Fundamentals of Combustion (Part 2)

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Lecture - 68

Droplet Combustion in Convective Environment

Let us start this lecture with a thought process from Henry Ford.

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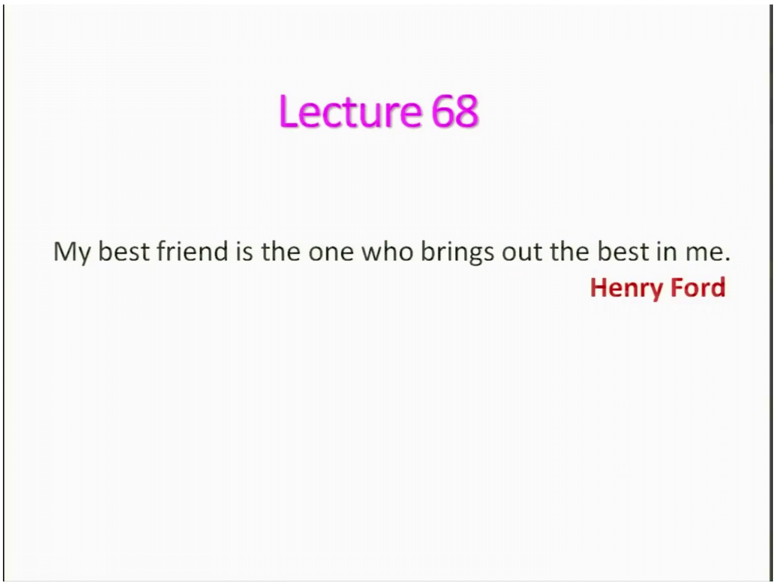

Lecture 68

My best friend is the one who brings out the best in me.
Henry Ford

My best friend is the one who brings out the best in me. And in the last lecture we basically derived the expression for mass flux rate from the droplet surface are the from the droplet right. And looking at little further how we can use it for determination of droplet burning rate and then we will be looking at also the droplet burning time, right.

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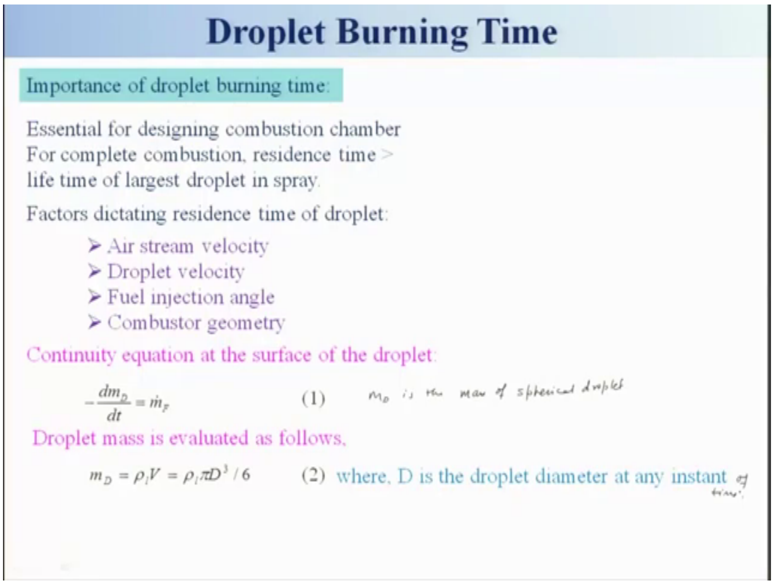

Droplet Burning Time

Importance of droplet burning time:

Essential for designing combustion chamber
For complete combustion, residence time > life time of largest droplet in spray.

Factors dictating residence time of droplet:

- Air stream velocity
- Droplet velocity
- Fuel injection angle
- Combustor geometry

Continuity equation at the surface of the droplet:

$$-\frac{dm_D}{dt} = \dot{m}_F \qquad (1)$$

m_D is the mass of spherical droplet

Droplet mass is evaluated as follows,

$$m_D = \rho_l V = \rho_l \pi D^3 / 6 \qquad (2)$$

where, D is the droplet diameter at any instant of time.

