

Fundamentals of Combustion(Part 2)
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Lecture – 74
Data Reduction – Scale and Temperature Factor

Let us start this lecture with a thought process from Benjamin Franklin, who says tell me and I forget, teach me and I remember, involve me, I learn. This is a very wonderful statement. So, far the learning process is concerned unfortunately that is not being practiced in modern time, in the education system. So, it is very important to get involved in the process. So, that you can learn and you need not struggle too much for that and let us now recall what we learnt in the last lecture? We basically tried to derive the equations for a diffusional process of carbon sphere, our objective is basically was to find out an expression for mass fraction of oxidizer and we had derived in the flag end, I told that we will going for the next lecture.

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At fuel surface ($r=R$), $Y_{Ox} = 0$

Eq (12) becomes:

$$\frac{\dot{m}_s R}{f D_{12}} = \ln \left[\frac{Y_{Ox,u} + \frac{1}{f}}{\frac{1}{f}} \right]$$

$$\Rightarrow \frac{\dot{m}_s R}{f D_{12}} = \ln \left[f Y_{Ox,u} + 1 \right] = \ln [B_c + 1] \quad (13)$$

Mass Transfer Number (Spalding)

$B_c = f Y_{Ox,u}$

For oxygen medium, $Y_{Ox,u} = 1 \Rightarrow B_c = f$

For air medium, $Y_{Ox,u} = 0.232 \Rightarrow B_c = 0.232 f$

$$\ln \left[\frac{Y_{Ox} + \frac{1}{f}}{Y_{Ox,u} + \frac{1}{f}} \right] = - \frac{\ln (B_c + 1) R}{r} = \ln (B_c + 1)^{-\frac{R}{r}} \quad (14)$$

$Y_{Ox} + \frac{1}{f} = (Y_{Ox,u} + \frac{1}{f}) (B_c + 1)^{-R/r}$

$$\Rightarrow Y_{Ox} = -\frac{1}{f} + (Y_{Ox,u} + \frac{1}{f}) (B_c + 1)^{-R/r} = -\frac{1}{f} + \left[\frac{Y_{Ox,u} + 1}{f} \right] (B_c + 1)^{-R/r}$$

$$Y_{Ox} = \frac{(B_c + 1)^{1 - R/r} - 1}{f} \quad (15)$$

So, what we will do? We will have to now apply the equation 12, we will have to use the equation 12 and apply the boundary condition that, we know that at fuel surface, that is basically r is equal to R and we know that Y oxidizer need not to be 0, but we are considering the mass fraction of oxidizer is to be 0 as a result the equation 12, becomes s

R by ρD 1 by 2 is equal to $\ln Y_{O_2}$ oxidize the infinity plus 1 by f and this is just 1 by f , because Y_{O_2} is 0 .

So, then if you look at, I can simplify and write it down as $s R$ by ρD 1 by 2 , if you look at is basically cancel it out here provided, if I multiplied with this f here, right and then about this one. So, this will cancel it out. So, what you will get? Is basically $\ln f Y_{O_2}$ infinity plus 1 and I can write down as $\ln B_c$ plus 1 and this B_c is basically known as mass transfer number and some people call it also the Spalding number right and keep in mind that, this is place a very important in this case B_c happens to be what? $F Y_{O_2}$ infinity, this is the B_c , what we have looked at it basically, this portion you know and when the surrounding medium is only oxygen.

What will happen if surrounding medium is only oxygen? Basically for oxygen medium Y_{O_2} infinity will be what? Nothing but your 1 , so therefore, B_c is basically nothing, but your f that is well air ratio and for air medium Y_{O_2} infinity is nothing, but your 0.2324 . So, therefore, B_c is nothing, but your $0.232 f$. So, this is we should keep in mind, we are basically now considering the air as a surrounding medium and we have already seen in earlier that $\ln Y_{O_2}$ plus 1 by $f Y_{O_2}$ infinity plus 1 by f is equal to minus $\ln B_c$ plus $1 R$ by r .

So, because what we are doing? We are basically using this equation 13 in earlier expression for the Y_{O_2} and we can get this and if we will simplify, this equation what we will get? We will get basically is nothing, but your $\ln B_c$ plus 1 and I can write down power to the R by r , if I take the antilog of this, what I will get? I will get, I can say this is 14. Taking antilog of equation 14, we will get basically Y_{O_2} plus 1 by f is equal to Y_{O_2} infinity, I am omitting one step, which is very simple one, that is B_c plus 1 minus R by r .

And if you look at that, I can get an expression for this, which is very easy to get that is, 1 by f and B_c , this will be minus 1 by f plus Y_{O_2} infinity plus $f B_c$ plus 1 minus R by $C R$ sorry capital R by small r and if you further simplify, what you will get? Basically if you look at $1/f$ is there you can take this, $1/f$ together and you will find, I can write on that basically Y_{O_2} 1 by f , if I take common of that, I will get of course, keep in mind that this, I can write down as plus in Y_{O_2} f , if I look at, I can write down f is into Y_{O_2} infinity nothing, but your B_c , I can write down in this place right.

