Fundamentals of Combustion (Part 2)
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Lecture - 69
Droplet Combustion in Convective Environment and Introduction to Spray Combustion Mode

Let us start this lecture with a thought process that your deadliest enemy is the best friend who compels you to realize your potential while soiling his own soul. All the time we will be worried about enemies, but they are your friends.

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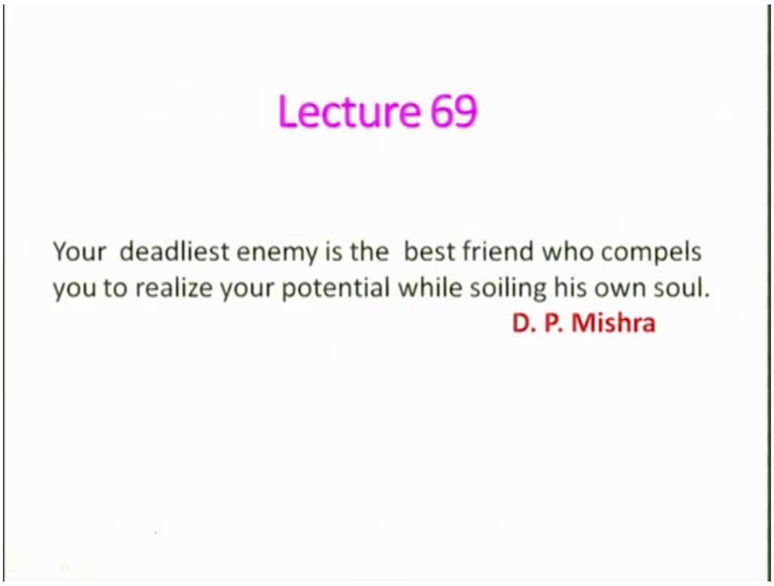

So, let us recall what we learnt in the last lecture, we basically looking at how to handle the mass burning rate of the droplet under convective environment and we looked at a basically heat balance as the surface of it and then trying to look at it. It is a very you can say adopt solution.

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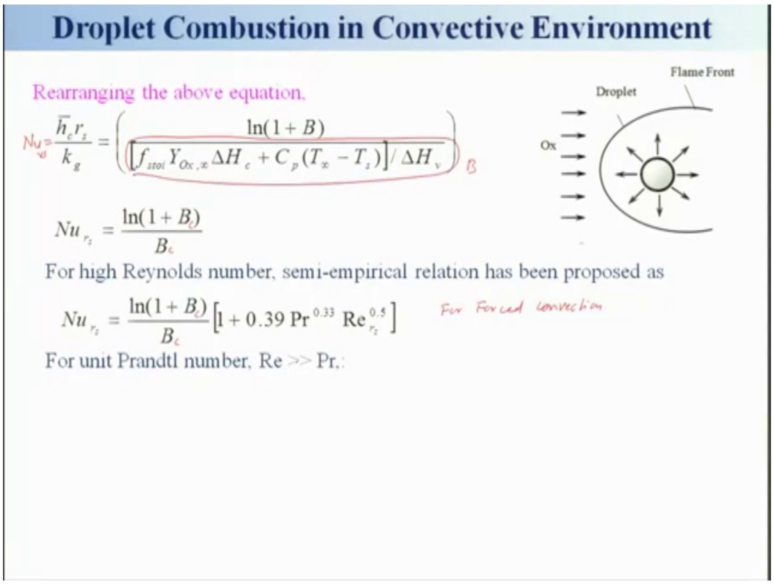

So, rearranging the earlier equation, we can get basically h c r s by k g and this is ln 1 plus B and these terms you already we had discussed last time. So, if you look at this is nothing but your Nusselt number based on the radius at the surface radius of the droplet right and if you look at this is this term is nothing but your it is a basically B right. And so, therefore, I can get Nu r s is basically ln 1 plus B divided by B, B is the transfer number, for combustion we can write c basically, for evaporation it will be different right.

And this is basically what we have looked at is some kind of a simplified way of looking at it. It does not include the Reynolds number or the Prandtl number although this will be valid for that what we have seen right. So, for high Reynolds number, we can plug in some of the relation from the heat transfer to into this right and develop a semi empirical relation which has been proposed as Nu ln 1 plus B divided by B.

