Fundamentals of Combustion (Part 2)
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Lecture - 48
Determination of Laminar Burning Velocity for Premixed Flames

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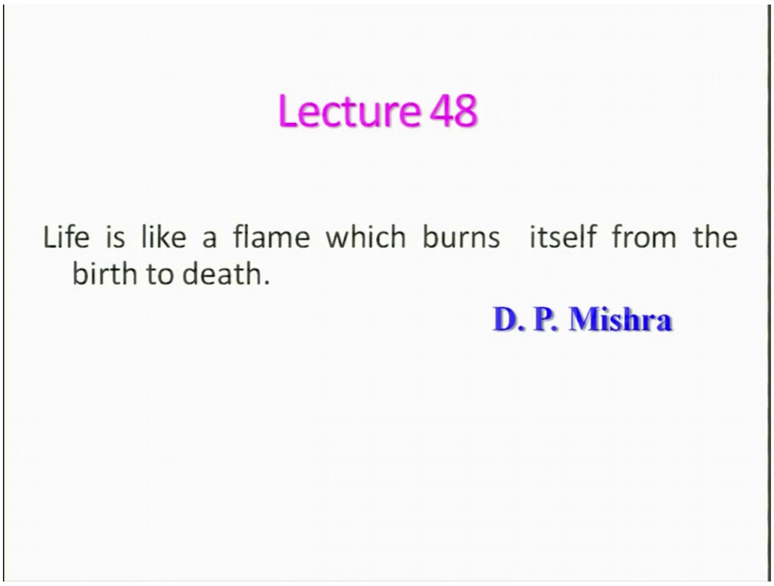

Lecture 48

Life is like a flame which burns itself from the birth to death.

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Let us start this lecture with a thought process life is like a flame which burns itself from the birth till death. So, in the last lecture, we basically derived an expression for burning velocity of one dimensional pre-mixed flame of course laminar burning velocity.

Today what we will do will basically take an example how to determine the burning velocity in actual situations, right that we will consider.

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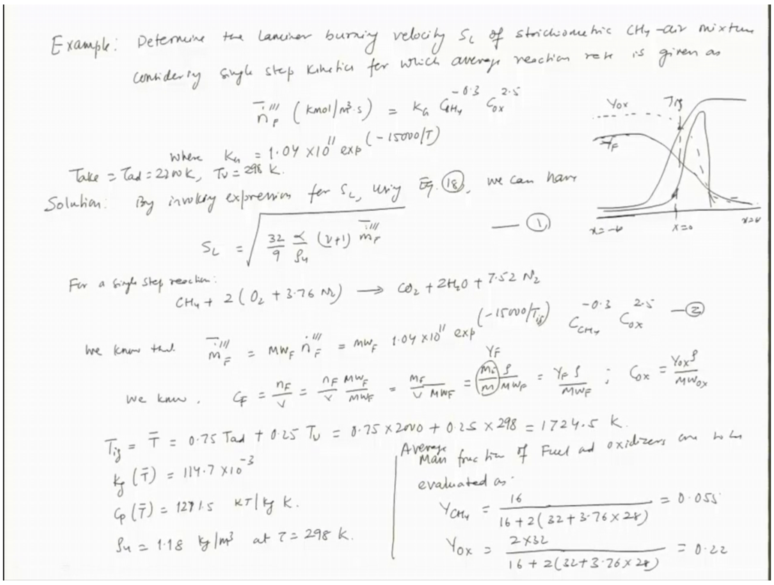

Example: Determine the laminar burning velocity S_L of stoichiometric CH_4-air mixture considering single step kinetics for which average reaction rate is given as

$$\bar{\dot{n}}_F''' \; (kmol/m^3 \cdot s) = K_G \, C_{CH_4}^{-0.3} \, C_{ox}^{2.5}$$

where $K_G = 1.04 \times 10^{11} \exp(-15000/T)$

Take $= T_{ad} = 2200\,K$, $T_u = 298\,K$.

