Fundamentals of Combustion (Part 2)

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Lecture – 46
Laminar Flame Theory for Premixed Flames

Let us start this lecture with the thought process a tiny spark can cause a mighty fire. So, in the last lecture we basically initiate a discussion how to derive a relationship for burning velocity by considering a 1 dimensional laminar steady flame right.

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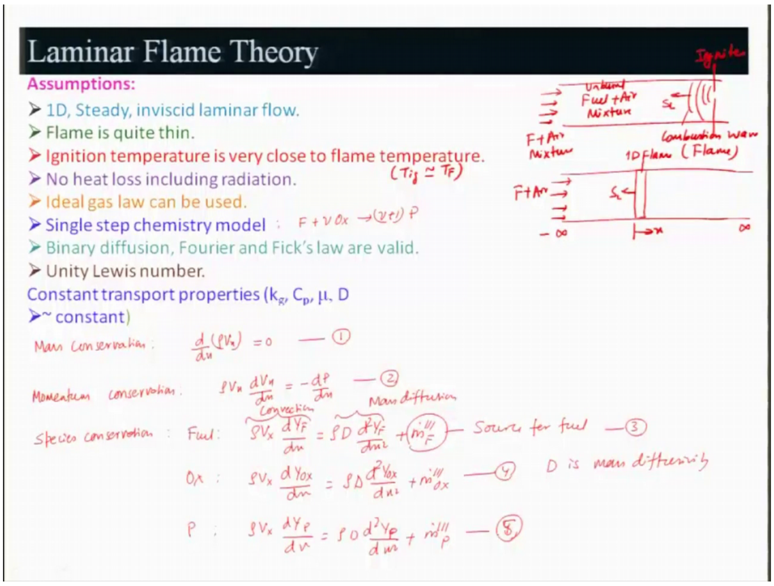

Laminar Flame Theory

Assumptions:

- 1D, Steady, inviscid laminar flow.
- Flame is quite thin.
- Ignition temperature is very close to flame temperature. $(T_{ig} \simeq T_F)$
- No heat loss including radiation.
- Ideal gas law can be used.
- Single step chemistry model : $F + \nu\, Ox \rightarrow (\nu+1) P$
- Binary diffusion, Fourier and Fick's law are valid.
- Unity Lewis number.

Constant transport properties (k_g, C_p, μ, D
- ~ constant)

Mass conservation : $\frac{d}{dx}(\rho V_x) = 0$ — ①

Momentum conservation : $\rho V_x \frac{dV_x}{dx} = -\frac{dP}{dx}$ — ②

Species conservation : Fuel : $\underbrace{\rho V_x \frac{dY_F}{dx}}_{\text{Convection}} = \underbrace{\rho D \frac{d^2 Y_F}{dx^2}}_{\text{Mass diffusion}} + \boxed{\dot{m}'''_F}$ — Source for fuel — ③

Ox : $\rho V_x \frac{dY_{ox}}{dx} = \rho D \frac{d^2 Y_{ox}}{dx^2} + \dot{m}'''_{ox}$ — ④ D is mass diffusivity

P : $\rho V_x \frac{dY_P}{dx} = \rho D \frac{d^2 Y_P}{dx^2} + \dot{m}'''_P$ — ⑤

And we have made the assumptions and I just repeating some of these thing, that is 1 dimensional steady inviscid laminar flow and flame is quite thin and no heat loss including radiation ideal gas law can be used, single step chemistry model we will be using and of course, unity Lewis number and constant properties will be considering right. And if you look at, what we will be doing? We will be basically looking at the all this mass conservation equations for our case right.

So, let us consider the mass conservation, mass conservation equation for the 1 dimensional steady flow what it would be? It would be nothing but your by d x rho V x. If I say in the x direction only right is equal to 0 right and let say this is equation 1, the momentum conservation for 1 dimensional steady inviscid laminar flow that would be rho V x d V x d x minus d P by d x, because the viscous term is not there and it is the steady group right equation 2.

And species conservation and we will be considering the single step chemistry right. So, if you look at single step chemistry model that is fuel is reacting with the nu moles of oxidizer going to nu plus 1 of product right. That means, I am having fuel oxidizer and product I will have to having I will have to invoke 3 equations, basically one is for the fuel is rho V x; V x is the velocity along the x direction and the mass fraction of fuel d Y by d x is equal to rho D; square of Y F by d x square plus F right.