And as I told, this you can you can say combustion 1 plus 0.39 Prandtl number power to 0.33 and Re is Reynolds number based on the r s power to the 0.5 and this you might have familiar with this thing right. This is generally valid to call it mimics the experimental data. Well, so far the force convection is concerned, this is for the force convection right and if want to deal with basically natural convection, you can express in terms of Grashof number right and we will be not discussing about that.

And now, if I say that this Reynolds number is much larger than the Prandtl number right. And then, you will see that this expression can be simplified further that is I can write down.

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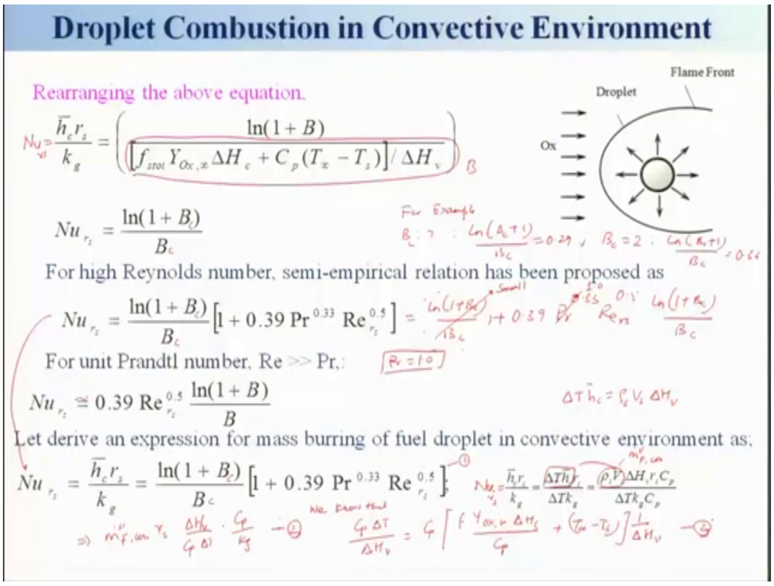

Ln 1 plus B c divided by B c plus 0.39 Prandtl number 0.33 Re r s 0.5 and ln 1 plus B c divided by B c right. And if you look at the first term here, this will be quite small as compared to second term right. Why it is so?, Because if you look at it is ln and then this is a very, very small and this will be quite small right, I will take an example. So, let us see that if B is 7, for example, B value is 7. Then what will be ln? Let us say B c. B c plus 1 divided by B c that will be 0.29.

Well, let us take another example B c is basically 2. So, then this term will be ln B c plus 1 by B c will be very, very small again 3 6 right because, this is a logarithmic term. So, therefore, it get compressed and then very small. So, therefore, this is small you can neglect it and if you neglect that one, so, you will also say that this will be equal to 1 if not really. So, you will get a Nu r s is approximately equal to this one right, you can use this provided the Prandtl number is equal to 1. A Prandtl number is very, very small right.

And so, you can see now, what we will do? We will basically look at these expressions right and see that derive some relationship or a mass burning rate for convective environment right. So,, so, let us derive an expression for the mass burning fuel in convective environment as. So, we know this Nu r s we are taking this relations here right, it is c r s by k g is nothing but ln 1 plus B divided by B. Keep in mind, I am just using sometimes B, sometimes B c, but it is c stand for combustion 1 plus 0.9 sorry 1 plus 0.39 Prandtl number power to 0.33 Reynolds number power to the 0.5.

So, let us look at certain things and then find out like how we can relate. For example, if you look at this h c r s by kg right already we have done right and this is your Nusselt number basically right and that is what delta t H c is your convective heat transfer coefficient r s divided by delta t k g. We are just putting delta t here and delta t there. That is all you are doing and in place of these we know that delta t H c is nothing but your rho s v s delta H v right.

So, I can in place of this, I can use this one right we have already done that and rho s v s delta H v r s and I am just multiplying C p in the numerator and C p in the denominator and delta t remains same right. And keep in mind that what is this term? This is basically mass flux of fuel right of course, with convection right that we are considering because Nusselt number is there.