Solution: By invoking expression for S_L, using Eq. ⑱, we can have

$$S_L = \sqrt{\frac{32}{9}\frac{\alpha}{\rho_u}(\nu+1)\,\bar{\dot{m}}_F'''} \quad \text{— ①}$$

For a single step reaction:

$$CH_4 + 2(O_2 + 3.76 N_2) \longrightarrow CO_2 + 2H_2O + 7.52 N_2$$

We know that $\bar{\dot{m}}_F''' = MW_F \, \dot{n}_F''' = MW_F \; 1.04 \times 10^{11} \exp(-15000/T_{ig}) \; C_{CH_4}^{-0.3} \, C_{ox}^{2.5}$ — ②

We know, $C_F = \frac{n_F}{V} = \frac{n_F}{V}\frac{MW_F}{MW_F} = \frac{m_F}{V\,MW_F} = \overset{Y_F}{\boxed{\frac{m_F}{m}}}\frac{\rho}{MW_F} = \frac{Y_F\,\rho}{MW_F}$; $C_{ox} = \frac{Y_{ox}\,\rho}{MW_{ox}}$

$T_{ig} = \bar{T} = 0.75\,T_{ad} + 0.25\,T_u = 0.75 \times 2000 + 0.25 \times 298 = 1724.5\,K$.

$k_g(\bar{T}) = 114.7 \times 10^{-3}$

$C_p(\bar{T}) = 1271.5 \; KJ/kg\,K$.

$\rho_u = 1.18\,kg/m^3$ at $T = 298\,K$.

Average Mass fraction of Fuel and oxidizers are to be evaluated as:

$$Y_{CH_4} = \frac{16}{16 + 2(32 + 3.76 \times 28)} = 0.055$$

$$Y_{ox} = \frac{2 \times 32}{16 + 2(32 + 3.76 \times 28)} = 0.22$$

Determine the laminar (Refer time 01:00) burning velocity SL of stoichiometric CH4 air mixture considering single step kinetics for which average reaction rate is given as kilo mole (Refer time 02:00) meter cube second, right is equal to KG CH 4 0.3 and C oxidizer 2.5 where 1.04 into 10 power 11, right. Basically we will have to find out SL. So, what you call in working expression for SL using equation, what would be question that is it.

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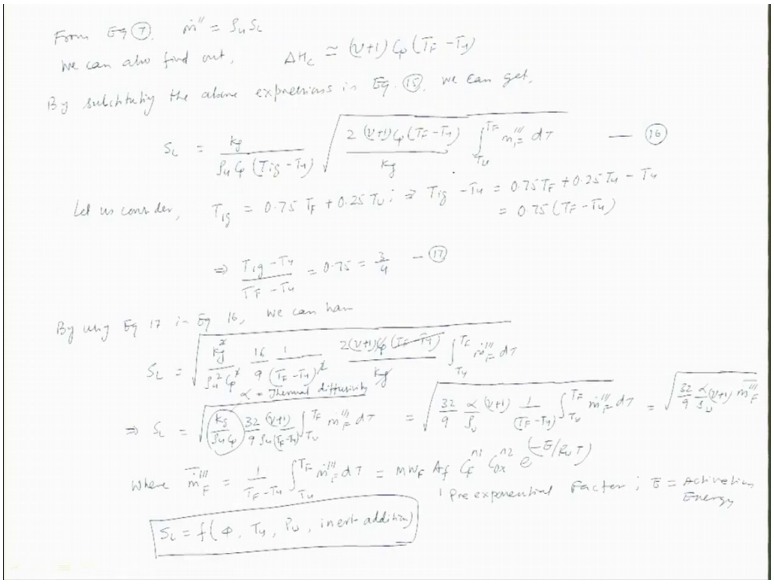

From Eq ⑦. $\dot{m}'' = \rho_u S_L$

We can also find out, $\Delta H_c \simeq (\nu+1) C_p (T_F - T_u)$

By substituting the above expressions in Eq. ⑮, we can get,

$$S_L = \frac{k_g}{\rho_u C_p (T_{ig} - T_u)} \sqrt{\frac{2(\nu+1) C_p (T_F - T_u)}{k_g} \int_{T_u}^{T_F} \dot{m}_F''' \, dT} \quad \text{— ⑯}$$

Let us consider, $T_{ig} = 0.75\,T_F + 0.25\,T_u$; $\Rightarrow T_{ig} - T_u = 0.75\,T_F + 0.25\,T_u - T_u = 0.75(T_F - T_u)$

$$\Rightarrow \frac{T_{ig} - T_u}{T_F - T_u} = 0.75 = \frac{3}{4} \quad \text{— ⑰}$$

