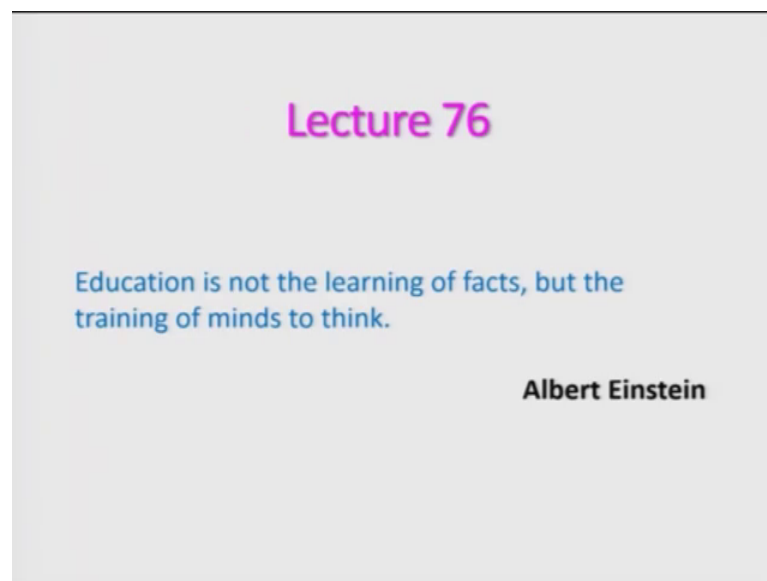


**Fundamentals of Combustion (Part 2)**  
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**Lecture – 76**  
**Carbon Sphere in Convective Environment**

Let us start this lecture with a thought process by Albert Einstein; who says, education is not the learning of facts, but the training of minds to think.

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Unfortunately, modern education is not providing an opportunity to develop a good mind, rather it is emphasising on learning of facts. So, let us recall what we learnt in the last lecture. In the last lecture we basically look that how does the lifetime of a fuel particle varies and we also looked at the time requirement for initial droplet diameter burning for carbon and kerosene, and then we put we also discuss about the coal sphere and being a the volatilized or paralyzed fuel sphere. And later on we moved into the temperature to derive a expression for the temperature profile and we had derived the expression for temperature in between.

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$$m \dot{s} R^2 \left( \frac{dT}{dr} \right) = kg r^2 + m \dot{s} R^2 \Delta H_c \quad \text{--- (4)}$$

By continuity, we know;  $m \dot{s} r^2 = m \dot{s} R^2$

By re-arranging Eq. (4), we can have

$$m \dot{s} R^2 (dT - \Delta H_c) = \frac{kg r^2}{r} dr \quad \text{--- (5)}$$

By simplifying it further, we can have

$$\int_T^\infty \frac{dT - \Delta H_c}{T - \Delta H_c} = \int_r^{r_\infty} \frac{m \dot{s} R^2}{kg r} \frac{dr}{r}$$

$$\ln \left( \frac{T - \Delta H_c}{T_\infty - \Delta H_c} \right) = \frac{m \dot{s} R^2}{kg D_12} \left( \frac{1}{r} - \frac{1}{r_\infty} \right) \quad \text{--- (6)}$$

As we have derived an expression for

$$\frac{m \dot{s} R^2}{kg D_12} \frac{1}{r} = \ln \left[ \frac{T_\infty - \Delta H_c}{T - \Delta H_c} \right] \quad \text{--- (7)}$$

By combining Eq. (6) & (7), we can have

$$\ln \left[ \frac{T_\infty - \Delta H_c}{T - \Delta H_c} \right] = \frac{kg D_12}{m \dot{s} R^2} \ln \left[ \frac{T_\infty - \Delta H_c}{T - \Delta H_c} \right] \quad \text{--- (8)}$$

$L = 1$   
 $D_{12} = \alpha_g$

That expression is basically, let me write down again; that is,  $r^2 C_p T$  is equal to  $kg r^2 \frac{dT}{dr} + m \dot{s} R^2 \Delta H_c$ . Keep in mind this we had given equation 4 if I remember correctly. And if you look at these remain constant, right by continuity we know that  $m \dot{s} r^2$  is equal to nothing but your  $m \dot{s} R^2$ . So, we can write down here basically in this place, I can write down  $m \dot{s} R^2$  right, remain same. So, by re arranging equation 4, we can write down, we can have that is  $m \dot{s} R^2$ .

