Fundamentals of Combustion (Part 2)
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Lecture – 73
Lecture 33: Diffusional theory for Carbon Combustion

Let us start this lecture with a thought process.

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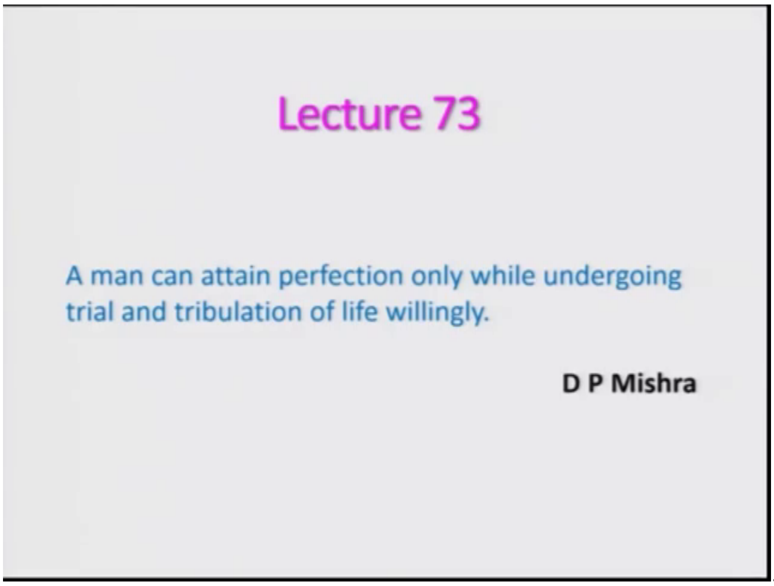

A man can attain perfection only when undergoing trial and tribulation of life willingly. Most of us we do not want to have trial and tribulations and which are very much part of life and when we will gleefully undergo the transformation during this trial and tribulation we will definitely get a lot of strain to have perfection in our life which is the main objective of human life. So, let us recall what we learnt in the last lecture, we basically look that the Combustion of Solid Fuel and in the process we learnt that there are two kinds of regime, one is the kinetic controlled regime of combustion other is the diffussionally controlled combustion and.

Today we will be discussing about the Diffusional Theory of Carbon Combustion.

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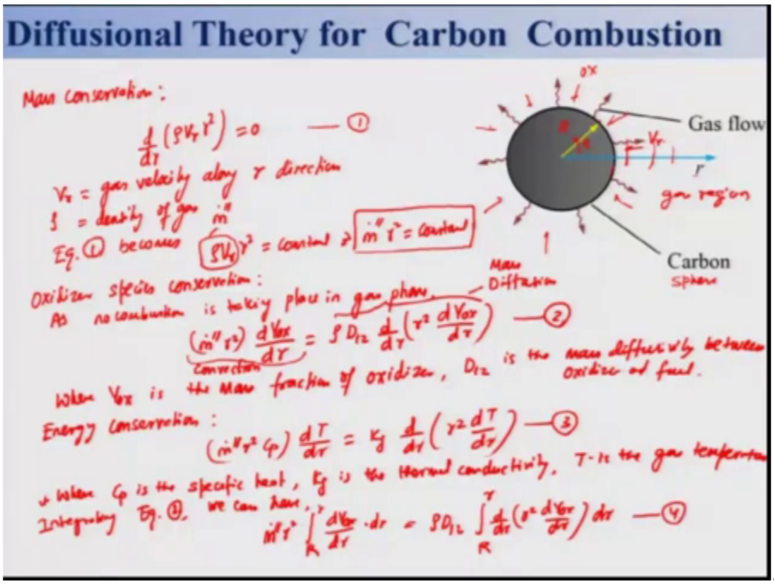

If you recall that we had discussed about various assumptions, what we can make to simplify the problem and one of them of course, is that we are considering carbon combustion and if you recall that it is one of the simplest one because it is not paralyzing fuel; that means, the combustion what we will be taking place on the surface of the carbon sphere what I have shown here and unlike in a paralyzing fuel in which there will be volumetric devolatilization. Here the carbon will be devolatilize on the surface this is your surface of the carbonate cell therefore, it is very simpler one to analyse beside this we have assume that the particle to be spherical nature.

