Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture – 10
Introduction to Airplane Performance, Equation of Motion

Dear friends, welcome back, in our previous lectures we discussed about aerofoil and the corresponding characteristics of aerofoil, we find out what is the lift curve slope, what is CD0, CD minimum and then Cm0 or CM Ac about aerodynamic, CMAC which is about aerodynamics centre and we observed it is for most of their falls, it falls within 22 to 26% of the code and we calculated how to get CLL alpha 3D, given CLL alpha 2D, right, we related that.

Right now, let us look at here what are the external forces acting on the aircraft. The whole aim of this exercise is to find out what is the weight of a system or weight estimate; the initial estimate of the weight of the system which is by adding up the weights of individual components, right that is the weight breakup of the UAV and then what is the design CL at which you want to fly from which you can figure out what should be the corresponding area or the planform area of this UAV.

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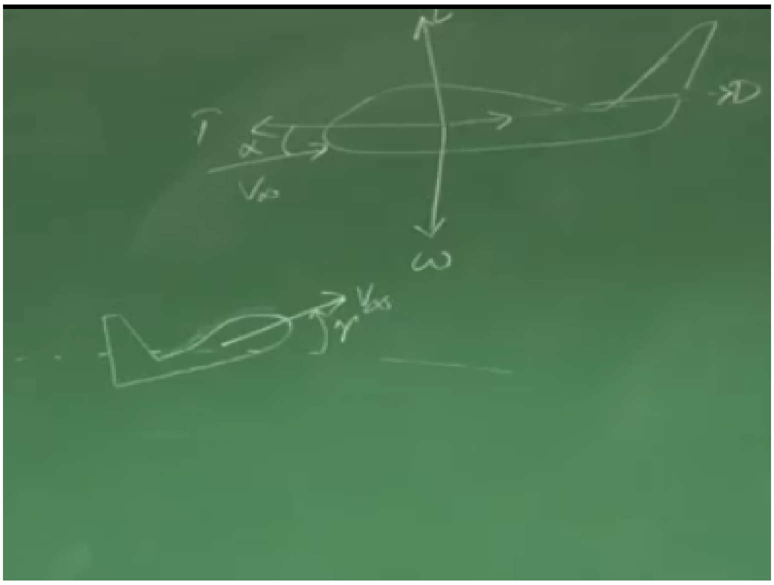

Now, let us look at what are the various external forces acting on this aircraft. So, as we discussed thrust is the force that makes this aircraft to fly at the desired velocity, we have weight acting perpendicular; vertically down and due to this thrust, the aircraft flies at a velocity V infinity and because of which we have aerodynamic forces lift and drag, okay, we have lift and drag acting along V infinity.

Now, when you see V infinity like this, what does it mean? When you draw V infinity like this, it represents the direction of motion of the aircraft, so when you say velocity, it should have both magnitude and the direction, right. So, in what direction the aircraft moves otherwise, what is the aircraft path, right? Now, let us discuss concept called flight path angle, right.

See you have your UAV moving at a velocity V infinity, right, so with respect to local horizontal, the angle made by this V infinity is known as gamma flight path angle, right. Now, is there a relationship between flight path angle, angle of attack and orientation of the aircraft, right?

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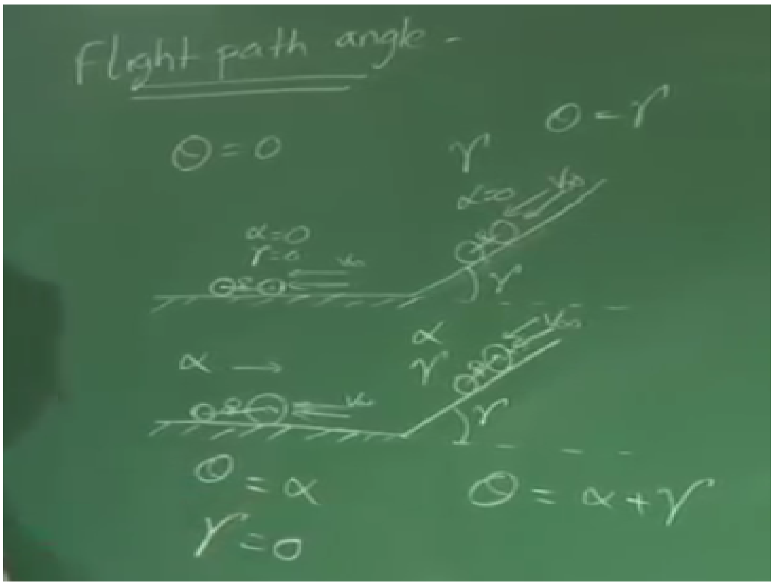

So, to understand that let us take up a small example, let us consider a level road, okay. Say, I have a bike with equal front and rear tyre, say this 2 wheels of this bikes are connected by means of this road, since we have equal front and rear tyre, this rod will be parallel to the road here, say I am sitting on this bike, if I start moving in this direction, the free stream velocity that I face will be parallel to my path in general so, my path is parallel to the road here, so the free stream velocity is parallel to me, say this is my V infinity.

Now, let us assume this is my reference line and since it is parallel to this path, right, since I have an equal front and rear tyre and the free stream is also parallel to this V infinity, parallel, to this road, so the angle of attack that I face with respect to this free stream is 0, I do not see assuming this is my; this rod is my reference line, I do not face any angle of attack and moreover, the path that I am following is parallel right.

So, the angle made by this V infinity with respect to this plane road is 0, gamma is 0 here, since it is a horizontal road, right, so I do not have any flight path angle or the; and as well as since my path is parallel to the ground at the same time, I do not see any angle of attack in this case. Now, say suddenly I encountered a flyover which is inclined to local horizontal or the initial horizontal road by an angle gamma, right.

So, I; with the same bike I start moving on this flyover right, so the free stream now will be since I am moving in this direction, the free stream will be opposite to me, right again, I am moving parallel to this flyover, the free steam will be parallel to this fly over, say this is my V infinity, right. Since I have level tyres like I mean equal diameter tyres, I still face 0 angle of attack here, alpha is 0, what I have but my free stream velocity is inclined at an angle gamma which is nothing but the path in which I am moving, right.

So, the path in which I will be moving is gamma here with respect to horizontal, now say on the same road, I have changed my bike, say I have a bigger front wheel and a smaller rear wheel, so let us say we connect these front and back tyre by means of a road, say I am sitting on this, I start moving in this direction, right, so I will start facing stream opposite to me and which is parallel to this ground, since I am moving parallel to this.

So, but in this case what I see is an angle of attack alpha because this free stream V infinity is making an angle with my reference line alpha here, so in this case, I have an alpha, right, and once I, since I am travelling on the same road, right, I will face an flyover after a while, say this is gamma again, now I am travailing with a different bike compared to this case, right with the bigger front wheel and the smaller rear wheel.