This is basically diffusion term like mass diffusion and this is your source term, mass for fuel mass conservation of fuel per unit volume and this is of course, your convection term right. And this is equation 3; similarly, oxidizer rho V x, given D is the mass diffusivity. And the product will be rho V x d product that is equal to rho D plus and m dot triple dash, product it is 4, this is 5.

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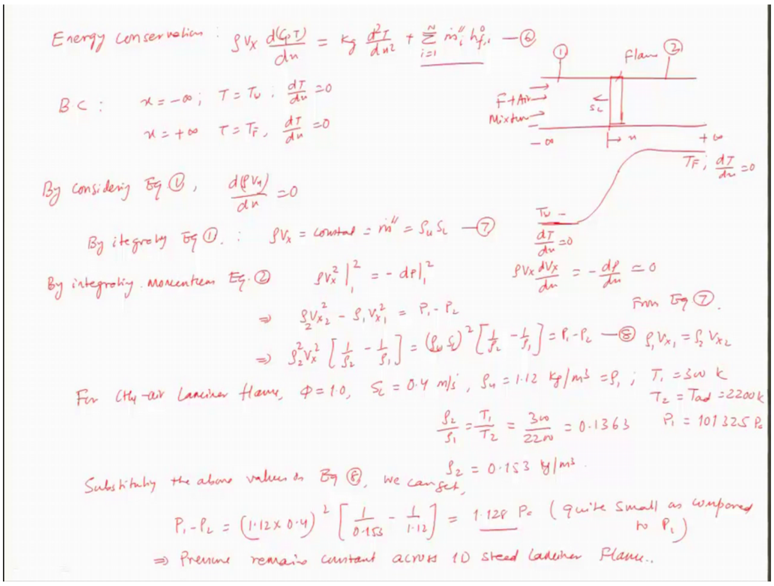
Energy conservation : $\rho V_x \frac{d(C_p T)}{dx} = k_g \frac{d^2T}{dx^2} + \underline{\sum_{i=1}^{N} \dot{m}_i''' h_{f,i}^0}$ — ⑥

B.C : $x = -\infty; \; T = T_u \, , \; \frac{dT}{dx} = 0$

$x = +\infty \quad T = T_F \, , \; \frac{dT}{dx} = 0$

By considering Eq ①, $\frac{d(\rho V_x)}{dx} = 0$

By integrating Eq ①. : $\rho V_x = \text{constant} = \dot{m}'' = \rho_u S_L$ — ⑦

By integrating Momentum Eq. ② $\rho V_x^2 \Big|_1^2 = -dP\Big|_1^2$ $\qquad \rho V_x \frac{dV_x}{dx} = -\frac{dP}{dx} = 0$

$\Rightarrow \rho_2 V_{x_2}^2 - \rho_1 V_{x_1}^2 = P_1 - P_2$ $\qquad$ From Eq ⑦.

$\Rightarrow \rho_2^2 V_x^2 \left[\frac{1}{\rho_2} - \frac{1}{\rho_1}\right] = (\rho_u S_L)^2 \left[\frac{1}{\rho_2} - \frac{1}{\rho_1}\right] = P_1 - P_2$ — ⑧ $\quad \rho_1 V_{x_1} = \rho_2 V_{x_2}$

For CH_4-air Laminar flame, $\Phi = 1.0$, $S_L = 0.4 \text{ m/s}$, $\rho_u = 1.12 \text{ kg/m}^3 = \rho_1$; $T_1 = 300 \text{ K}$

$T_2 = T_{ad} = 2200 \text{ K}$

$\frac{\rho_2}{\rho_1} = \frac{T_1}{T_2} = \frac{300}{2200} = 0.1363$ $\qquad P_1 = 101325 \text{ Pa}$

$\rho_2 = 0.153 \text{ kg/m}^3$.

Substituting the above values in Eq ⑧, we can get,

$P_1 - P_2 = (1.12 \times 0.4)^2 \left[\frac{1}{0.153} - \frac{1}{1.12}\right] = \underline{1.128} \text{ Pa}$ (quite small as compared to P_1)

$\Rightarrow$ Pressure remains constant across 1D steady laminar flame..

And similarly we can write down the energy equation; rho V x C P T d x is equal to k g d square by d x square plus m dot i h f i naught, is not it? This is basically i is equal to 1 to N that in this case N will be 3 well oxidizer and the product.