So, if you recall that it is very important to estimate the droplet burning time why? Because, when you are designing a combustors, you should see that all the droplets should be all consumed before it is going out of the exit of the combustor. Otherwise, it will be a problem in the not only the emission, but also the energy inefficiency and other things.

So, and as I told it is essential for designing combustor for complete combustion residence time right, must be greater than the lifetime of the largest droplet in the spray, because, you may say look there are several droplets. What you will be considering when you are designing will say whatever the largest droplet is there; I will have to design for that. And if that is taken care then rest of the things will be taken care right. And this is of course, a little adoc kind of things and, but that is essential to be using this as a design tool.

And factors that dictates, the factors that dictate the residence time of the droplets are basically what, like the air stream velocity right which we have not considered and we will be considering it and droplet velocity and fuel injection angle and combustor geometry all those thing will be taking.

So, if you look at the continuity you will find out the mass of this droplet is basically consumed. So, therefore, it is getting a receded, the mass given is therefore, I am putting it negative is equal to the mass of the fuel which is getting out outward it is moving out from the flame surface. And this is basically equation 1 and keep it mind that this m D is the mass of spherical droplet right; which is changing with respect to time, because the radius will be changing therefore, mass also will be changing.

And I can write down that m D is nothing but your mass will be rho l into v right, rho l is the density of the fuel or the liquid fuel, and V is the volume of the sphere is pi D cube by 6 right. And as I told that D is basically the droplet diameter at any instant of time right, at any instant of time. And it will be changing is not it, the diameter will be changing it will be receding.

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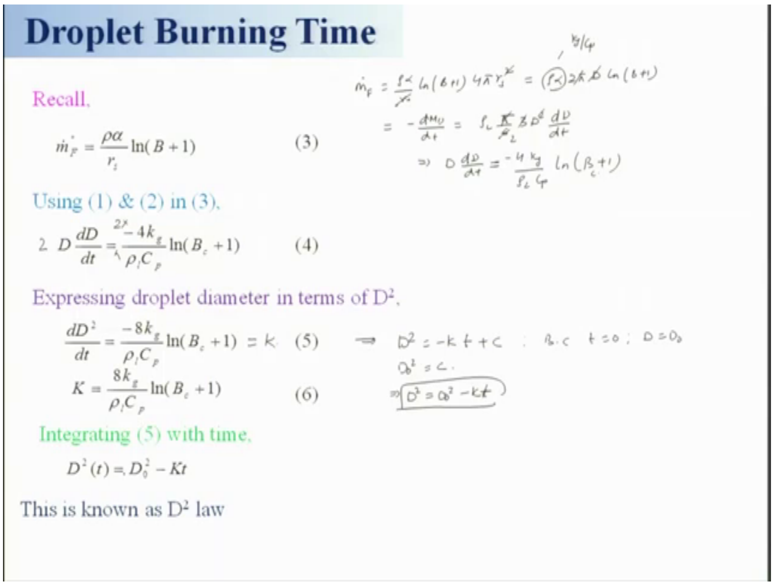

Droplet Burning Time

Recall,

$$\dot{m}''_F = \frac{\rho\alpha}{r_s}\ln(B+1) \qquad (3)$$

Using (1) & (2) in (3),

$$2\,D\frac{dD}{dt} = \overset{2\times}{}\frac{-4k_g}{\rho_l C_p}\ln(B_c+1) \qquad (4)$$

Expressing droplet diameter in terms of D².