Now, again the free stream will be parallel to my path and my path is along this flyover which is inclined at an angle gamma that means, my frees stream V infinity gives the direction of my path, if I know what is the angle measure with respect to this free stream that means that angle = the angle at which my flight; my vehicle is actually moving, right, so in this case, since I have a bigger front wheel and smaller rear wheel even, so I will face a; in this case, I will see certain angle of attack right as well as I am moving on a inclined path which is gamma.

So my, what is my total orientation now? Let us; let theta represent the total orientation with respect to which is measured with respect to horizontal, right. So, in this case, what is my orientation? 0, in this case, gamma with respect to horizontal, so in this case, my orientation is alpha, right, here it is my orientation is alpha + gamma, so similar to this case, say when you are travelling on a level road, right, you still have an orientation which does not mean that since you are sitting in this at this angle, it does not mean that you are moving in this direction, right.

That means you can still have an angle of attack when you are travelling horizontal or parallel to the ground, right, even when you have, when you are flying at a level flight condition, you still have an angle of attack, so in which the gamma is 0 in this case, right because gamma is measured with respect to local horizontal right.

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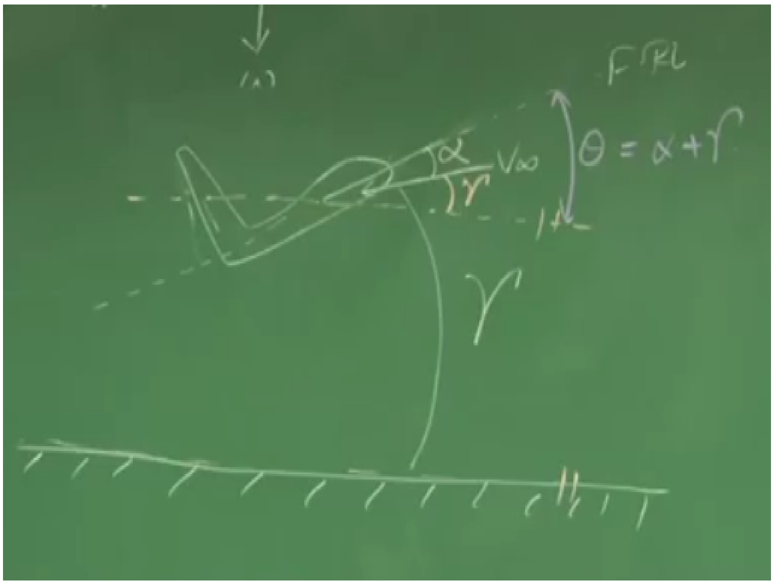

So, now let us come back to this aircraft, say this is my FRL; fuselage reference line, so my angle of attack is defined with respect to this fuselage reference line, say this is my V infinity so what does it mean, so my aircraft is actually moving in this direction, right along that is why you face V infinity opposite to you so, the free stream will be opposite to the direction of your motion right.

So, this angle, say is the angle of attack, right, say this is my local horizontal or say this is my ground right, so the angle made by this V infinity with respect to this local horizontal is gamma, right, so if I can represent that local horizontal here okay, these 2 are parallel, so this angle is gamma, right and this is alpha, now the orientation of the aircraft is theta which is alpha + gamma, so gamma gives the direction of your motion.

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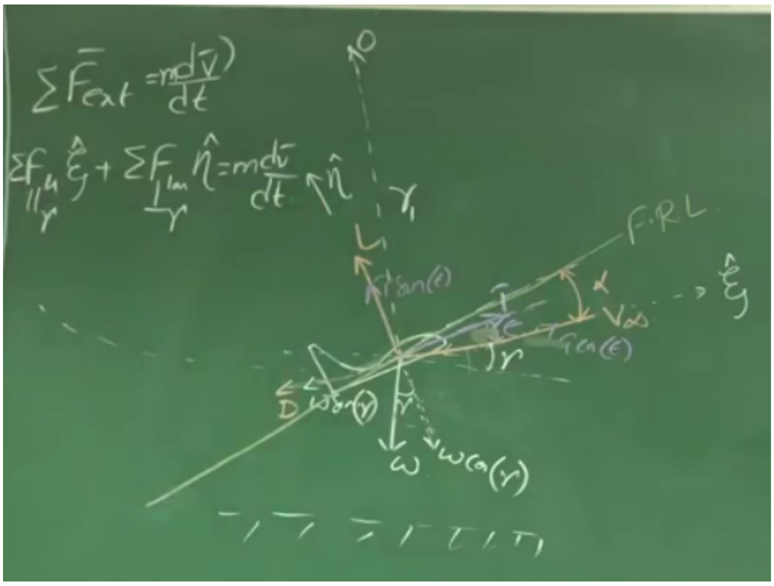

Let us consider an aircraft moving in a vertical plane, right moving in this vertical plane constraint a motion of this UAV to this vertical plane. So, let w act perpendicular to the local horizontal, right is a weight acting perpendicular to local horizontal, let T be the thrust vector which is along the fuselage reference line FRL and say the aircraft is moving with the velocity V infinity, right and this angle is known as angle of attack.

So, with respect to local horizontal, so the aircraft is moving with V infinity in the direction gamma right here, yeah, now what will be the direction of drag, along V infinity, this is drag and you have lift which is perpendicular to V infinity, this is lift. Now, let us now write down equations of motion along the flight path and perpendicular to flight path, right, so that is along V infinity and perpendicular to V infinity.

So, the component of w which is acting along perpendicular to V infinity but in negative direction of lift is w cos gamma and the component of weight acting along D is w sin gamma, right where this angle is gamma. We know V infinity and this negative lift line are perpendicular, so we know this angle is 90 degrees, this is 90, so you know, you have this as gamma, right, this is your local horizontal and weight acts perpendicular to local horizontal.

So, this angle is 90 degrees again, okay but this angle is gamma that V infinity makes an angle gamma with respect to local horizontal, so what about this angle? 90 – gamma, this becomes like 90 – gamma, so this particular portion is 90 – gamma and this is 90 degrees, so 90 – 90 – gamma will give you gamma, this is your; so, this is gamma, right, so the component of w acting along negative lift is w cos gamma and along the drag is w sin gamma.

And say this is your thrust misalignment; see it is not necessary that the thrust is along the fuselage reference line, okay. So, let us make it bit complicated, so thrust is not along FRL, right, say this is your thrust line and this is your reference line, let me take an another chalk, this is your fuselage reference line, right, this is your FRL, right, so this thrust is misaligned, so now the angle of attack will be with respect to fuselage reference line, so this is your V infinity, so this angle is alpha, right.