And if you look at if you want to solve these things you need to have a boundary condition right. What are the boundary condition you are having? That x is equal to minus infinity, let me draw that because, if I look at this is my length x. So, my flame would be like this is minus infinity, this is plus infinity, right.

And if I take a may be a temperature profile to illustrate this point right, it will be this is T F, this is T U right. And what happens to here? At minus infinity the gradient this will be d T by d x is equal to 0 and similarly here d T by d x is equal to 0, here yes or no? So, I can write down that this is basically infinity T is equal to T U and d T by d x is equal to 0; x infinity, T is equal to T F, d T by d x is equal to 0, right.

And if you look at like we will have to now consider how to because these are coupled equations right and it is also non-linear in nature because the term is source term is the non-linear, right. So, what we will have to do? We will have to see that simplify it and let us look at the mass conservation by considering equation 1 that is mass conservation right. What I can then; I can integrate that and that is rho V x by d x is equal to 0 that if I integrate it equation 1, I can get basically, rho V x right is equal to constant is equal to mass flux, is not it?

Mass flux remaining constant for 1 and that is equal to if I say this is S L right; that means, this flow which will be taking place is basically nothing but your rho U S L right because, the fluid which will be fuel plus air in our case I am considering or mixture we will be moving with, but same velocity that of the burning velocity. Are you getting? Then that is nothing but rho U S L, yes or no? So, this we will be using that later on, let us say this is equation if you consider that this will be 6 I guess right, is it right this 6?

Student: Yes sir.

This will be 7. And let us look at the momentum equation, momentum equation right. By integrating momentum equation that is 2 right between station 1 and 2, let us say I am saying this is station 1 and this is let us say station 2, something you know have a.

If I will do that what I will get? I can get this is if I you know integrate this equation what I will get? I will get the rho V x square right into the station 1 to 2 is equal to d P of course, this will be minus 1 to 2 right. I can write down that whatever we have done already, I think let me write down that is rho V x square I can say 2 2 minus rho 1 V x square 1 is equal to P 1 minus P 2 right.

So, if I mean I have already done for when we are talking about detonation when we are talking about combustion wave analysis right. That is nothing but here I can say rho square 2 V x 2 square is equal to 1 by rho 2 minus 1 by rho 1 right.

Can I not write then this? Is not it? Because I am taking rho 2 here multiplied by there that things right. And considering that is rho 1 V x 1 is equal to rho 2 V x 2, from the continuity from equation 7. So, is equal to P 1 minus P 2 or I can write down this is nothing but your rho U S L whole square into 1 by rho 2 minus rho 1 is equal to P 1 minus P 2, we will substitute some values right. So, how what will be the difference between these two you know pressure between station 1 and station 2? Is it a very big quantity or a small quantity?

Let us consider the for methane air laminar flame right and phi is equal to 1. I will consider S L as 0.4 centimeter, no 0.4 meter per second basically 40 centimeter per second 0.4 meter per second.

And rho U I can take as 1.12 kg per meter cube, density. I have taken air, but it depends on that, but it may be some order of that ok. Now if you look at rho 2 which is nothing but rho 1 in this case right and rho 2 by rho 1 is equal to T 1 by T 2 from the ideal gas law keeping that pressure is a remaining constant, but for the time being.

But now what will be this is let us say T 1; T 1 is 300 Kelvin and T 2 will be adiabatic temperature I am considering. What will be that? 2200 Kelvin ok? If I consider that 300 divided by 2200 Kelvin, it is coming something 1363 and similarly the density 2 will be 153 kg per meter cube.

Now if I substitute this I can say equation 8. Substituting the values the above values in equation 8, we can get what is that; P 1 minus P 2 will be rho 1.12 into S L is 0.4 whole square 1 by 0.153 minus 1 by 1.12, you will get is a very very low value, you will get that is something 1.128 Pascal.

And if you look at what is P 1, P 1 is 101325 Pascal. As compared to the pressure right, the P 1 minus P 2 is very very small is quite small as compared to P 1. If I take atmospheric pressure, if I take higher pressure it may be little bit more, but however, this is very small. Therefore, pressure it implies pressure remains constant across 1 D steady laminar flame. Therefore, there is no need to consider the momentum equation.