$$\frac{dD^2}{dt} = \frac{-8k_g}{\rho_l C_p}\ln(B_c+1) = k \qquad (5)$$

$$K = \frac{8k_g}{\rho_l C_p}\ln(B_c+1) \qquad (6)$$

Integrating (5) with time,

$$D^2(t) = D_0^2 - Kt$$

This is known as D² law

$$k_g/C_p$$

$$\dot{m}_F = \frac{\rho\alpha}{\cancel{r_s}}\ln(B+1)\,4\pi r_s^{\cancel{2}} = \rho\alpha\,2\pi D\ln(B+1)$$

$$= -\frac{dm_D}{dt} = \rho_L\frac{\pi}{6_2}3D^2\frac{dD}{dt}$$

$$\Rightarrow D\frac{dD}{dt} = -\frac{4k_g}{\rho_L C_p}\ln(B_c+1)$$

$$\Rightarrow D^2 = -kt + C \quad ; \quad \text{B.C} \quad t = 0;\ D = D_0$$

$$D_0^2 = C.$$

$$\Rightarrow \boxed{D^2 = D_0^2 - kt}$$

So, now what will have do? We will have to connect these things and recall that we had derive these expression m dot double dash F is equal to rho alpha divided by r s is ln plus B. Now from these I can find out what would be the m dot F, because we will have to find out m dot F, m dot F is nothing but your rho alpha by r s what I will do I will have to just multiply the surface area, is not it 4 pi r s square, this will cancel it out and what you will get is basically rho alpha right 2 pi D right and ln b plus 1. That is the thing what you will be getting right. And this is r s is it will be changing also right, it is changing right.

So, therefore, this will be 2D you know to pi b. And what we will do we will combine these equations right, if I will do that, what I will do? I will use this equation 1 and 2 and 3 right, I will get basically I can write down here m dot this thing is equal to minus d M

D by dt. And this is nothing but your rho L pi by 6 and 3 D square D by dt right. So, this will cancel it out and this pi will cancel it out that side. And anything else will cancel it out and the D also will be cancel it out this D will be cancel it out yes or no one D from this side to that right.

Then you will get I can write down that D by D by dt is nothing but your, this will be 2 will be going that 4, 4 in the alpha L, I can write down what kg by C p can I not write down. So, kg by rho L C p ln B plus 1, I can say this is C basically due to combustion just to make that it is not due to vaporization right, just to differentiate, B can be same B can be used for combustion and vaporization right. Of course, some kind of term will be eliminated when you are talking about combustion and also a vaporization will take care of that.

So, therefore, I will have to basically now express this droplet diameter right in terms of D square and that I can write down this as I can write down 2D right, I can do here itself 2D is nothing but 2 into that right. So, what I will be getting is basically D square is, because I can take this inside and dD square dt is equal to 8 kg by rho C p ln B plus 1 right. And keep in mind that this I can call it has a K like that right, K and K is nothing but your 8 kg rho L C p ln B C plus 1 right. And if I integrate this equation 5 right, what I will get? I will get basically D square t is equal to D naught square minus K t, right.

What is the right how I am getting? Basically what I am doing, I am getting from this equation I am getting D square is equal to minus K t plus C right. With boundary condition t is equal to 0 what is D? D is equal to D naught. So, when this t is 0 this is D naught square is nothing but your C right.

Therefore, this is I can write down D square is equal to D naught square minus Kt right. This is the expression what I am getting and this is a very simple relationship right and this is known as D square law. And people have also conducted experiment they have observed for most of the single fuel droplet under quiescent atmosphere follows this one roughly.

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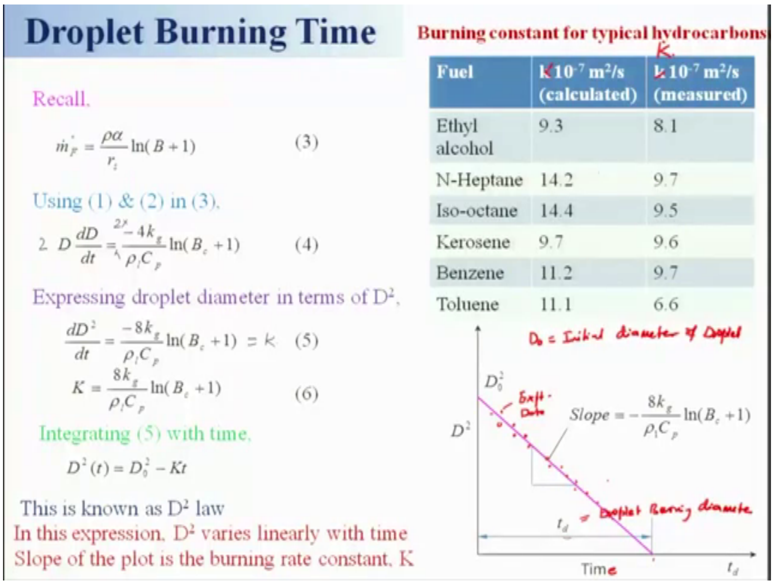