Now, this T is misaligned with V infinity by an angle epsilon, so your T is not along; not acting along the direction of your motion, right. So, now the component of T along your direction of motion is T cos epsilon and perpendicular to the direction of motion which is along the direction of lift is T sin epsilon. Now, let us write equations of motion for this case. Say, this particular aircraft is rotating in this vertical plane about a centre point O, which is of radius r1, right; this is performing a loop at the same time.

Now, according to Newton's second law, the total external force is acting = rate of change of linear momentum assuming mass is constant, d/dt of V bar, right. So, what are the external forces here? Thrust, lift, drag and weight that are acting on this aircraft, so now let us consider a unit vector along lift direction is eta and unit vector, which is parallel to flight path angle is zeta; zeta cap, right.

So, this is your positive direction of your eta cap and along the lift and this is the positive direction of zeta cap. So, the total external force is; force which is parallel to gamma * zeta cap + sigma F, you should say it is the total external force, right, so sigma F which is perpendicular to gamma that is perpendicular to gamma is perpendicular to free stream velocity here along eta direction, right = m * dV bar / dt.

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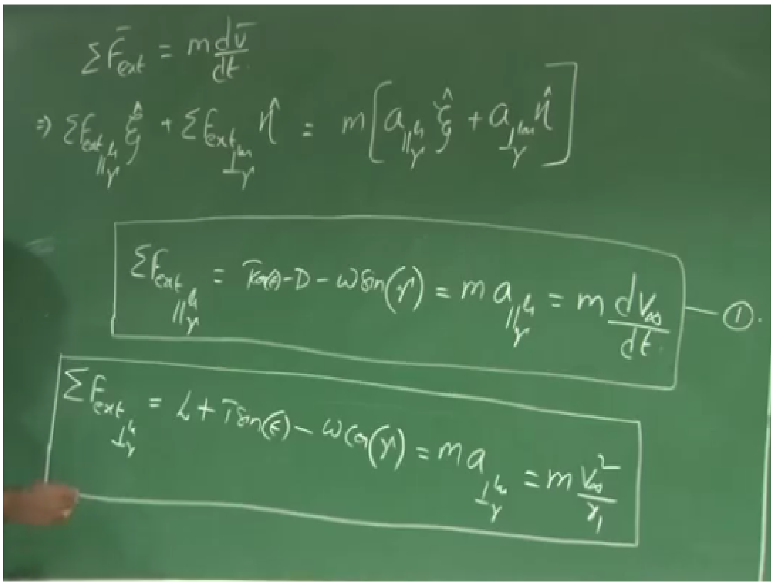

Let us rewrite those equations here, so the total external forces, F external = rate of change of linear momentum right, so while handling this equations, we assume that the aircraft is a point mass acted upon this 4 external forces, right. So, this is sigma F external, which are parallel to gamma and the corresponding unit vector along gamma is zeta cap + total external forces which are acting perpendicular to gamma eta cap = mass * acceleration, which is parallel to gamma * zeta cap + acceleration, which is perpendicular to gamma * eta cap, right.

Now, let us consider the forces or the equation of motion along the flight path angle which is by comparing the coefficients of zeta cap, right, so total sigma F external parallel to gamma =; what are the forces? Positive forces here is like, what are the total force is acting along zeta cap? T cos epsilon D and w sin gamma, so T cos epsilon is in the positive direction D and w sin gamma in the; or acting along the negative eta cap direction, zeta cap, sorry.

So, the total external forces are T – D T cos epsilon – D – w sin gamma, this = mass * acceleration which is parallel to gamma, since V bar; V infinity is a velocity along this direction, the acceleration will be and it assume, I mean the aircraft here is assumed as the point mass, so the acceleration we can represent it as dV infinity/dt. So, say this is your first equation; equation number 1.

Now, consider the second equation, zeta F external which is perpendicular to gamma = what are the forces acting perpendicular to flight path angle are lift and T sin epsilon or along the positive direction of eta cap, right along the positive direction of this axis and w cos gamma along the negative direction, so what you have as external force is perpendicular to flight path are T; L + T sin epsilon lift at a component of thrust due to its misalignment – w cos gamma = mass * acceleration perpendicular to gamma, right, flight path.

Now, we assume that this aircraft is rotating about this, about O, right which means O is the centre of this rotation which is at a radius r1, so what will be the perpendicular acceleration here; V square/r, right, so what do we do with these 2 equations, what is the motive?

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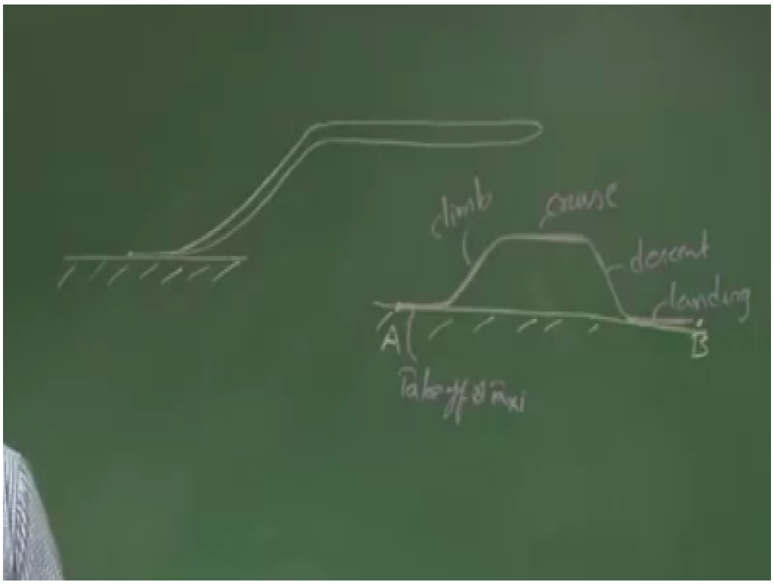

Consider a typical profile of a UAV, say this is your home right, it initially has to take off, this is taxi, and take off and then climb and then cruise, right and then take a turn and cruise back to home, descent and landing it required position or the launch position or they can be another mission, where you want to send unmanned system from one location to the other location which also involves, it take off cruise and landing, say this is your point B, this is your point A.

So, if you want to send this UAV to another location, which also involves a take off, climb, cruise, descent, and landing, take off a taxi, climb, cruise, descent and land, right, so these are the typical mission for any UAV or in fact, if you want to generalise for any aircraft, right. Now, what should I do if I want to perform this mission, say I have an aircraft, first of all I need to know what should be either the fuel or the battery that I need to carry, so that I can perform this particular mission.

When you say is there a fuel or a battery it means you need to carry so much energy, right, so first you need to know what is the requirement of this particular mission or the requirement of the system to perform this particular mission, right and also in order to perform this mission, you need to produce thrust, right and it is a variable thrust because it has to undergo various phases, so the thrust has to vary.