By using Eq 17 in Eq 16, we can have

$$S_L = \sqrt{\frac{k_g^{\cancel{2}}}{\rho_u^2 C_p^{\cancel{2}}} \frac{16}{9} \frac{1}{(T_F - T_u)^{\cancel{2}}} \frac{2(\nu+1)\cancel{C_p}\cancel{(T_F - T_u)}}{\cancel{k_g}} \int_{T_u}^{T_F} \dot{m}_F''' \, dT}$$

α = Thermal diffusivity

$$\Rightarrow S_L = \sqrt{\boxed{\frac{k_g}{\rho_u C_p}} \frac{32}{9} \frac{(\nu+1)}{\rho_u (T_F - T_u)} \int_{T_u}^{T_F} \dot{m}_F''' \, dT} = \sqrt{\frac{32}{9}\frac{\alpha}{\rho_u}(\nu+1)\frac{1}{(T_F - T_u)}\int_{T_u}^{T_F} \dot{m}_F''' \, dT} = \sqrt{\frac{32}{9}\frac{\alpha}{\rho_u}(\nu+1)\,\bar{\dot{m}}_F'''}$$

Where $\bar{\dot{m}}_F''' = \frac{1}{T_F - T_u}\int_{T_u}^{T_F} \dot{m}_F''' \, dT = MW_F \, A_f \, C_F^{n1} \, C_{ox}^{n2} \, e^{(-E/R_u T)}$

Pre exponential Factor ; E = Activation Energy

$$\boxed{S_L = f(\phi, T_u, P_u, \text{inert addition})}$$

Oh I have not given an equation. This will be 18, equation 18. You can have 2 by 9 alpha by rho u nu plus 1 average.

So, this is a single step reaction, right. So, we are using a single step chemistry. So, I can write down that is methane is reacting with basically two of oxidizer 3.7 6 N 2 going to 2 plus 2 water plus 7.52 N 2, right. This is basically for single step reaction. And we need to evaluate basically you know all these properties like new we can find out what it would be, right. We will have to also find out basically the mass fractions of that right because we will have to evaluate this M dot triple dash F.

So, we know that M dot triple dash F average is equal to nothing, but M WF n F, right. So, that is M W F into Kg. Kg is basically 1.04 10 11 and I can write down x p 15,000 by T into C H. So, if you look at I can express that CA concentration in terms of the mass fractions, right that will be or let me write down, right. Now, CH 4 0.3 into C ox 2.5, but we know the concentration basically C F I can write down n F by volume is nothing, but your n F by volume molecular weight of fuel divided by molecular of fuel, right and this is nothing, but your mass of fuel and V is if you look at this is Y v right into MWF and this I can write down M F by M into rho by MWF is equal.

This is nothing, but here Y f. So, Y f rho by MWF, right. So, similarly I can write down as Y oxidizer row by MW oxidizer. So, if I say this is equation 1 and this is 2, then I can put these values are concentration equation 2 and I can get. Keep in mind that this reaction is basically occurring at where it will be something like the ignition temperature you can say, right or I can if you look at this if I say my X is 0 and this corresponding T ignition, this is my reaction rate.

So, kind of things, so naturally what will be happening this here my temperature will be T ignition. So, T ignition we already know. I am saying this as basically some average temperature between the reaction zone in the reaction zone, where there is equal to 0.75 which is nothing, but the ignition right, T adiabatic plus 0.25 T u. So, T adiabatic is not given, but if you look at T adiabatic for this thing will be around 2,200 Kelvin, right rate 0.76 into 2000 Kelvin Kelvin plus. In this example, of course the T u is not given, but in real situation will be given.

You can say take T adiabatic is 2200 Kelvin Kelvin and T u is equal to 300 Kelvin. So, 0.25 into 300, it will become something 1724.5 Kelvin. Oh I am sorry. So, this is coming basically 298. So, this at this temperature, this is corresponding to basically the ignition, right at that you will be evaluating otherwise the temperature is changing. Where to evaluate that is the question. So, therefore, this is corresponding that and you will be evaluating that portion, some average temperature will be taking and this average temperature is right.

And also the properties you will be evaluating there right evaluating at this, for example like if I want to find out the properties Kg of T average will be 114.3, right and Cp you will be taking as basically 271.5 this will be kilo joule per kg Kelvin, however the row u you will have to evaluate at the again the row average. You will be in row u in this case properties, you will be finding out as a row u is equal to 1.18 Kg per meter cube and this is evaluated at T is equal to 298 Kelvin, right. So, these properties of course will be given in your example, but here you will have to take at that average temperature.