So, that I can get $B_c + 1$ divided by f right and $B_c + 1$ R by r . So, that becomes basically, if I take it out this will be Y_{O_2} is nothing, but your $B_c + 1$, $1 - \text{capital } R$ by r minus 1 divided by f . So, this is your expression for Y_{O_2} , you can see this is basically the equation 15.

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Theory For Single Coal Combustion

At moderate temperature and for small particle, the oxygen mass fraction at the surface is given by

$$\frac{\dot{m}_s R}{\rho D_{12}} = \ln\left(\frac{Y_{O_2,s} + 1/f}{1/f}\right) = \ln(B_c + 1)$$

B_c is the mass transfer number $B_c = f Y_{O_2,s}$

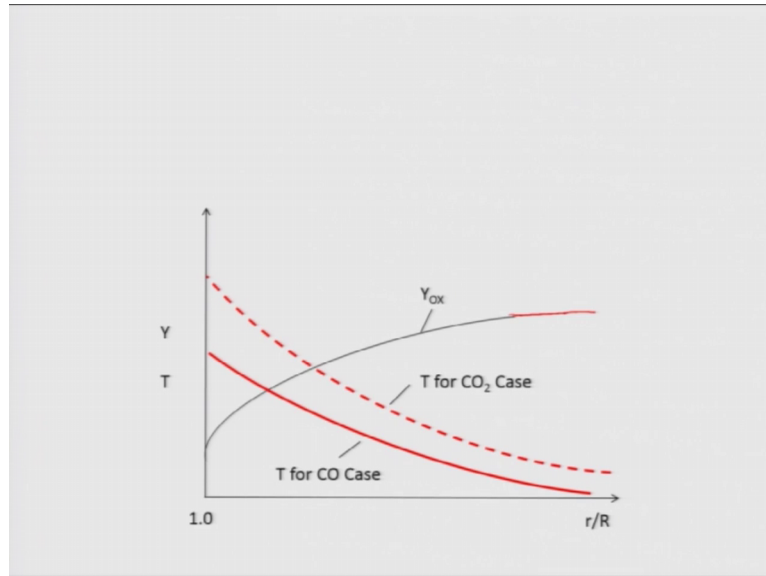
$$Y_{O_2} = \frac{(B_c + 1)^{1-R/r} - 1}{f}$$
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Fuel	ρ_{fuel} (g/cm ³)	MW_{fuel}	B.P _{fuel} (°C)	f	B_{oxygen}	B_{air}
Aluminium	2.70	27.0	2.467	1.12	1.12	0.26
Boron	2.34	10.8	2.550	0.451	0.451	0.105
Carbon	1.50	12.0	4.827	0.75	0.750	0.174
Carbon	1.50	12.0	4.827	0.375	0.375	0.087
Magnesium	1.74	24.3	1.107	1.107	1.520	0.353
Zirconium	6.44	91.2	3.578	3.578	2.850	0.662

So, now, Y_{O_2} is a function of this mass transfer number and also the fuel and oxidizer ratio and which is a function of R at moderate temperature for small particle oxygen mass fraction, you will get basically, we have seen this thing earlier just to summarize and and this B_c is the mass transfer number.

I have already talked about it and derived it. So, I can also get a expression Y_{O_2} is $B_c + 1$, $1 - R$ by r minus 1 with f , this is the expression, what we had derived just now right and this is your basically equation 15, we have already now, if you look at basically various fuels like Aluminium and Borons like carbon of course, we have consider. So, you will find that different boiling point of the fuel, I have given here and different mass fraction of the fuel will lead to various B_c , basically this is about the mass transfer in oxygen and air

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And if you consider the burning time of the solid right, which will take care of that and then you will find that if you will now plot this thing, you will get a expression basically oxygen, which is at the 0 here or a very small values and then it changes with respect to time and then it reaches asymptotic values at infinity. It is remaining constant and of course, the temperature and then for different things, we can get later on, but what we will do now? We will basically look at about the carbon burning rate.

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Carbon Burning Rate:

$$\dot{m}_c = -\frac{dm_c}{dt} \quad \text{--- (1)}$$

where m_c is the mass of carbon particle

$$m_c = \rho_c V_c = \rho_c \frac{\pi D^3}{6} \quad \text{--- (2)}$$

radius of carbon (fuel) sphere D is diameter of sphere at any instant of time

As derived earlier $\dot{m}_c = \dot{m}_c = \frac{8}{3} D_{12} \ln(B_c + 1) \quad \text{--- (3)}$

$$\dot{m}_c = \dot{m}_c A_c = \frac{8}{3} D_{12} \ln(B_c + 1) \times 4\pi \left(\frac{D}{2}\right)^2 = \frac{32}{3} \pi D_{12} \ln(B_c + 1) D^2 \quad \text{--- (4)}$$