And I can take this  $C_p T$  and then minus  $\Delta H_c$  is equal to  $kg r^2$  I can write down  $\frac{dT}{dr} C_p T, C_p I$  can minus  $\Delta H_c$ . So, if you look at so, I can write down basically  $C_p T$  here  $1$  by  $C_p$  here. Now, I can simplify further it further, we can have  $\frac{dT - \Delta H_c}{T - \Delta H_c}$  divided by  $C_p T - \Delta H_c$  I can have; is equal to  $R^2 \frac{dr}{r^2}$ , I can write down and this is  $kg$  by  $C_p$ .

And now I can integrate it, this one from  $T$  to  $T$  infinity. And similarly here I can integrate with respect to basically  $r$  to  $r$  is equal to infinity. And keep in mind that we can simplify here  $kg$  by  $\rho C_p$  I can write down basically  $\alpha_g$ ;  $\alpha_g$  is nothing but your  $kg$  by  $\rho C_p$ . And this is your thermal; so, we will get by integrating this is basically  $\ln \left( \frac{T - \Delta H_c}{T_\infty - \Delta H_c} \right)$  is equal to and I can write down  $kg$  by  $C_p$  is nothing but your  $\alpha_g$  by  $\rho g$ , I can say this is  $\rho g$  And minus  $1$  by  $r$  and this integration from  $r$  to infinity.

And that is equal to if you look at when  $r$  is equal to infinity that will be 0. So, this become basically I can say nothing but your  $s R$  square  $R$  s square divided by  $\alpha g$  by  $\rho g$   $1$  by  $r$ . So, this is basically I can say equation 5. And we had earlier derived an expression for similar term in terms of mass fraction of oxidizer. So, as we had derived an expression for this term; like in as the  $m \dot{s} R$  square by  $\rho g$  diffusivity  $1$  by  $r$  is equal to  $\ln Y_{ox}$ . This is basically  $T_2 T_{\infty}$ ,  $Y_{ox \infty} + 1$  by  $f Y_{ox} + 1$  by  $f$ . Let us say this by combining this equation and also we can write down equation 5 and 6, we can have; keep in mind that this when will integrate this term, I will get basically  $\ln C_p T_{\infty} - \Delta H_c$ ,  $C_p T_{\infty} - \Delta H_c$  and this term will come.

So, which is nothing but your equal to in place of this, I will be replacing this, I will get is equal to  $\rho D$   $1$  by  $2 \rho g$  divided by  $\rho g \alpha g$  is it into  $\ln Y_{ox \infty} + 1$  by  $f Y_{ox} + 1$  by  $f$ . So, this will cancel it out, I can say this is equation 7. Now we got an expression basically in terms of temperature and also the mass fraction of oxidizer. Now, if I take this antilog and keeping in mind that Lewis number is equal to 1 in this expression, Lewis number is equal to 1 we have consider Lewis number equal; that means, the basically diffusivity is nothing but your  $\alpha g$ ; that means, this is going to 1.

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By simplifying Eq ⑤ and taking antilog, we can have

$$\frac{C_p T - \Delta H_c}{C_p T_{\infty} - \Delta H_c} = \frac{Y_{ox, \infty} f + 1}{Y_{ox, \infty} f + 1} \quad \text{--- ⑤}$$

We had derived earlier that

$$\frac{Y_{ox, \infty} f + 1}{Y_{ox, \infty} f + 1} = (B_c + 1)^{-R/r} \quad \text{--- ⑥}$$

By combining Eq ⑤ and ⑥, we can have

$$\frac{C_p T - \Delta H_c}{C_p T_{\infty} - \Delta H_c} = (B_c + 1)^{-R/r} \quad \text{--- ⑦}$$

$$\Rightarrow \frac{C_p T}{C_p} = \frac{\Delta H_c}{C_p} + \frac{(C_p T_{\infty} - \Delta H_c)(B_c + 1)^{-R/r}}{C_p} \quad \text{--- ⑧}$$

$$= T_{\infty} = \frac{\Delta H_c}{C_p} + (T_{\infty} - \frac{\Delta H_c}{C_p})(B_c + 1)^{-R/r} \quad \text{--- ⑨}$$

At fuel surface,  $T = T_s$ ;  $B_c = 1$ .  $B_c = f Y_{ox, \infty}$

Eq. ⑨ becomes

$$T_s = \frac{\Delta H_c}{C_p} + (T_{\infty} - \frac{\Delta H_c}{C_p})(f Y_{ox, \infty} + 1)^{-1} \quad \text{--- ⑩}$$

$$T_s = \frac{C_p T_{\infty} + f \Delta H_c Y_{ox, \infty}}{C_p (f Y_{ox, \infty} + 1)} \quad \text{--- ⑪}$$

