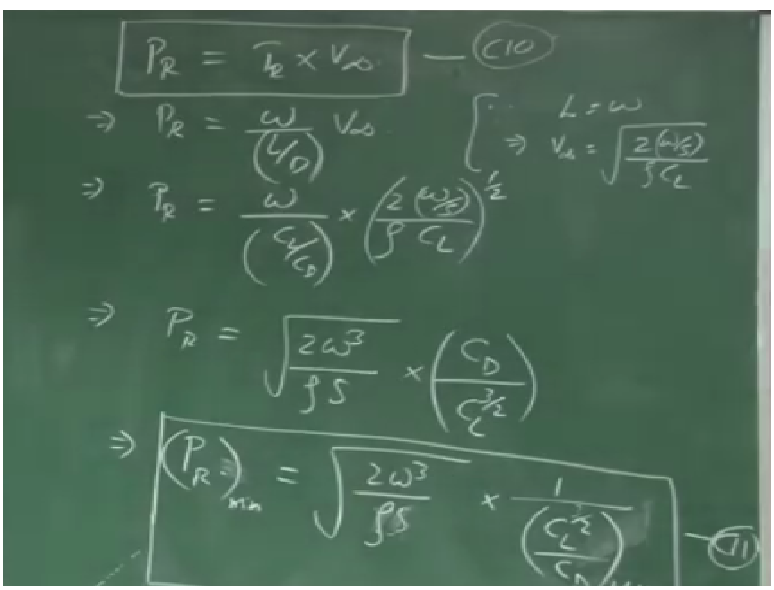

What we have, power required = thrust required * velocity, right, what is thrust required? w/ L/ D * velocity, what is w/L/; w/ CL/CD velocity is also a function of CL since L = w, V infinity =

root over twice wing loading / rho * CL, we substitute that here, what you have is 2 * w/ S/ rho * CL power CL, right to the power of 1/2, this implies power required = square root over 2w cube /rho S * CD/ CL power 3/2.

This implies power required if you want the minimum power required condition, what you need to have is rho S * 1/CL power 3/2 / CD maximum, so the minimum power required of this UAV can be obtained when you have CL power 3/2 CD maximum condition. So, let us see how to; what is the corresponding CL power 3/2/ CD maximum condition.

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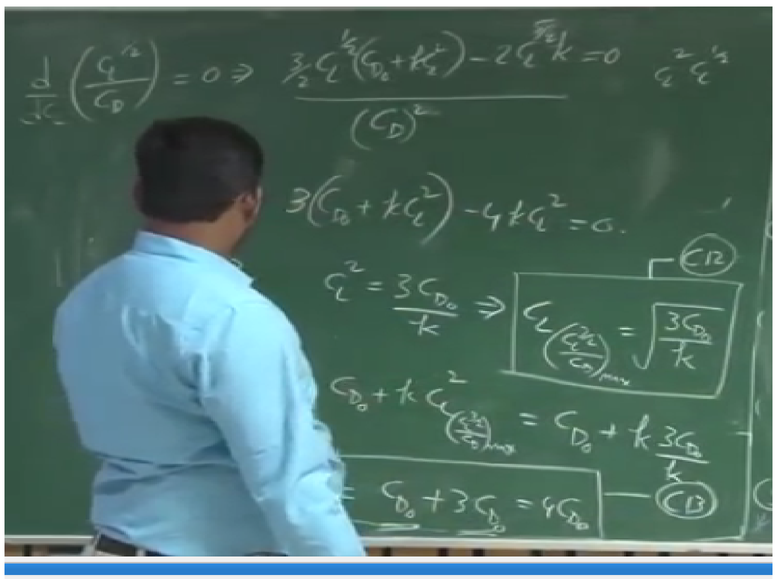

What is CL power 3/2/ CD max that can be obtained by differentiating this CL power 3/2/CD/CL and equating it to 0, this implies 3/2 CL power 1/2 * CD which is CD0 * k CL square – 2CL 2kCl power 3/2 * CL, this is 3/2 * CL power 5/2 = 0/ CD square, this = 3CD0 + k CL square = -4 k CL square = 0, am I correct? CL square + CL, this CL power 5/2 is CL square * CL power 1/2 right, x power a * x power b, x power a + b, right.

This = CL square = 3CD0/ k that implies CL, which implies CL for CL power 3/2 / CD max = root over 3 CD0/ k, this is your, which equation is c10; this is c10, this is c11, consider this as c12, right. What will be the corresponding CD? What is the corresponding CD for CL power 3/2 CD maximum? CD0 + k CL square, where the CL should be CL power 3/2/ CD max condition,

this = CD0 + k * CL square for the CL power 3/2/ CD max is 3CD0/k which turns out to be CD for CL power 3/2/CD max = CD0 + 3CD0 which is = 4CD0.