By using Eq (1) & (4), we will get

$$\dot{m}_c = \rho_c \pi \frac{8}{3} \frac{dD^2}{dt} D \quad \text{--- (5)}$$

By using Eq (3) and (5), we can get: $\dot{m}_c = \frac{32}{3} \pi \frac{1}{2} \frac{dD^2}{dt} = \frac{32}{3} \pi D_{12} \ln(B_c + 1) \quad \text{--- (6)}$

By integrating Eq (6) we will get:

$$\int_0^t \frac{dD^2}{dt} dt = \int_0^t \frac{8 D_{12} \ln(B_c + 1)}{K_c} dt \quad \text{--- (7)}$$

At $t = 0$, $D = D_0$ - Initial diameter of sphere

$$\Rightarrow D^2 = -K_c t + C \quad \text{--- (8)}$$

At $t = 0$, $D^2 = C \Rightarrow \boxed{D^2(t) = D_0^2 - K_c t} \quad \text{--- (9)}$

of time

So, let us look at carbon burning rate; that means, if I take a sphere and then we will have to find out carbon and I am considering in the R direction at the surface this is nothing but your, R. So, we are considering that how much of the carbon particle is getting

consumed? This is basically carbon sphere, we are considering and this can be any other fuel as a matter of fact, but keep in mind that this need not to be pyrolysing, this is basically a non- pyrolysing fuel.

So, if I will consider the mass rate of consumption of the fuel, in this case it is carbon is nothing, but your change in mass of the fuel with respect to time, where mass is the m_c is the mass of carbon particle and that has to be evaluated as m_c is nothing, but your ρ_c , that is the density of carbon and the volume of carbon, this is basically density of carbon. Carbon is basically fuel sphere and V_c is the volume of fuel sphere, what we are considering? That I can write down as $\rho_c \pi D^3 / 6$ and D , D is basically the diameter of sphere, at any instant of time; that means, this will be changing as it get consumed. So, it will be changing.

Now, we know that of earlier that, we have derived as derived earlier, we that find out as a s is equal to carbon is nothing, but a ρ this is of course, the g I am just trying to differentiate that this ρ is nothing, but your gas phase and $D^{-1/2}$ is 2 by $D \ln B_c$, because R was there in place of R , I am writing basically 1 by R was there. So therefore, 2 by $D \ln B_c$ plus 1 . So, now if you look at what is this? $m \dot{c}$ $m \dot{c}$ is nothing, but your $m \dot{c}$ into A_c , that is the carbon surface area, which is nothing, but your $\rho_c g D^{-1/2}$ and 2 by $D \ln B_c$ plus 1 into area. Area will be basically $4 \pi r^2$, it is by 2 whole square.

So, this will cancel it out and if you look at that, you will get is basically you will get this D will cancel it out, you will get $\rho_c g D^{-1/2} 4 \pi D \ln B_c$ plus 1 . So, we know that basically m_c is nothing, but your, and if I look at this as equation, I can say that, this is a new equation number. I am putting it here 1 and this maybe the 2 equation number and this may be 3 . So, therefore, let us look at now, the equation 1 and then put this equation 1 A here and we will get $m \dot{c}$ by using equation 1 and 1 A , we will get $m \dot{c}$ is $m \dot{c}$ is nothing, but your $\rho_c \pi 3$ by $6 d$, D^2 square by dt , which is basically equation 4 , I can say and that is nothing, but by using by using equation 3 and 4 , we can get $\rho_c \pi$ by this is basically 2 1 by $2 d$ by d^2 square t is equal to $\rho_c g D^{-1/2}$, $4 \pi D \ln B_c$ plus 1 .

So, if you look at this π will cancel it out and keep in mind that this is ρ_c and. So, therefore, we can write it down basically as D^2 square by dt is 8 , nothing, but your $\rho_c g D^{-1/2}$. So, this D D will cancel it out by $\rho_c \ln B_c$ plus 1 . So, I will get basically

equation 5 by integrating equation 5, we will get $\int_0^t dt = \int_{D_0}^D \frac{D^2}{8 \rho_g D^2} \ln \left(\frac{B_c + 1}{B_c} \right) dt$. So, if you look at we will use this is basically, equation 6 we know that initial condition at $t = 0$, the D will be D_0 that is your initial diameter of sphere.

So, therefore, when you will get this one, you will basically get that as D^2 , you can say the D^2 is equal to I can say this is as a constant this, I can take as a constant K_c $K_c t$ plus, this is another constant. So, then implies that D_0^2 is equal to basically C and therefore, you will get is D^2 law and $D_0^2 - t$ is equal to $D_0^2 - K_c t$. So therefore, this is the very equation, what we have done? This is known as D^2 law and it is similar to that of the, what we had derived for the derived for the liquid droplet combustion. So, keep in mind that this K_c is basically the, involve lot of constant and also the transfer number. So, we will stop over here and in the next lecture we will be basically looking at how this D^2 law can be used to evaluate, the burning time lifetime of the lifetime of the carbon sphere or any other solid fuel and we will see various aspect in the next time.

Thank you very much.