And moreover, we will be talking more in terms of a propeller driven aircraft, right, so we need to produce enough power, right to perform this particular mission, so first of all, we need to install an engine, which is capable of producing the required power by the system, right, how do you know that, how do you get to know? Or in other words, which engine you want to install for this particular aircraft to perform this particular mission.

And what should be your fuel weight or overall weight breakup,, how do you get to know for a given mission requirements, if this, if I want to build a UAV, what should be the weight break up or what should be the overall take-off weight, how do you estimate it and finally, how do you design the wing of this UAV, right, so the answer comes from this equations, so these 2 equations are going to help us to address all these questions.

Before proceeding further, for any design to start with or say if you consider a bike, right, so you see there are many specifications, right, it accelerates within so much time and it has got an engine with so much power, right, if the maximum speed is this much and the minimum speed like or the most economic speed is 40 to 60 or something, so all these are ratings of the systems, right.

Do you think whether this ratings have come after the design as a result of the product or they were considered before design, if you ask me I will say yes, I want these requirements from the system or I want these specifications to be the output of this system and I have designed my system and I selected the engine accordingly, so that means you have to specifies your requirement beforehand, before initiating the design.

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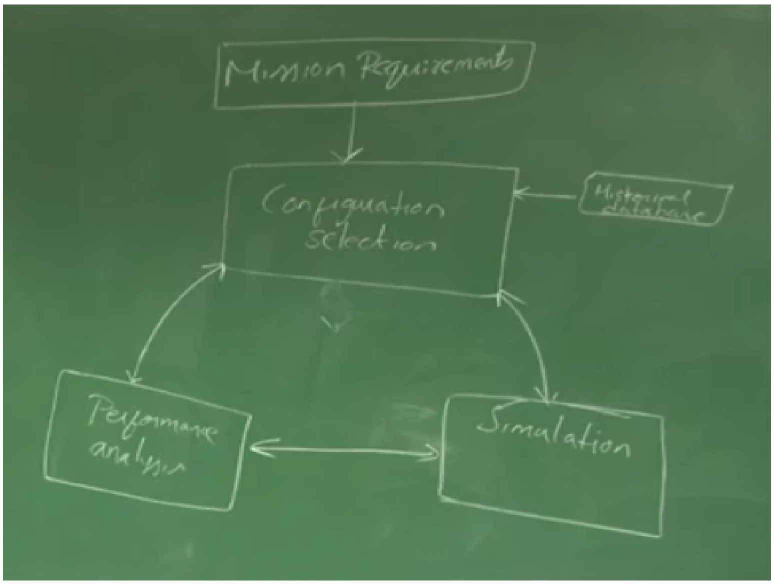

So, what you call it as mission requirement, so mission requirements are taken as an input to the design process right say, initially you take this mission requirements and we use historical data base to perform configuration selection, so what does this configuration selection includes, you need to design what should be the wing planform or you going for a wing alone configuration.

Do you have a fuselages or have you blended the fuselages into the wing or do you need a tail right, if you need why and what should be the size of that tail and how far the wing and tail should be placed, why there is a T tail, why there is a conventional tail, so all these questions to be addressed, right, so that is the configuration selection here. So, in this course we will do, we will look at a glimpses of all these things, right, whatever we are going to discuss in this flowchart.

Once you select the configuration, so that means the output here is the 3 dimensional 3D model, right, 3D model, where you have the aerofoil selection and the wing planform geometry and the tail sizing and the control surface, so you can get it as an output from this configuration selection and see whether this configuration selection satisfies the mission requirements or not, it is like performance analysis, right.

You also work with simulation say, certain parameters if you want to have the system, certain parameters, so that you can have a better control over the system and better performance, right in terms of aerodynamic parameters which are generally absorbed by simulation, so this in order to achieve that those parameters, what should be the configuration, it is not that just from the configuration you do the simulation, it is that whatever the parameters that satisfies the simulation, right.

And which are in general, addresses most of the; address most of the problems like control and all, right, so if you want such parameters to be reflected in the system, so what should be the configuration. Similarly, you have assume some mission requirements, right in which you might have specified some performance parameters, so this again a vice versa, if you want this performance parameter, what should be the wing size or the wing loading or the thrust loading.

Or say, if you want such a rate of climb, what should be the engine that you need to install on these for these particular UAV, what should be the specific fuel consumption, so all these comes here, so mission requirements will dry the entire design process, right and then again, they are interlink, simulation and performance again they are interlinked.

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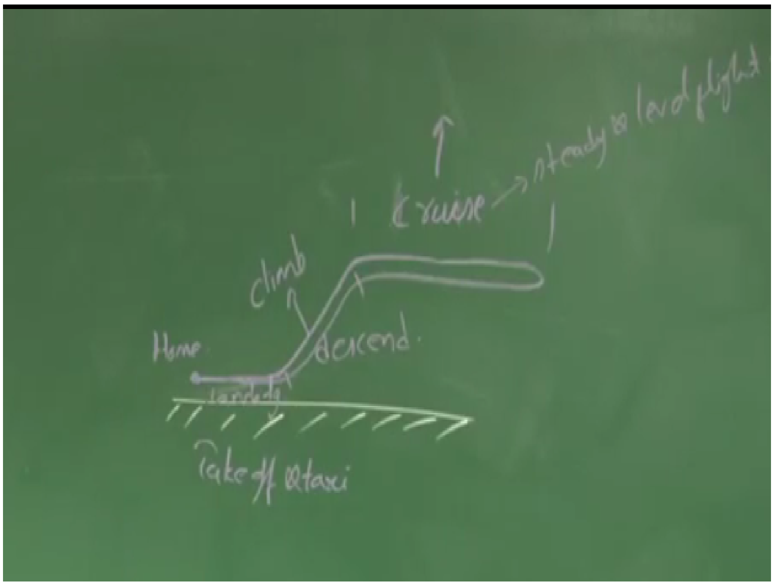

Coming back to this typical mission profile, we witnessed there are various phases during the particular mission that UAV has to perform, so here let us say, this is your home from which you are launching your UAV, it has to accelerate to the desired velocity and then, so during this acceleration phase, we can turn it as take off and taxi, right, so and then it will climb to the desired altitude and it performs a mission that you required.

So, this mission is mostly a cruise or a steady and level flight, right and then it will come back to the approach point and then start approaching the home by descending to the ground;

by descending to the required; by descending and then it will take an approach, descend and landing. So, if you look at this entire profile, so the maximum time this UAV will spend to spend performing cruise, right.

Now, you have to design your system whereas, the climb hardly takes around 2 minutes because it is an unmanned, it can have bigger higher rate of climbs, right and descend will hardly gliding flight most; in most of the cases and then landing yeah of course may be for a couple of minutes, so the major part is during cruise, the major part; this UAV has to perform is cruise right, where it involves may be 2 hours of flight, 3 hours of flight or 40 minutes of flight.