Now, what we will do? We will basically now evaluate the M dot F and for that we need to evaluate the mass fraction of the methane and the oxidizer and what we will be doing, we will be taking because if you look at the mass fraction of the fuel, let us say this is your mass fraction, it will be this is your Y oxidizer and this is my fuel kind of things, right. This is why F right and this will be you know it will be highest year and lowest in the product side, right. This is your X is equal to infinity, X is equal to minus infinity right and then, where you will evaluate? Basically you will have to evaluate the average value somewhere here, right which will be taking because each along the X direction is changing.

So, you are basically trying to evaluate average property. So, we will be evaluating the properties at average. The mass fractions of fuel and oxidizers had to be, mass fraction of fuel had to be evaluated as. Let us first you know evaluate the mass fraction of fuel and oxidizer from basically this expression in the chemical reaction, that is Y CH 4 will be 16 divided by 16 plus 2 into 32 plus 3.76 into 28.

Thus, that will be coming around 0.055 and this mass fraction at the inlet or X is equal to minus infinity. Similarly what will be oxidizer? It will be 2 into 32 divided by 16 plus 2 into 32 3.76 into 28. It will be something 0.22. Rest of the of course will be oxygen nitrogen, right however, when we will use this thing in expression for the mass congestion rate of the fuel right per unit volume in equation 2, then I will have to use the average values, right.

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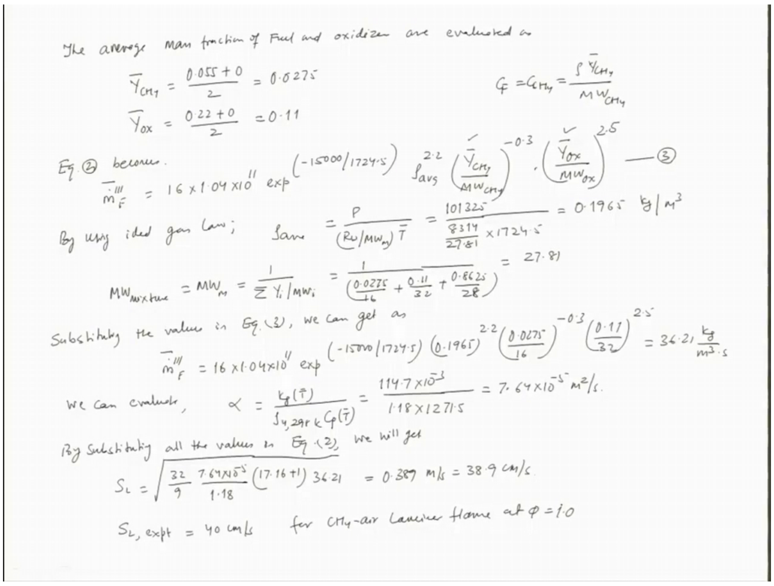

The average mass fraction of Fuel and oxidizer are evaluated as

$$\bar{Y}_{CH_4} = \frac{0.055+0}{2} = 0.0275 \qquad C_F = C_{CH_4} = \frac{\rho \bar{Y}_{CH_4}}{MW_{CH_4}}$$

$$\bar{Y}_{ox} = \frac{0.22+0}{2} = 0.11$$

Eq. ② becomes.

$$\bar{\dot{m}}'''_F = 16 \times 1.04 \times 10^{11} \exp\left(-15000/1724.5\right) \rho_{avg}^{2.2} \left(\frac{\overset{\checkmark}{\bar{Y}_{CH_4}}}{MW_{CH_4}}\right)^{-0.3} \cdot \left(\frac{\overset{\checkmark}{\bar{Y}_{ox}}}{MW_{ox}}\right)^{2.5} \quad \text{— ③}$$

By using ideal gas law;

$$\rho_{ave} = \frac{P}{(R_u/MW_m)\bar{T}} = \frac{101325}{\frac{8314}{27.81} \times 1724.5} = 0.1965 \ kg/m^3$$

$$MW_{mixture} = MW_m = \frac{1}{\Sigma Y_i/MW_i} = \frac{1}{\left(\frac{0.0275}{16} + \frac{0.11}{32} + \frac{0.8625}{28}\right)} = 27.81$$

Substituting the value in Eq. (3), we can get as

$$\bar{\dot{m}}'''_F = 16 \times 1.04 \times 10^{11} \exp\left(-15000/1724.5\right) (0.1965)^{2.2} \left(\frac{0.0275}{16}\right)^{-0.3} \left(\frac{0.11}{32}\right)^{2.5} = 36.21 \frac{kg}{m^3 \cdot s}$$