Combustion parameters  
Ambient conditions

So, therefore, we can simplify this equation 7 and also taking antilog by simplifying equation 7 antilog we can have  $C_p T_{\infty} - \Delta H_c$   $C_p T_{\infty} - \Delta H_c$  is equal to  $Y_{ox \infty} f + 1$  divided by  $Y_{ox \infty} f + 1$ . We had derived earlier that  $Y_{ox \infty} f + 1$   $Y_{ox \infty}$

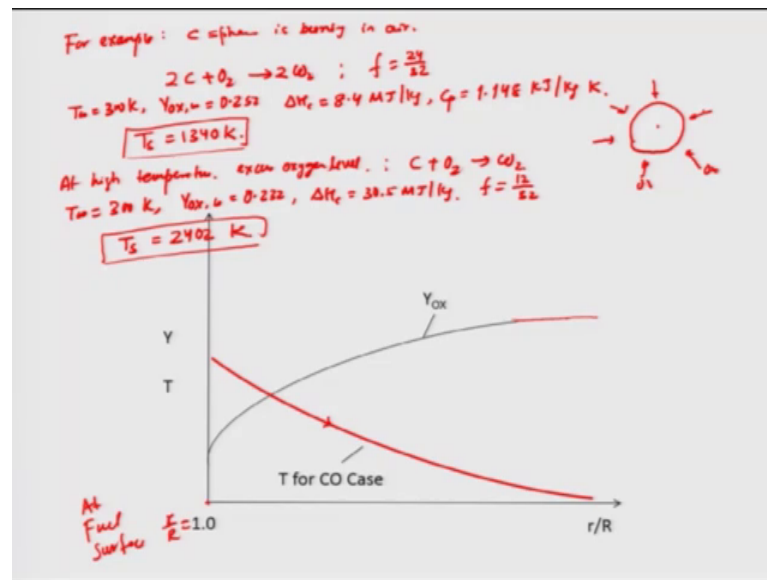
infinity  $f + 1$  is nothing but your  $B_c + 1$  power to the  $r$  by  $R$ . So, this is your basically 8, I can say this is 9 by combining equation 8 and 9, we can have  $C_p T_\infty \Delta H_c$   $C_p T_\infty \Delta H_c$  is equal to  $B_c + 1 R$  by  $r$ .

So, if I will this is my equation 10, if I simplify further, I can get an expression for  $C_p$  temperature  $C_p T$  is equal to  $\Delta H_c + C_p T_\infty - \Delta H_c B_c + 1 R$  by  $r$ . And we can basically simplified further, and get some expression like that; that is I will get because if I divide this by the  $C_p$  this equation,  $C_p$  here,  $C_p$  here similarly,  $C_p$  with this I will get  $T$  is equal to  $\Delta H_c$  by  $C_p$  is equal to  $T_\infty - \Delta H_c$  by  $C_p B_c + 1 R$  minus  $r$ . So, this is the expression, this is 11. Now, if you consider this at the fuel surface,  $T$  is basically  $T_s$  and fuel surface means what?  $R$  by  $r$  equal to 1.

And  $B_c$  is we know that is  $f Y_{ox}$  infinity and equation becomes  $T_s$  is nothing but your  $\Delta H_c C_p$  and plus  $T_\infty \Delta H_c$  by  $C_p$ , I can write down here, that in place of  $B_c$  I can write down  $f Y_{ox}$  infinity plus 1, and this will be minus 1. And if I simplify this expression, I will get  $T_s$  is equal to basically I can do that, that I will get I will just write down the final expression; that is  $T_\infty + f \Delta H_c Y_{ox}$  infinity divided by  $C_p f Y_{ox}$  infinity plus 1. So, this is the expression you will get the surface temperature 12. Now keep in mind that, this expression temperature is varying with respect to radius.

And of course, it is a function of values like for example,  $T_s$  is a function of what is the function of  $\Delta H_c$  also function of  $f$ , sorry,  $T_s$  is a function of the  $f$  and  $\Delta H_c$  and also the  $T_\infty$  and  $Y_{ox}$  infinity and this property is basically combustion parameter. And these are ambient conditions so, that means,  $T_s$  is basically function of  $f$  and also  $\Delta H_c$ , and it will be dependent also the  $T_\infty$  and  $Y_{ox}$ .  $F$  is basically fuel and oxidizer ratio.

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For example, if we will consider the carbon sphere is burning in the presence of air in air, and to produce carbon monoxide at a moderate temperature and with deficient oxygen level.