So, there from that we can find it out right and if that is the case I can write down that as basically as convection and r s right and delta H will be by I can say C p delta t because, this is delta t I am taking into C p by k g. I am just rewriting these terms right and keep in mind that these C p delta t divided by delta H v is what is that that already we have done, that is basically C p we know that C p delta t by delta H v is equal to C p F y oxidizer infinity delta H c divided by C p right plus t infinity minus t s and 1 over delta s v right. This for delta t, I can write it down right.

So, therefore, I can now club this relationship. Let say, I can say this is equation 1 and this is equation 2. I can because both are same, both are basically no sorry this is the not the case, this is the case ok. This is the case and I can write down here this 1 and let us say, this is 3 right. And I can use equation 1 2 and 3.

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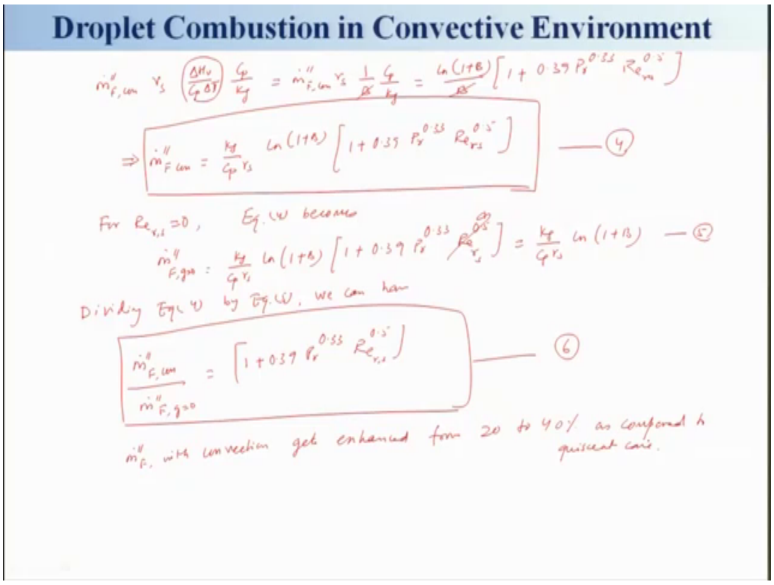

I can write it down as basically F convection is basically if you look at right, this is nothing but your B right ok. Let me write it down. This is I had already written down r s this is delta H v by C p delta t C p by k g. I am rewriting right and this C p by delta t is nothing but your this is 1 over B right. I can write down m dot F convection r s is nothing but 1 over B and C p by k g is equal to what is that that is basically ln 1 plus B divided by B into 1 plus 0.39 Prandtl number 0.3 Re r s 0.5.

So, these will cancel it out and you can get basically a relationship which will be convection is equal to right is equal to k g. I can take to this side kg by C p and r s and then, rest of the terms right that is ln 1 plus B 1 plus 0.39 Prandtl number 0.33 Re r s 0.5.

And now, this is a very nice term for convection you look at it contains all the terms right. Both Reynolds number and other things I can say this is equation 4. Now, if Reynolds number is 0; that means, it is basically quiescent atmosphere and what you call 0 g, then for Re r s is equal to 0, right. Then, equation 4 becomes m dot F g is equal to 0 is kg by C p r s ln 1 plus B 1 plus 0.39 Prandtl number 0.33 r s 0.5. So, this will be 0.

So, then this will be basically k g by C p r s ln 1 plus B right. This is equation 5 dividing equation 4 by equation 5, we can have m dot F convection divided by F g is equal to 0 is equal to very simple term, that is 1 plus 0.39 Prandtl number 0.33 Re r s 0.5 that is all. Because, this term will be cancel it out and this is a very simple expression which is quite elegant in nature right.

And, keep in mind that because of convection, this mass burning rate will be increased. Generally, people get this you know mass burning rate with convection gets enhance from around 20 to 40 percent as compared to quiescent case.

In other words, the a convection plays a very important role for enhancing the mass burning rate and which is of course, the obvious thing and that is being derived here. So, if you look at a you can use this thing expression for solving these problems and keep in mind that these are not very accurate ok. But, however, it can be used for design purposes because, it is not available.