In real situation it need not to be and we are considering all other effects along the other direction if you take a sphere that we can take a coordinate system of r theta and pi and along this theta direction right, we have neglected the variation of various properties. So, also in the five the only along the r direction which I have shown here like the properties will be varying and keep in mind, that we are considering this sphere basically under acquiescent atmosphere; that means, there is no motion of fluid around it, which is not the case in practical application unless otherwise you are conducting experiment in laboratory in a very controlled condition or controlled environment then only it is possible otherwise it not and we are assuming also the steady state burning of the carbon sphere rather it is quasi steady state as I have mentioned earlier.

So, therefore, you know it is also simplified and we are considering the diffusional process because we are considering that burning of carbon will be dictated by the amount of oxygen, which will be diffusing from the ambient condition right towards the carbon surface right. So, for this purposes what we will do, we will basically invoke the various conservation equation and first of all we will considered the mass conservation equation Mass Conservation. If you consider mass conservation for one dimensional situation you will get of course, under the steady state condition.

You will get d by d r rho v r r square is equal to 0, keep in mind that the velocity with which the fluid will be or the gas will be going out here with V r, V r is the gas velocity along r direction and row is the density of gas and if I if I say this is equation 1 if I the equation 1 basically if I integrate and then you will find that it is basically implies, that that is equation one I can write down becomes row V r r square is equal to constant, what is the meaning if you look at row V r this term is nothing, but your mass flux term right that is mass flux mass flow rate per unit surface area at various direction.

At various location from the centre of the sphere if you see that into the r square that is area will be remaining constant right. So, therefore, it is constant, what is indicating? It indicating that mass flow rate at each r location it will be remaining constant; however, the mass flux will be inversely proportional to r square therefore, mass flux will be changing with respect to r and the radius increases therefore, the mass flux will be decreasing, that you should keep in mind and as I told that oxygen plays a very important role for the combustion of the carbon sphere that we are considering now as we are only interested in diffusional kind of combustion.

So, therefore, we will have to invoke the oxygen conservation equation right. So, if you consider basically Oxidizer species conservation keep in mind, that the momentum equation we are not considering here because of fact that in one dimensional it turns out to be pressure remain constant along the vertical r direction so, therefore, it is not really required. So, and it is atmospheric combustions that we have already seen for the liquid droplet combustion. So, therefore, we are not considering the momentum conservation equation; however, we will be considering the oxidizer species conservatic equation.

And considering that combustion takes place in the gas phase for no combustion rather as no combustion is taking place in gas phase, then the species conservation equation turns out to be balance between the convection and diffusion, that is basically m dot double dash r square this is nothing, but your mass flow rate which will remain constant I can say from the continuity equation d r is equal to row D 1 by 2 d by d r r square d y o x by the r keep in mind that where Y x is the mass fraction of oxidizer.

And if you look at the D 12 is the mass diffusivity between oxidizer and fuel and of course, density I have already defined density of the gas and this region is basically gas region around the sphere in this equation 2 if you consider that this term is basically due to convection, like and this term is basically diffusion term you can consider as the Mass Diffusion term and as if told earlier, that oxidizer species equation basically the balance between the convection and diffusion of the oxidizer and as there is no chemical reaction takes place in the gas phase.

Now, we will consider the energy conservation equation in the similar fashion and similar condition that is nothing, but your nothing, but the balance between the heat conduction and also the also the heat convection. So, that is the m dot r square c p and d t by d r is equal to k g by d r r square d t by d r keep in mind that the c p is where c p specific heat and k g is the thermal conductivity of course, T is the gas temperature and keep in mind the thermodynamic properties are remaining constant along with the radial direction which is not true because the c p is a function of temperature.

So, also the thermal conductivity, but; however, for simplicity reason we have kept it constant and therefore, I have taken out from this differentiation of the first term and so, also the second term. So, now, what we will do and this is basically the equation 3. What we will do? We will have to integrate the equation 2 with the following boundary condition, the boundary condition what we can and look at it basically later on let us integrate the equation 2 integrating equation 2 we can have is basically what we can get is m dot double r square.