So, the ratio of times like; the ratio of time that it is spend to for cruise and to the other phases of flight is very high or in other way, the ratio of other phases of flight and the cruise time is very less, right. So, now whatever the design that you have to perform is for this cruise flight. So, now let us look at what should be the power requirement or the wing required, right to say, what should be the wing required to lift the UAV of weight w, right.

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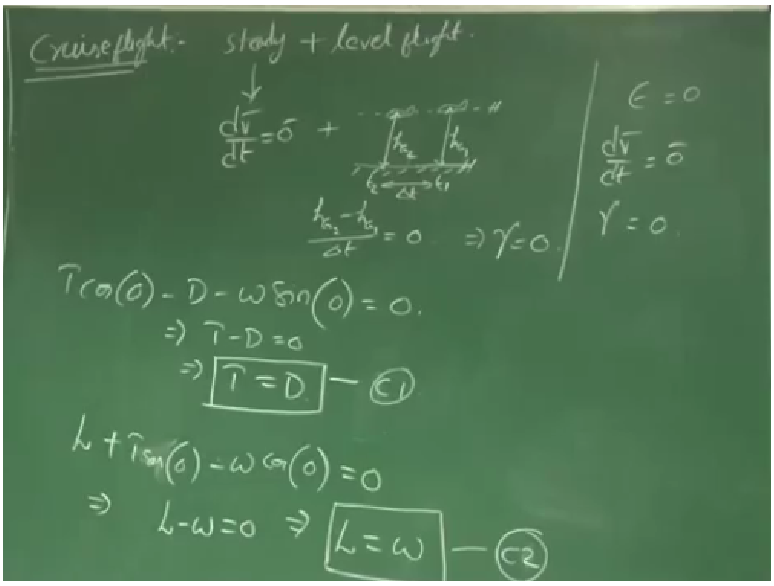

So, we have to design this wing for this cruise, now let us look at cruise flight, so what is this cruise flight? It is steady + level flight, so what do you mean by steady, the constant velocity flight, right and level flight, so earlier we discussed, right what is level flight, consider this is your ground, say this is your UAV flying at a geometric altitude of Hg at time t1, now over a delta t, say t2, you again measure the altitude of this UAV, say it is hg2 and hg1.

So, over this delta t, this hg2 – hg1 remains same, right, if it remains same, then you say it is a level flight. So, from; if you have level flight, that means the flight path will be parallel to the local horizontal, so gamma is 0 that means gamma = 0, right and further assume there is no thrust misalignment which is epsilon 0, so the total assumption CRR, it is a steady flight and level flight without any thrust misalignment.

Now, let us look at how these equations of motion in this vertical plane reduces to; substituting those epsilon 0 and steady condition and level flight condition in equation 1 and 2, we have, since a steady flight, the total; the acceleration is 0 both parallel and perpendicular components of acceleration as 0 there, so there is no misalignment further, so cos epsilon cos 0 T cos 0 – D – w sin of gamma is 0 again here is = 0.

This implies T – D = 0 that is T = D, see this is my equation; equation c1, which is equation for cruise 1, all right and now substituting the same condition for equation 2, what you have is L + T sin 0 – w cos 0 = mass * acceleration; perpendicular acceleration to flight path angle which in this case is 0, right, this is L – w = 0, which implies lift = weight of the system. If these 2 conditions are satisfied, you can have a level flight, right which is a steady and level flight.

Now, say if you have a mini jet engine, if you want to install on it, right, now thrust = drag and lift = weight, right. Now, what are the velocities that you with; at which you can achieve this cruise flight or in other words, what should be the range of velocities that are possible to have a cruise flight, right?

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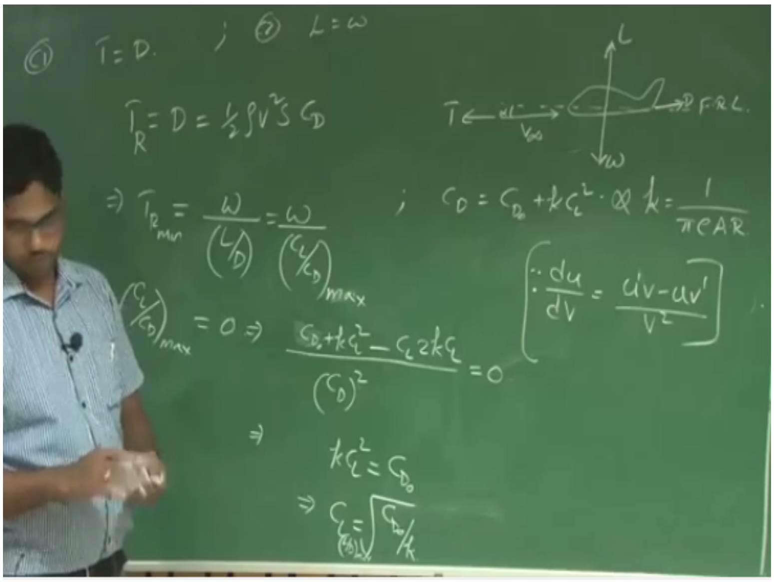

So, you have T = D from c1 and from c2, you have L = w, see what is T = D? So, in this case, how does the aircraft look like? Say this is your FRL; fuselages reference line and this is your V infinity that means, the aircraft is moving in this direction and this is your alpha and weight is acting perpendicular to the free stream, sorry, to the local horizontal and lift is acting perpendicular to free stream.

And you have thrust along the free stream, since there is epsilon is 0 and you have thrust along free stream and drag acting along the free stream, so this V infinity is parallel to local horizontal here that means the gamma is 0, right. So, in this case, if you want to have a level flight whatever the drag that is generated by the system should be overcome by the thrust that means, drag is the requirement of the system to move at that particular velocity, right.

For example, so T is a; T is D; T = D * 1/2 rho V square S * CD, okay. Drag is not the requirement in fact, drag is the systems output, right is the response, I mean it is the output from the system when moving at that particular velocity. Now, this is the; this is the negative force right, now the system has to generate thrust, which is equivalence to this drag in order to have this level flight.

That means, this is required by the system to perform this particular level flight at this velocity V infinity, right. So, what I can do is thrust required = w/L/D, right, so dividing equation 1 and 2, by performing a simple manipulation like T/w = L/D, this implies, thrust required by the system = w/ L/D; D/L, sorry, D/L, which is L/D that is = w/ CL/CD, here which is w/CL/CD.