Momentum equation is basically turns to be 0 right, yes or no? If I say; that means, my momentum equation, my momentum equation is rho V x d V x by d x minus d P by d x and this happens to be 0 right, there is no need really to consider, but change in pressure is across the x is very very 0, so this is 0. So, therefore, there is no need to consider the momentum equation.

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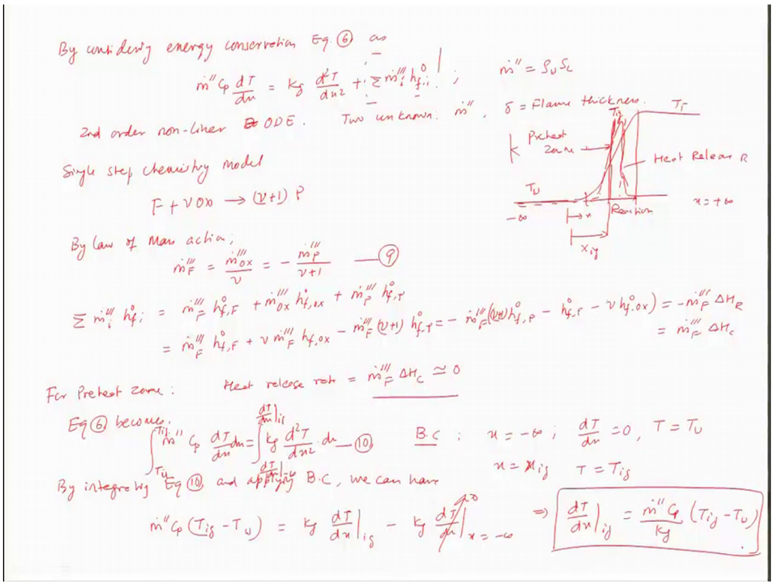

By considering energy conservation Eq ⑥ as

$$\dot{m}'' C_p \frac{dT}{dx} = k_g \frac{d^2T}{dx^2} + \Sigma \dot{m}_i''' h_{f,i}^0 \quad ; \quad \dot{m}'' = \rho_u S_L$$

2nd order non-linear ~~E~~ ODE. Two unknown: $\dot{m}''$, δ = Flame thickness.

Single step chemistry model

$$F + \nu\, Ox \rightarrow (\nu+1) P$$

By law of Mass action;

$$\dot{m}_F''' = \frac{\dot{m}_{ox}'''}{\nu} = -\frac{\dot{m}_P'''}{\nu+1} \quad \text{—⑨}$$

$$\Sigma \dot{m}_i''' h_{f,i}^0 = \dot{m}_F''' h_{f,F}^0 + \dot{m}_{ox}''' h_{f,ox}^0 + \dot{m}_P''' h_{f,P}^0$$

$$= \dot{m}_F''' h_{f,F}^0 + \nu \dot{m}_F''' h_{f,ox} - \dot{m}_F'''(\nu+1) h_{f,P}^0 = -\dot{m}_F'''\left((\nu+1) h_{f,P}^0 - h_{f,F}^0 - \nu h_{f,ox}^0\right) = -\dot{m}_F''' \Delta H_R$$

$$= \dot{m}_F''' \Delta H_c$$

For Preheat zone: Heat release rate $= \underline{\dot{m}_F''' \Delta H_c \simeq 0}$

Eq ⑥ becomes,

$$\int_{T_u}^{T_{ig}} \dot{m}'' C_p \frac{dT}{dx} dx = \int_{\left.\frac{dT}{dx}\right|_{-\infty}}^{\left.\frac{dT}{dx}\right|_{ig}} k_g \frac{d^2T}{dx^2} dx \quad \text{—⑩}$$

B.C: $x = -\infty$; $\frac{dT}{dx} = 0$, $T = T_u$

$x = x_{ig}$ $T = T_{ig}$

By integrating Eq ⑩ and applying B.C, we can have

$$\dot{m}'' C_p (T_{ig} - T_u) = k_g \left.\frac{dT}{dx}\right|_{ig} - k_g \cancelto{0}{\left.\frac{dT}{dx}\right|_{x=-\infty}} \Rightarrow \boxed{\left.\frac{dT}{dx}\right|_{ig} = \frac{\dot{m}'' C_p}{k_g}(T_{ig} - T_u)}$$

So what we will be now considering is basically we will be considering the energy equation and finding out a relationship for burning velocity right. So, let us now write down the energy equation, energy conservation equation. This is equation is what number?

Student: 6.