So, naturally you can use it. So, and it will not be very accurate because of fact that right. This wake region is not being taken care by this analysis right. So, therefore, it will be not right and wake will be changed depending on how it is. So, this for laminar and then turbulent you know all those things will be here.

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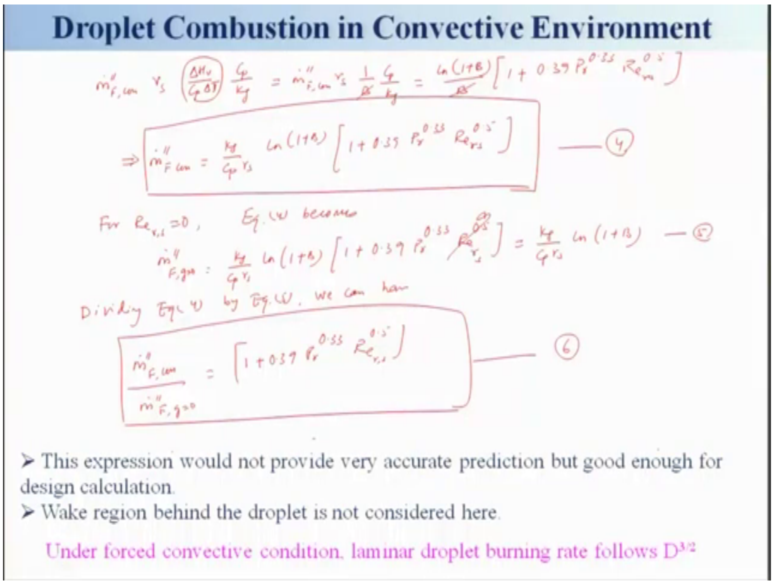

And very interestingly, if you use find out the D square law you will find out for a force convection laminar droplet burning rate follows by D power to 3 by 2 as compared to D square in case of quiescent atmosphere, this is something 1.5 and that is 2 and if we enhance the turbulence level, then it will be 1; are you getting. So, the it become almost linear the changes in the droplet burning time or droplet life time that you can say.

So, with this, we will look at we have looked at how to take care this droplet, you know, combustion in a convective environment. Now, we will be looking at basically how we will do in a handle the spray combustion in a very simplified way the we will be using a simple.

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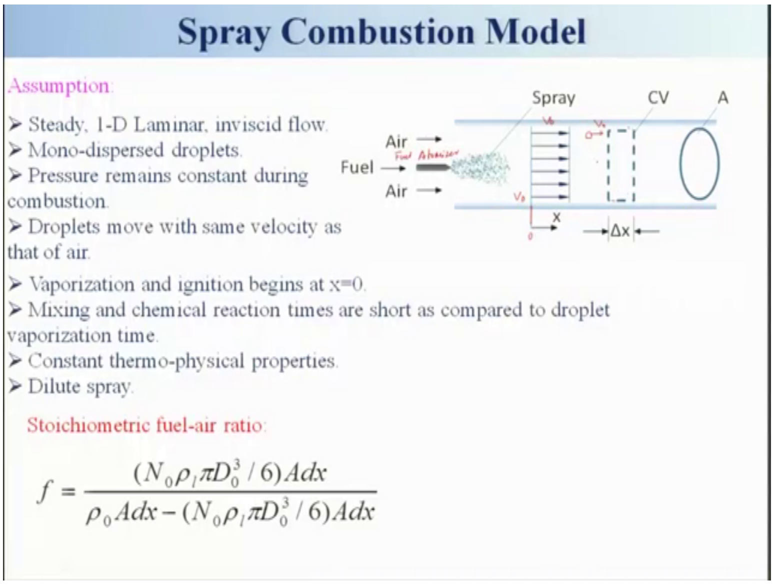

Spray combustion model. You assume that this is a 1 dimensional tube and of course, is a circular. It is only varying along the x direction. Keep in mind that this injector or the atomizer right. This is the atomizer as a fuel atomizer right is far away from this exposition where we are looking at everything and it says that it is good enough. So, that the mono dispersed droplets are available and keep in mind that mono dispersed droplet really is not possible to have it unless you produce in the lab in a controlled conditions right. But, however, for simplicity we are considering.