So, as per the reaction it will be  $2C + O_2$ , it is going to basically  $2CO$ . And in this case what will be  $f$ ?  $f$  will be basically 24 by 32. And if I consider  $T_\infty$  is 300 Kelvin and  $Y_{O_2, \infty}$  will be 0.232. And if I consider the heat of combustion as 8.4 mega joule per kg and  $C_p$  value I need. So,  $C_p$  I will take 1.148 kilo joule per kg Kelvin, I will get a surface temperature which is 1340 K. And I can consider another extreme situation; where the large quantity of oxygen is diffusing to the carbon particle, right oxygen is diffusing large quantities not deficient and it is at high temperature right. Then we will consider basically the reaction to occur at high temperature and excess or oxygen level.

So, we will say that the reaction which will be occurring is  $C + O_2$  going to the carbon dioxide. In this case,  $f$  will be 12 by 32, and we will take the same condition  $T_\infty$  is 300 Kelvin  $Y_{O_2, \infty}$  is 0.232 but however,  $\Delta H_c$  maybe higher So, we will consider as 30.5 mega joule per kg. If I substitute all these values in equation for the surface temperature that we are derived earlier, we will get the  $T_s$  is 2402 Kelvin, right. We will evaluate that. And keep in mind that this is a we have estimated from this

analysis; however, the actual temperature if will measure, it will be much lower as compared to this estimated surface temperature of carbon sphere.

A question arises why it is because a fact that we have not considered the radiation losses from the fuel surface which is quite exorbitant and also the properties which we have taken as a constant. So, therefore, basically temperature will be much lower. So, let us look at how does this temperature varies in the gas phase for this your case. So, it will be decreasing with respect to r and this is your basically fuel surface this is corresponding to r by R equal to 1 and this is the fuel surface right, at fuel surface this point. So, it is decreasing asymptotically decreasing and we can see that. Now, we are looked at this in the quotient atmosphere, but let us look at what happens when it is in a convective atmosphere.

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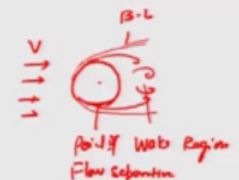
Carbon sphere in convective environment:

The experimental results for convective heat transfer coefficient  $h_c$  are:

$$\frac{h_c}{C_{p,g}} = \frac{D_p}{D_o} \left[ 2 + 0.6 Re_{D_o}^{1/2} Sc^{1/3} \right]$$

$D_o$  Initial diameter of fuel  
 $C_{p,g}$  specific heat of gas

$Re_{D_o} = \frac{\rho V D_o}{\mu} = \text{Reynolds Number}$   
 $Sc = \text{Schmidt Number} = \frac{\nu}{D_o}$  — kinematic viscosity  
 $V = \text{Relative velocity between the sphere and fluid}$



Like means carbon sphere in convective environment so, if you look at in real situation what happened? It is not that will have to burn the any fuel sphere or fuel particle in a quotient atmosphere, but mostly with flow. So, if I assume there is a flow which is taking place, let us say this is a uniform flow with v, then what will happen? There will be some boundary layer will be formed right, this is your boundary layer.

And also the flow will be slightly separated out, and there will be wake formations, wake region, because this is the point of point of flow operation. And of course, for this you want to analyse it is quite difficult, and you will have to basically solved momentum

equation and 2 dimensional at least or maybe 3 dimensional it is quite complex. And which is difficult to tractable along with the chemical reaction. Therefore, people have conducted experiment, and they have come up with a basically empirical result and the experimental results for heat transfer coefficient.

What is more important? The how much heat being transferred from here, if we evaluate, then we can also find out what is the droplet burnings and the things. So, we can do that result for the convective heat transfer coefficient  $h$ , as  $h$  by  $C_p g$  this is basically gas is equal to  $\rho g$  diffusivity and  $D_{naught}$  is the initial droplet diameter  $2$  plus  $0.6 Re D_c$  this is half, and this is  $1$  by third Schmidt number. So,  $d$  is basically initial diameter of fuel, and  $C_p g$  is specific heat of gas. And if you look at Reynolds number is based on diameter, this could have been the diameter  $D_{naught}$  I can write down,  $\rho V D_{naught}$  by  $\mu$ . This is your Reynolds number. And Schmidt number is nothing but your kinematic viscosity divided by diffusivity.

And  $V$  is basically one can consider the actual velocity or in a real situation you can consider the relative velocity between the sphere and fluid because this particle also will be moving. So, therefore, it is important to take the relative velocity, and keep in mind that you will by using this expression one can get a similar to the  $d$  square law, it is not  $d$  square law, but you can get some expression. By that you can really use it in a actual calculation. So, with this we will stop over, and in the next lecture we will be moving into a new topic combustion and environment.

Thank you very much.