I can take it out because that remain constant we have seen this is d y o x by d r into d r and we are integrating between the surface this is the surface r to r is equal to row D 12. Again I can take out because I am keeping this as a constant row D 1 to d by d r r square y by x d r into d r this is at the surface of the carbon to any r. So, I can say this is equation 4.

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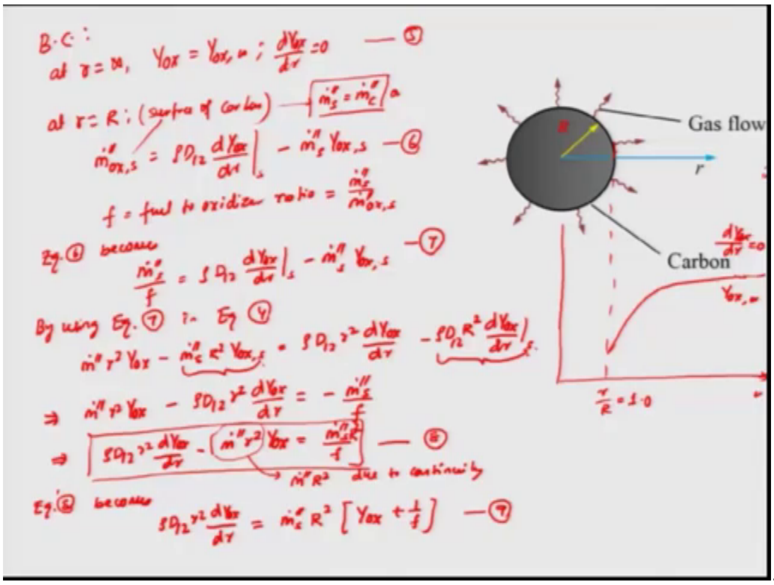

Now, what we will do we will have to look at the apply basically boundary condition for this equation 4 if you look at boundary conditions and what are that at r is equal to infinity; that means, very far away this is you r very far away around infinity, where there is not much changes is occurring. So, what we can say we can say Y o x is equal to Y o x infinity and also the gradient will be 0 if I just plot this from here. The centre and at this point this is basically r by R equal to and this is infinity if I consider that oxygen level will be basically it is attaining something infinity value is remaining constant as Y o x infinity and this gradient is 0 at this point like you can say that d y o x by d r is equal to 0.

And at r is equal to R that is at the surface this is basically at surface of carbon. What we will get? We will get basically mass flux of oxidizer at s is nothing, but your diffusivity and flux Y o x d r at I can say that is a surface which is nothing, but your r is r small r divided by capital R is equal to 1 minus m dot s Y o x at surface, basically one can say also that this is nothing, but your m dot s is equal to m dot carbon at surface at carbon surface basically I can write down that also. So, this is your constitutive equation and we can say that this is basically 5 and then this is equation 6 and there is another thing.

So, we need to look at it one is the stoichiometric equation that is f is nothing, but your fuel to oxidizer ratio is m s divide by that oxidizer at s, s means at the surface of this carbon that is s we are using symbol this is basically s means surface of the carbon. So, therefore, I can write down equation 6 becomes equation 6 s by f is nothing, but your 1 2 d Y o x d r surface minus m dot s Y o x s. So, I can say this is basically equation 7 and when we will consider these by integrating this equation.

And substituting that in equation 4; that means, by substituting equation 7 in equation 4 we can get By using equation 7 in equation 4 we can get this way that is basically m dot r square Y o x minus s R square Y o x and in the beginning I am not substituting equations I am just writing. So, that it will be simplified row D 1 2 r square d Y o x by d r minus row D1 2 R square the surface d Y o x by d r at s. So, if you look at this I can write down that basically I can take this out basically row d, and then these term.

And this term if you recognise that basically nothing, but your m dot s by f this d terms together. So, I can write down this m dot r square Y o x minus row D 1 2 r square d Y o x by d r is equal to basically minus m dot s by f. In other words I can write down that as row D 1 2 r square d Y o x d r minus I can just rewriting that r square Y o x is nothing, but your m dot s by f. So, this is we got it and this I am saying it is equation 8 and keep in mind that this is remaining constant according to continuity equation look at this one.