Now, what should be the minimum thrust that the air craft has to produce, so that you can still have a level flight, right, so this minimum; thrust required minimum = maximum L/D, right, so if we have this L/D maximum, you can get the corresponding minimum thrust required for this particular UAV of weight w. So, what is CL/CD? Maximum, how can you get? Differentiate with respect to CL and equate it to 0.

Why CL, why not CD? CD is a function of CL, right, CL is a variable here, so where CD = CD0 + k * CL square, right, so this implies CD square * CL okay * CD0 + K CL square – CL * 2k CL = 0, how we have got this? Du/dv = u dash v – uv dash/ v square, multiplying by CD square on either side of these equation, what you have is; k CL square = CD0, yeah this is 2k CL square and this is k CL square, you have to take it to the right hand side, you will get k CL square = CD0.

This implies CL = root over CD0/k, where you know k is since and k = 1/pi e AR, you know how to calculate aspect ratio and you need to know what is a Oswald's efficiency of UAV, right, then you will be able to find out what is the corresponding CL condition for L/D maximum, L/D max or CL/CD maximum ratio, this is the CL condition.

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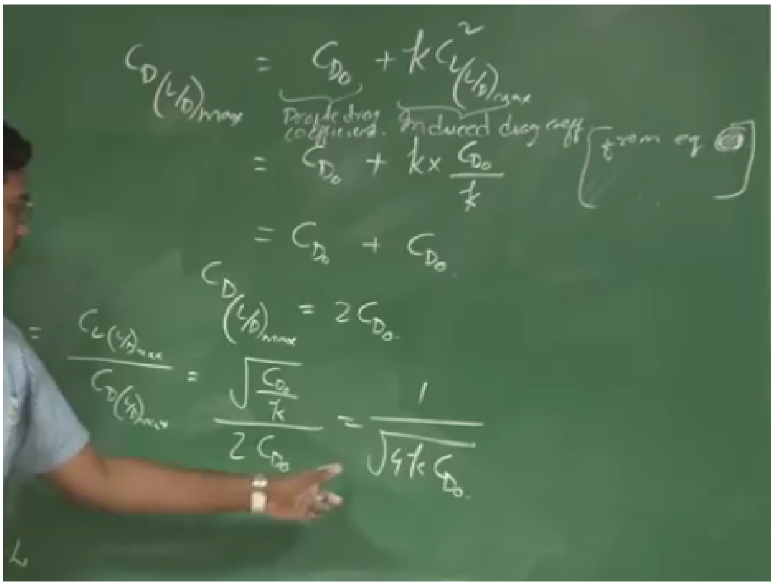

Now, what will be the CD for L/D max? CD = CD0 + K * CL square CL for L/D max, max square, this is = CD0 + what is CL for L/D max, it is CD0/k, so this say equation 1 for this case, from equation 1 or say c3, cruise equation 3, so this is c3, k * CD0/k which is = CD0 +

CD0, okay, so this particular term is known as induced drag coefficient; induced drag coefficient, this is profile drag coefficient, right.

In this case, during L/D max condition, your profile drag = induced drag, right, so the total drag coefficient here is for L/D max which is equivalent to 2CD0, right. Now, what is L/D max = CL for L/D max and CD for L/D max, this = square root over CD0/k/ CD0, 2CD0 this = 1/root over 4k CD0, you see here that is L/D max is 1/ root over 4k CD0. Now, how do you get minimum thrust requirement of this system?

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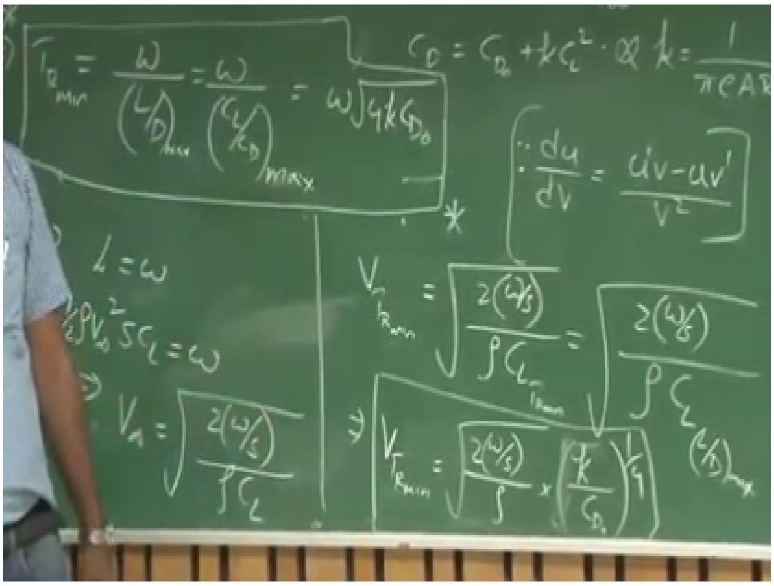

We need to substitute this L/D max, CL/CD max condition here, so this = w * square root over 4k CD0, so thrust required minimum is obtained by L/D maximum, which is w under root over 4k CD0, so you can calculate what is the minimum thrust requirement to achieve this level flight, if you know the information about the weight of this UAV and the induced drag correction factor and CD0, profile drag coefficient, right.

If you know this information, you will be able to figure out what is the minimum thrust required? Now, say if you want to fly at this minimum thrust required, how do you do or say will the pilot understand fly at this w * root over 4 k CD0, now we need to translate to the, to his language, right, so what he can understand is what should be the velocity of flight, right. Now, we will figure out what should be the corresponding velocity of flight to achieve this level flight with minimum thrust requirement.

We have c2, from c2 we have L = w, so this = V infinity, which is 1/2 rho V infinity square = w; SCL; sorry, 1/2 rho V infinity square SCL = w and V infinity = root over twice the wing loading/ rho * CL. Now, velocity for thrust required minimum, V for Tr minimum = root over twice the wing loading and the over density of the air at that particular altitude right * CL for Tr minimum.

What is the value of CL for the corresponding thrust required minimum, this = square root over 2 times w/S/ rho *; what is CL for Tr minimum, CL condition for L/D maximum, so when is this thrust required is minimum, when you have L/D maximum, how do you achieve this L/D maximum, when CL is the corresponding CL and CD for L/D max, so the corresponding CL for L/D max is root over CD0/k, right.

So, this implies velocity for thrust required minimum = root over 2w/ S/ density * root over CD0/k that is k/CD0 raised to the power of 1/4, this is the corresponding velocity at which you need to fly if you want to maintain minimum thrust requirement, right and you still have a level flight. Say, if you do not want to fly at this minimum velocity or minimum thrust required velocity, since your mission may demand you to fly at a different velocity right.