So, we will be making certain assumption before carrying out the analysis; one is steady one dimensional laminar inviscid flow right and that we do all the time and the mono dispersed droplet. As I told mono dispersed means, the droplet size distribution will be same right and that is the condition and of course, you can extend this for the poly dispersed droplet. But, that will be quite complex and pressure remains constant along the x direction during combustion or along the length of the combustors right which is shown here and that we have already done several times.

So, this is not a bad assumption as such and droplets move with the same velocity as that of the air because, the drop the velocity here at this location this is basically 0 and the velocity will be V naught this is the you know mixture velocities right, fluid will be moving and there is no lagging or the relative velocity between the droplet and this thing because what happened if it is V naught here right and droplet may not move at the same velocity.

But, in this case, what we are saying? It is also moving with V naught that is the assumption we are making; that means, there is no difference ok. But, in real situation, it will not be never of course, if it is a too small droplet, it may possible that it will be the lagging between the droplet and the your fluid will be not there or will be minimum, but ok.

So, vaporization and ignition begins at x is equal to 0. This is your x right. Nothing is happening here, it is just mixing. But, actually vaporization can occur here itself. Once you atomized in the very you know after this fuel injected from the orifice of the fuel atomizer, then it can be started vaporizing right, once it is formed.

But, however, we are saying it is still when this and that is for the simplification and mixing and chemical reaction times are as compared to droplet vaporizes in time, there is basically droplet vaporization will take more time because, it is governed by the diffusion right and chemical reaction is very fast. Because, we will be using all the time the thin flame approximation this and also this will be diffused and control. So, therefore, this is the valid assumption one can say.

And constant thermo physical properties which will be discussing we will be using there because, there is no other way you can handle and this is a dilute spray right, dilute spray means the droplets will be it is not a dense spray right. The number of droplets per unit volume will be very, very less. You know, in the case of dilute spray, other number density and stoichiometric fuel air ratio we are considering.

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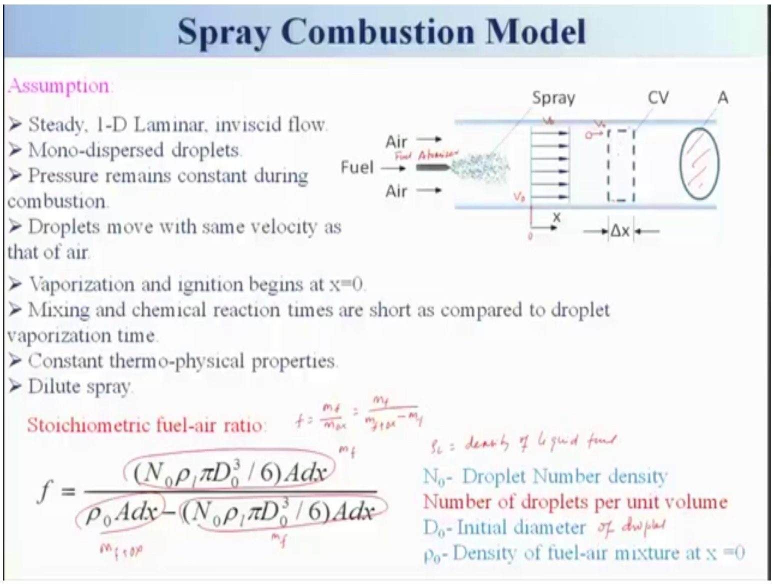

Then, we can take this control volume here right and the we can find out basically this is the fuel air ratio is nothing but your f is equal to mass of fuel divided mass of oxidizer right. And that can be really derived from here and by this expression it looks to be quite complex. But, keep in mind it is very simple because N naught is the droplet number density; that means, number of droplet per unit volume there is N naught into rho l rho l is the basically density of liquid fuel right and D naught is the initial diameter of droplet. And A I have taken this cross sectional area and delta x, I have taken this control volume.

So, you can look at it and this is basically this portion is mass of f and this is basically rho naught keeping density of fuel air mixtures right mass of fuel air mixture fuel plus you can say f right or oxidizer right oxidizer and this is the mass of fuel right, this is your mass of fuel.