This I can write down as m dot R square. Now if I will do that then I equation 8 becomes and this I can write down due to continuity, which mass flow rate remains constant along the radial direction. So, then I can write down row D 1 2 r square d Y o x by d r is equal to basically m dot s R square I can take it out there will be some r square here R square then what I will get I will get Y o x plus 1 by f. So, this is the equation I am getting and let me call it as a question 9. Now what we will do we will basically integrate again this equation to get an expression for mass fraction of oxidize and if we will.

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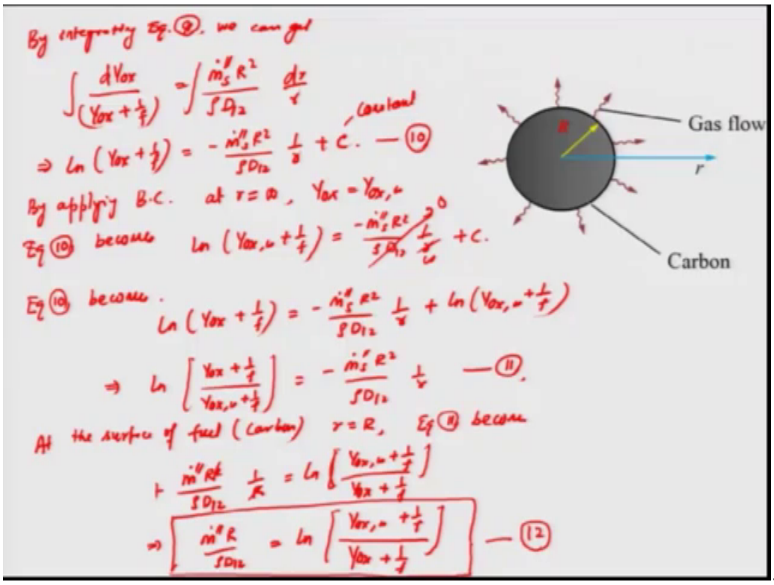

By integrating 9 we can get x by Y o x plus 1 by f equal to s R square row D 1 by 2 d r by r and this is we are integrating. So, what we will get, we will get basically L n Y x plus 1 by f is equal to minus m dot s R square row D 1 by 2 1 by r plus C and this is equation 10, what we need we will have to apply the boundary condition to forgetting this C constant this is basically a constant. So, by applying B C boundary condition particularly at r is equal to infinity we know that Y o x is nothing but your Y x at infinity. So, if we will we do that what we will get a question 10 becomes L n Y o x infinity plus 1 by f because this term will be 0 and when this term will be 0 is equal to r is infinity.

So, therefore, these term if you look at this term I am just writing for simplicity 1 by r, r is infinity this is basically infinity therefore, this will be 0 is plus C. So, therefore, equation 10 becomes L n Y o x plus 1 by f is equal to minus s r square D 1 by 2 1 by r plus L n for x infinity plus 1 by f. So, if you will just take this term to the left hand side I will get L n Y o x plus 1 by f divided by Y o x infinity 1 by f is equal to m dot s R square by row D 1 2 1 by r.

And at the surface of fuel in this case carbon that is r is equal to R. So, equation 11 becomes I can write down basically that is R square by row D 1 2 1 by R of course, this will be minus right, and if you consider this as L n if I will I can write down just opposite of that. So, that Y o x infinity plus 1 by f divided by Y o x plus one by f keep in mind that this become plus. So, therefore, this will cancel it out.

I can write down this is nothing, but your r by row D 1 2 is equal to L n Y o x infinity plus 1 by f divided by Y x plus 1 by f this is your question 12. So, we now have basically got a relationship between the various terms like mass flux mass flux of which is max relationship between the mass fraction of oxidizer and also the mass flux and infusibility. In the next lecture what we will do we will look at also, what happens at the fuel surface and then we will derive an expression for mass fraction of oxidizer now will stop over.

Thank you very much.