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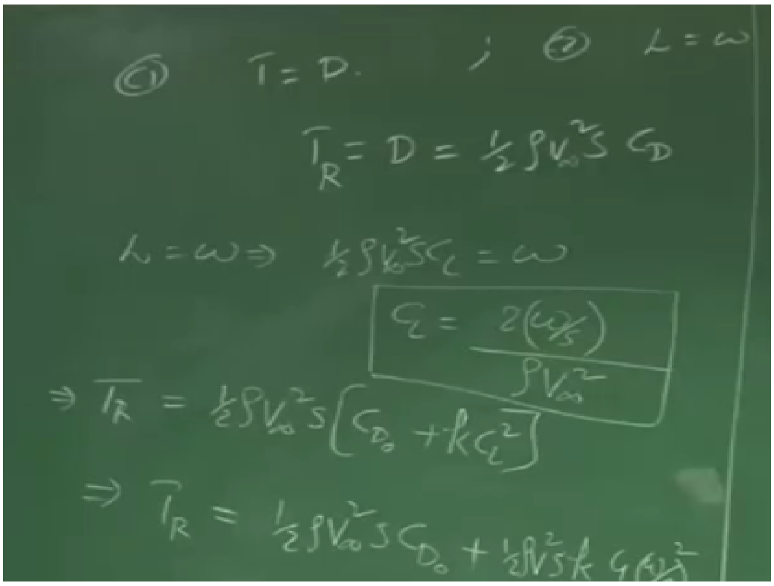

So, what should be the solution, what will be the corresponding thrust required there, right, we will discuss that right. Now, let us have a closer look at this equation c1, thrust required = drag which is 1/2 rho V square S CD and from c2, we have L = w, this implies 1/2 rho V square S CL = w, where CL = 2w/S/rho * V infinity square. Now, thrust required = 1/2 rho V infinity square S * CD0, we express CD as in the form of the drag polar CD0 + K CL square.

So, you can get CL from here, substitute this CL in this equation, so thrust required = 1/2 rho V infinity square S CD0 + 1/2 rho V infinity square S k * CL square, which is 4w/S square/ rho square or rho infinity square V infinity power 4, right.

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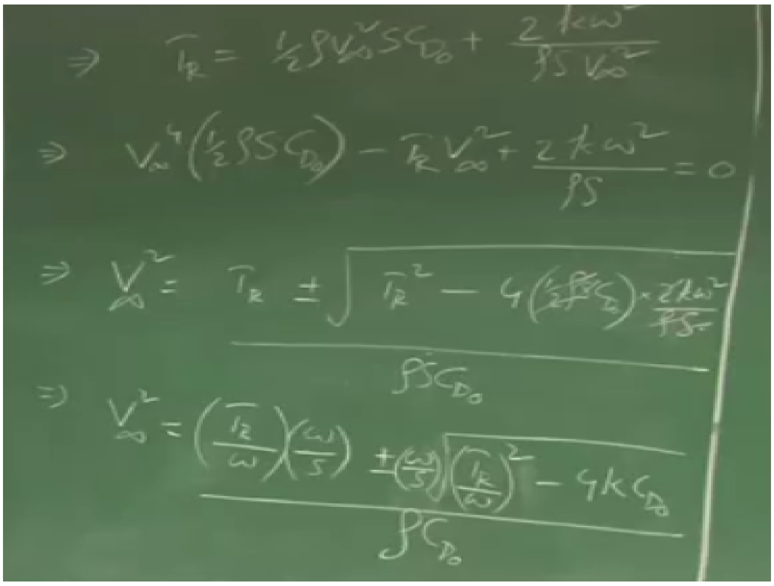

This = Tr 1/2 rho V infinity square, S * CD0 + 2k w square; 2 times k w square/ S/ rho, so V infinity raised to the power of 4 * 1/2 rho S CD0 – Tr * V infinity square + 2k w square / rho S = 0. So, what are the roots of this equation? So, V infinity square = -b + or – root over – b square – 4 AC; 4 * 1/2 rho S CD0 * 2 k w square/ rho S/ 2A rho S CD0, S, S cancels out, rho, rho cancels out, so am I correct.

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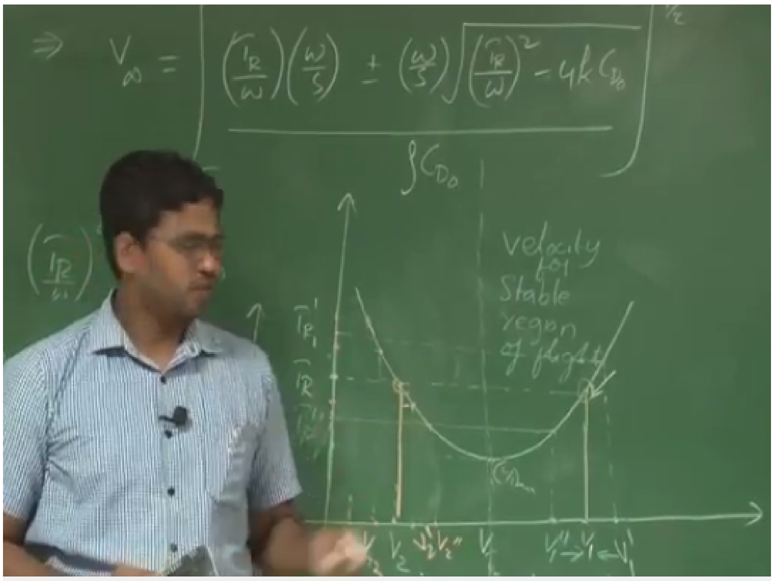

Tr/w, so thrust loading * wing loading + or – wing loading, the square root over Tr/ w whole square – 4k CD0 / rho, right. Now, let us look at how this thrust varies with velocity, thrust and V infinity is a typical variation of thrust with velocity, thrust required with velocity, right, so this is the point of minimum thrust requirement that is Tr minimum in the corresponding velocity for Tr minimum, which is obtained by L/D max condition.

So, the CL for L/D max is root over CD0/k, so you have to trim your aircraft to achieve that CL in order to fly at this particular V infinity, right, so that you are flying at a minimum thrust requirement condition, right and the corresponding angle of attack for trim we can find out by using the linear expression of CL, where CL = CL0 + CL alpha *alpha right, now this is your minimum thrust required condition.

Now, see if we observe here, for any; for a given thrust requirement, you can fly at 2 different velocities, v1 and v2, say if you want to fly at this particular thrust required condition Tr, so the options that you have is either you can fly at V1 and V2, so which velocity you need to choose, okay, let us divide this plot into 2 halves based upon the minimum thrust required condition.

So, by the way when this minimum thrust require occurs that means, you have a unique solution here, where if this discriminates = 0, you will get this unique solution, right, you will get a single velocity, so what is that discriminates? Tr / w whole square – 4k CD0 = 0, this implies Tr/w = root over 4k CD0, this = Tr = w * root over 4k CD0, so is it not the minimum thrust required condition, this is a minimum thrust required Tr minimum, minimum thrust required condition, right where L/D max is 1/root over 4k CD0.