So, I can write down the basically m f m f m f is equal to m f A plus oxidizer minus m f right. So, we can basically it is a very it can be simplified very easily. But, however, it has been done in such a way that if there is a distribution, you can integrate right. If there is a variation along the cross sectional area, you can integrate also right or if there is a variation right uh, you can do that.

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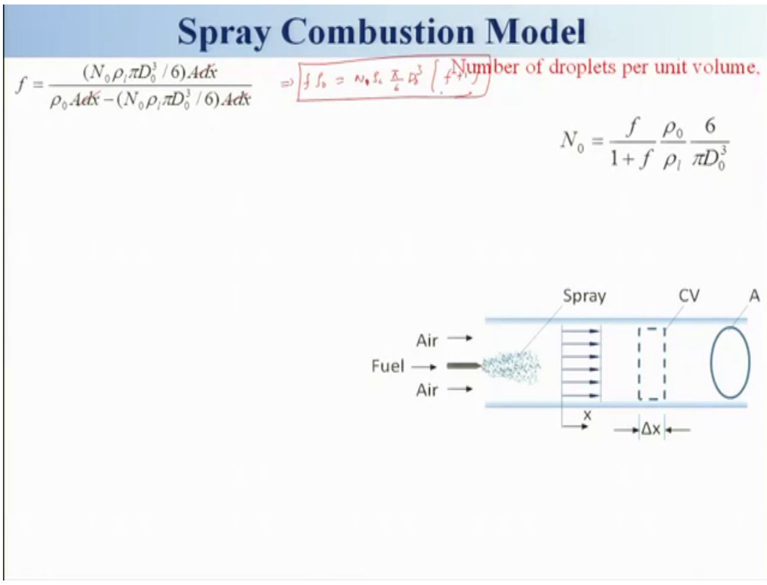

But, we will be looking at that that f I can write down as this right and I can write down this as basically f rho naught right. I can cancel this out right, this out I can cancel it out rho naught is equal to N naught rho l. I can take out and pi by 6 D naught cube is equal to f plus 1 because, I am just multiplying here and taking that out, I will get this expression right.

So, therefore, from this I can get very easily N naught number of droplet per unit volume will be nothing but your is equal to N naught is equal to f divide by 1 plus f rho naught by rho l 6 by pi D cube right. And, this is we will be using this expression I can say this is basically equation 1. Now, what it indicates? It indicates that this is basically you know, number density is a function of f and the initial density of the mixture and density of the fuel and of course, the initial diameter.

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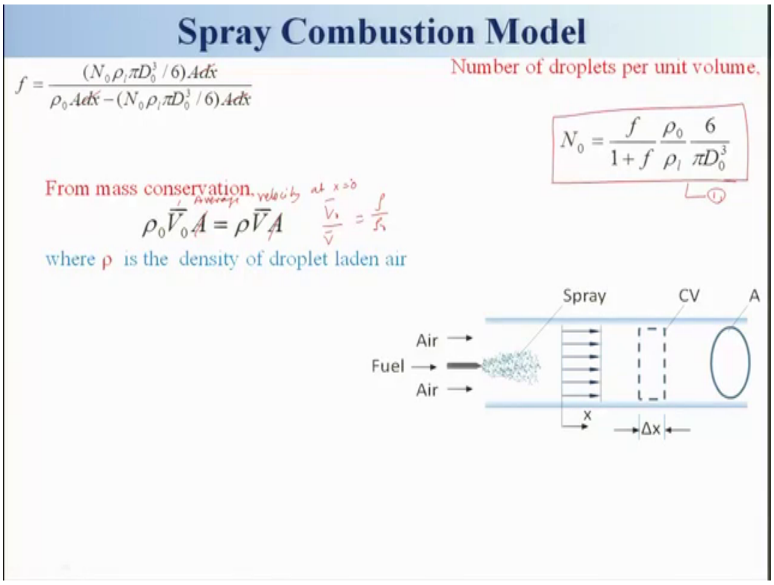

So, the mass conservation if you look at, I can this cancel it out, I will get basically V naught average by V is equal to rho by rho naught. But, what is this V naught average? This V naught average right, I can write down this is the average velocity at x is equal to 0, average velocity of fluid right. We will stop over here and we will discuss in the next because it will take more time.