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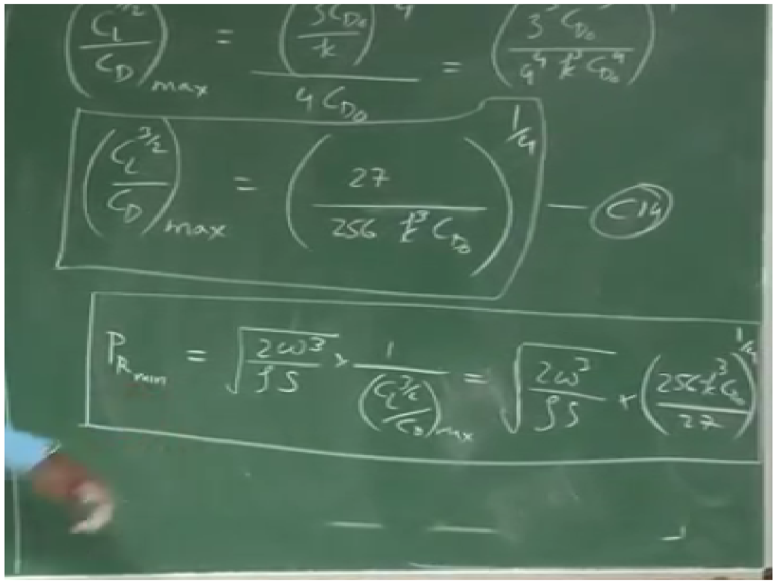

So, here if you observe the induced drag is 3 times a profile drag, you consider this as c13. So, what is CL power 3/2/CD max? this = what is CL here; 3CD0/k, 3CD0/k raised to the power of 3/4/ this is 4CD0, this = 1/4, so 3 cube * CD0 cube//4 power; 4 raised to the power of 4 * k cube * CD0 to the power of 4, so this = 3cube is 27/16, 16; 256, 256 *; /k cube * CD0 raised to the power of 1/4, this is your CL power 3/2/CD max condition, you can note it down as equation c13; c14, yes, equation c14.

Now, substitute this condition here, this power required, can you recall this equation, what is power required? Minimum = 2w/w; 2w cube/ rho S 1/ CL power 3/2/ CD max, this = square root of 2wcube / rho S * 256 k cube * CD0/ 27 raised to the power of 1/4, this is your minimum power requirement of the system, so you can estimate this minimum power requirement when you have the induced drag correction factor which can be calculated by means of planform geometry which is the, I mean planform geometry, given a planform geometry and Oswald's efficiency as well as profile drag coefficient and the weight of the aircraft as well as planform area, right.

So, once you have this parameters, you will get, you can estimate the minimum power required, so this particular minimum power required corresponds to this condition, so again we need to tell the controller that you have to fly at this particular velocity to achieve this minimum power requirement condition, right, we cannot; we cannot measure this power required on flight but rather we can measure, we have to convert this power required minimum condition to the measurable variable, right.

So, the velocity is one of the measurable variables here and the corresponding velocity for this condition is using c2, you can estimate, right

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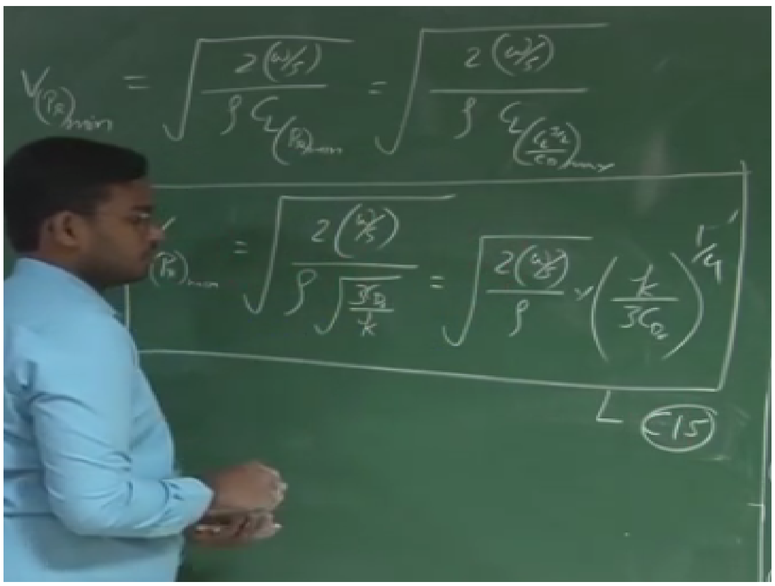

So, velocity for power requirement minimum = root over 2w/ S/ rho * CL for power requirement minimum PR minimum which is = square root over twice w/S/rho * CL for CL power 3/2 / CD maximum that implies velocity for minimum power requirement of propeller driven aircraft UAV = square root over 2w/S/ rho * root over 3CD0/k, this = square root of 2w/S/rho * k/3CD0 raised to the power of; so this is the corresponding velocity for this level flight, when you are raising a propeller engine, so let this equation be c15, okay.