What is thrust required is w/ L/D, right, so this particular point corresponds to that particular, I mean, this points corresponds to this thrust required minimum condition and the corresponding velocity you can attain, so by if you equate it to 0, then you will have a unique solution from which you can find out what is the corresponding velocity of flight. Now, say if I want to fly at this thrust required condition, if I fly at then what is the velocity that I have to choose?

V1, there are 2 solutions here, right, you have + or – here, you have 2 solution, now which velocity you need to choose to fly?

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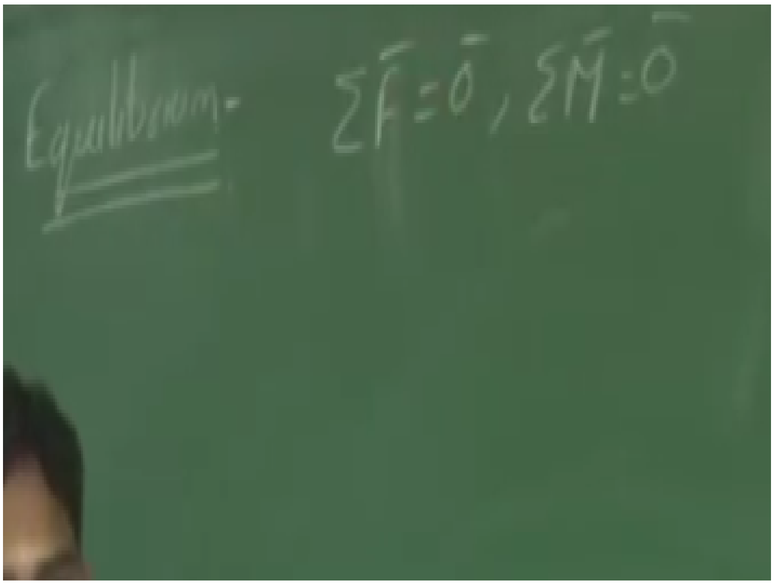

Now, let us define, so what is equilibrium? The state of the system about which resultant forces and moments are acting are 0, right, that means, there is no acceleration either linear or angular, right, either the body is moving at a constant velocity, now we can say cruise is a steady at level flight where there is a constant velocity right, which is = equilibrium, so if you are flying at different velocities that means, you are producing different thrust required from the engine, ultimately you have to give supply the thrust required to the system, right to the UAV.

So that thrust required need to be generated from the engine, right. Now, when you are flying at different thrust required condition and different velocities, all these points are equilibrium points as for as level flight is concerned, right, so at; aircraft have a range of equilibrium conditions, right. Now, what is equilibrium here; resultant forces and moments are 0, okay. Now, let us consider velocity v2, right.

Now, due to some; now during this condition, if you are flying at velocity V2 or during this discussion, let us assume the pilot from the ground station has set the throttle of this UAV at a particular setting, right has trimmed the UAV to a particular throttle setting. Now, we are not controlling the UAV, right, now say there is sudden gust, which has increased the velocity.

Now, say has a, see this is the direction of increased velocity, now say this increase in velocity will take you to this point, let us say, let this will be V2 prime, let this be V2 prime, so this is the point that has increased the velocity, so the corresponding thrust required is Tr prime or Tr 2 prime at this case, so that means that the thrust required has decreased but the throttle is; the engine is delivering higher thrust and you are not touching the throttle right.

Now, the thrust required is decreased and the drag is decreased eventually that means, the drag is reduced, so the vehicle accelerate, there is an axis force that accelerates the system, this is the difference, so the vehicle try to further accelerate that means, the velocity further increases, in the corresponding; say this to V2 double prime and the corresponding thrust requirement decreases, V2 double prime, this is your Tr2 double prime, this has decreased further.

So, your drag decreases, the drag of the system decreases and this further accelerate, so you are not coming back to your initial equilibrium right, now say we assume that there is an increase in velocity because of the disturbance, now say there is a decrease in velocity because of the disturbance, so again we have come back to our original equilibrium say, this is my V2, Tr, Tr; V2, Tr is a corresponding coordinates of this, right of this point.

Now, say the velocity has reduced because of some external wind disturbances, right, since we are not touching the throttle here again, so this reduced velocity will result in an increased thrust here, the corresponding thrust requirement of this point is Tr, let this point be VA2, right and the corresponding thrust requirement is TrA2, right, so this has, I mean at this particular velocity, the thrust requirement has increased, which means the drag of the system has increased, right.

Now, this increase in drag will decelerate the system that means, the velocity further decreases, say it will come VA2 prime, say this is your VA2 prime; VA2 prime and the corresponding thrust is TrA2 prime, so this is your Tr A2 prime, so that means the thrust requirement has increased further, right which will decelerate the system further that means, the UAV is not coming back to its equilibrium by itself, either the pilot has to give a constant increase and decrease in the throttle even for a small disturbance.

Now, let us look at the other solution here, let this be, let this point be the other solution, why because this if you want to fly at this Tr either you can choose this velocity V, V2 or you can choose V1, right. Thrust, say this is the point of my interest thrust required at this particular location so, I can achieve this level flight either by flying V2 or V1, so from these equation, right, either V2 or V1.

Now, we understood if I fly at V2, I need to give a constant correction to this, right; it cannot come back to its equilibrium by itself. Now, say if I am flying at V1, see again let us repeat the same story right, you have a disturbance that has increased the velocity of the system that means, this is your increased velocity, so V1, V1 prime right, so this V1 prime will also increase the thrust required by the system, right.

Let this be Tr 1 prime, right, so this has increased the since, the thrust requirement increased, and moreover you are not touching the throttle of the system, you are not giving the extra energy to sustain this velocity, right, so the vehicle decelerates that means, the velocity decreases that means, you start coming towards this point, from this point to this point, right and now say, if your vehicle velocity is decreased, because of external disturbance from V1 to V1 double prime, okay.

Now, the corresponding thrust requirement is also reduced, so from this plot you can see the corresponding thrust requirement also reduced, so compare to Tr, Tr1 double prime is lesser, right, so since you are not touching the throttle again, this additional thrust that is available will accelerate the system to the trim velocity or the equilibrium; earlier equilibrium velocity that means, without any external control if there are any; if there is any disturbance, if you are flying at this point, the aircraft will automatically come back towards it equilibrium.

That is why to the right of this, to the right side of this line, it is known as stable region of flight that is velocity for stable region of flight, this is the unstable region of flight, right, so stability is an inherent property to come back towards its equilibrium once disturbed from it, right, it should come automatically. Now, why it is coming, why this curve looks like this, we will discuss in the coming lectures.