6 as I can write down this as C P I am taking out because C P is a constant, we are considering constant properties right. So, d T by d x is equal to kg d T x by d x square plus summation of m dot triple dash i. So, these equation we need to basically solve it, so that you can have a you know relations for that and keep in mind that m dash is equal to nothing but rho U S L, that already we have derived.

Now, this is basically a second order non-linear equations right. And it is having two unknown, this is you know second order non-linear equation, O D E; Ordinary Differencing Equation, right. And it is basically having two unknowns, right, what are two unknowns? One is mass flux is not known right of course, you do not know the temperature, but we will see how to and we will also the flame thickness, this is flame thickness is not known. It is similar to your boundary layer theory and where you know you can basically two unknown and four boundary conditions as to be satisfied.

And it is a Eigen value problem, so you can look at this Eigen values are m dot double dash and delta is a flame thickness right. And we will be looking at basically solving this equation either we can use a linear velocity profile and then try to solve it. And considering the physical phenomena into consideration right because, we have seen that; in the pre heat zone there is not much reaction is taking place.

That means there is not much heat release is occurring. Whereas, in the reaction zone we will have to also use some approximations that is the convention would not be very much predominant, right. If you look at I take this as the ignition temperature right T ignition. And this is my x this is basically minus infinity, this is x is equal to plus infinity and this point is T ignition and this is your T F, this is your T U.

Before venturing into that what I will consider like let us look at this term, how to handle this terms right? And for that we will consider the single step chemistry model F reacting with nu moles of 1 moles of fuel reacting with nu moles of oxidizer getting into nu plus 1 moles of product. And by this what we call law of mass action we know, F is equal to triple dash oxidizer divided by nu is equal to minus m dot triple dash product nu plus 1.

This I can call it as a equation 7, I can say 9 right. So, now let us look at this term, m dot triple dash i h f i naught is equal to triple dash F h f F oxidizer h f oxidizer plus triple dash product h f product. Now, I will just substitute these values as you know I will use this equation 9, I can write down F h f F there is no problem.

But here I will substitute that is nu m dot triple dash F right h f oxidizer plus product wise I will this will be basically minus triple dash F nu plus 1 h f product naught. Are you getting my point? I just I am using equation 9 for each of the you know mass; consumption of oxidizer for you know unit volume in terms of mass conservation of fuel per unit volume, right.

So, I can write down this as F as h I can basically, I can write down these as product minus h f F naught minus nu h f; no, nu h f right. So, this is minus I have taken and this is nothing but your I can say delta H R, is not it? I can write down that as or triple dash F into delta H C because heat of reaction for the exothermic will be negative; so negative negative positive; so this is the heat of combustion. Are you getting? I can write down that.

So, now, for the we will be considering this equation 6, right; for the pre heat zone if you look at this is basically I can say this is my heat release rate and this zone I am considering as my pre heat zone. And this portion I can consider as a my reaction zone right. So, for pre heat zone what we will consider? For pre heat zone, the heat release is very very low right; that means these term m dot triple dash sorry.

The heat release rate is very very 0 very very small; that means, F delta H C is approximately equal to 0, it is not 0, it is approximately equal to 0; that means, we are considering to be 0. So, as a result what you will get? You will get a relationship; that means this will be 0 in this case. So, I will get equation 6 becomes C P d T by d x is equal to kg d T by d x square right, because this term is 0.

Now I will have to solve this thing. What are the boundary condition here? The boundary condition is x minus infinity d T by d x is equal to 0. And T is equal to T U and x is equal to x ignition, this is the one, T is equal to T ignition, are you getting? Till ignition; so if I integrate this equation, let us say this is 10, equation 10.

What I will get? I will get that this basically, I will integrate this equation. And applying boundary condition we can have C P T ignition minus T U left hand side right. I will what I will do? I will say d x here, d x here integrate it, right. And this will be from T ignition to T U and this is also; you can say that d T d x ignition to d T by d x is equal to minus infinity.

So, that will be equal to kg, kg is a constant property. So, d T by d x is ignition minus kg d T by d x at x is equal to minus infinity, this is 0. So, therefore, I am getting d T by d x ignition is equal to m dot C P by kg T ignition minus T U. So, we are getting an expression for this the pre heat zone and we will stop over here. And in the next lecture we will be discussing about rest of the derivation for getting the laminar burning velocity ok.

Thank you.