We can evaluate,

$$\alpha = \frac{k_g(\bar{T})}{\rho_{u,29rk}\, C_p(\bar{T})} = \frac{114.7 \times 10^{-3}}{1.18 \times 1271.5} = 7.64 \times 10^{-5} \ m^2/s.$$

By substituting all the values in Eq. (2), we will get

$$S_L = \sqrt{\frac{32}{9} \frac{7.64 \times 10^{-5}}{1.18} (17.16+1)\, 36.21} = 0.389 \ m/s = 38.9 \ cm/s$$

$S_{L,expt} = 40$ cm/s for CH_4-air Laminar flame at $\phi = 1.0$

So, the average fraction of fuel and oxidizer are evaluated as Y CH 4 is equal to 0.55 plus 0 at the product side, right at X is equal to infinity. This will be 0 divided by 2.

So, that will get 0.0275 and Y of oxidizer will be 0.22 plus 0 divided by 2 that you will get 0.11. So, the equation 2 becomes I can write down as F average is equal to 16 molecular weight into 1.04 10 power 3 11 and XP 15000 divided by what I will be using 1724.5, right. I am substituting these values in the equation 2 and it will be basically row average.

This will be 2.2 Y CH 4 average by molecular weight of CH4 minus 0.3. Keep in mind that we are substituting CF, right or is equal to CH4 is equal to rho Y CH 4 average. Of course, I am taking into molecular of CH 4, right. This place of concentration I have taken row. I have taken out because the row is the average. You know this thing whichever coming, right into similar way I can also put Y average oxidizer MW oxidizer 2.5.

Keep in mind that we need to evaluate this row average, right row average by using ideal gas law. I can find out rho average is equal to P by R u MW mixture into T say in this case I will be using the mixture basically right into T average or I can say it is average.

The molecular weight of mixture is equal to molecular M. I am writing is nothing, but here 1 over Y i by MW i, right. I can write down that as 1 over 275 by 16 plus 0.11 divided by 32 plus rest will be I will be taking as nitrogen 0.8625 by 28 is equal to 27.81 and if I will do that that is basically pressure if you look at is given 101325. Pascals is the ambient pressure and it is given, right and is equal to 8314 divided by 27.81 into average temperature is 1724.5 is equal to basically coming as 0.1965 Kg per meter cube.

So, if you look at I know all the values U know why average here these molecular weight average I can evaluate that. This is basically equation 3. Substituting the values in equation 3, we can get as get is equal to 1.16 into 1.04 into 10 power 11 dimensional 15,000 1724.5 average. These 965 power 52.2 into 0.0275 by 16.11 by 30, right is equal to 36.1 Kg per meter cube per second and alpha we can evaluate alpha S Kg or T average by rho U 298 Kelvin CPT average is equal to 14.7 10 power 2 minus 3 1.18 into 11271.5 is equal to 7.64 into 10 power 10 minus 52 per second.

Now, we will substitute all the values in the equation 2, right. In equation 2, we will get SL is equal to 32 by 9 into this is 7.64 10 power 2 minus 5, density is 1.18 and Nu will happen to calculate as 17. This I have not done, but you can calculate that 17.16 plus 1 into 36.21. It will be coming around something 0.389 meter per second equal to 38.9 centimetre per second.

So, if you look at the approximate values or if you conduct experiment, you know it comes around experiment something around 40 centimetre per second for methane air. Laminar flame at phi is equal to 1. This is something you can say it is coming roughly, however you know lot of assumptions are being taken and then, some average properties are taken and this is not very exact, but it will give you some bulk back numbers, which will be helpful to understand and it encompasses all the features like it is a function of equivalence ratio and then, inlet temperature and pressure and also, the inert addition.

So, that is giving the flavour of all the things. It is a very simple analysis. Lot of assumptions we have made and we can utilize that for our calculations, however for the real situation you need to invoke the multi step chemistry and then, compute all the thing which can be done by numerical, right or asymptotic analysis. Again asymptotic analysis is an approximation where single step chemistry will be using, right will be used and their activation energy should be very high, then you do some kind of a approximation there and with this I will stop over here.

We in the next lecture, we will be discussing about how to relate this burning velocity to various other parameters that, we will be doing in the next lecture.