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Integrating (5) with time,

$$D^2(t) = D_0^2 - Kt$$

This is known as D^2 law

In this expression, D^2 varies linearly with time

Slope of the plot is the burning rate constant, K

Burning constant for typical hydrocarbons

Fuel	~~k~~ K 10^{-7} m^2/s (calculated)	~~k~~ K 10^{-7} m^2/s (measured)
Ethyl alcohol	9.3	8.1
N-Heptane	14.2	9.7
Iso-octane	14.4	9.5
Kerosene	9.7	9.6
Benzene	11.2	9.7
Toluene	11.1	6.6

So, if you look at these are the K capital values this is basically I can say this is there is some typographical error, this is capital K right this will be capital K. And the these are calculated values from the simple analysis 9.3 whereas, this is coming 8.14 and 14.2 is really not very far different, keeping in mind that simplicity of the analysis right, that you can keep in mind.

Now, if I plot this one, if I plot this thing, I will get D square by the time and this is basically the droplet time diameter right. This is droplet burning time this is droplet burning diameter. Now you will have to evaluate this how much time it will be taking. It looks to be linear, but actually it is not linear because it is goes by D square. So, therefore, you cannot really simply calculate let us say 100-micron droplet and it has gone let us say 2 second and then you are going for let us say 1 mm droplet and then this will no it is not that way.

So, therefore, that has to be taken care right. And this is very important to evaluate this t d. D naught is the initial diameter of droplet right and D is the diameter which will be changing and it become 0 at time t d right. So, that is the thing you should keep in mind right. So, right as I told earlier D square varies linearly with time and slope of plot is burning rate constant and you can have some experimental data you know you can have some people generally put some data as here. And then try to match this thing, these are you can say experimental data right. Need not to be these things, there will be some scattered and then you will have to say ok, this is the which is matching right that way it is being done.

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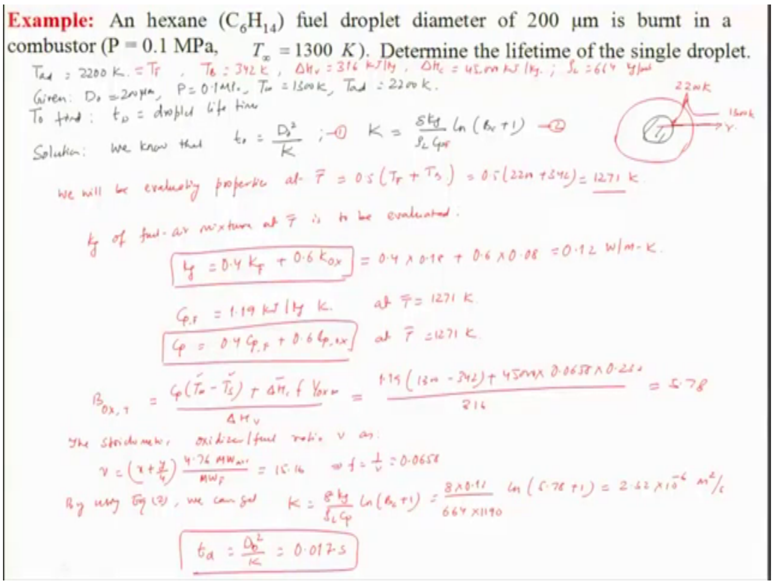

Example: An hexane (C_6H_{14}) fuel droplet diameter of 200 μm is burnt in a combustor (P = 0.1 MPa, $T_\infty = 1300\ K$). Determine the lifetime of the single droplet.

$T_{ad} = 2200$ K $= T_f$, $T_b = 342$ K, $\Delta H_v = 316$ kJ/kg, $\Delta H_c = 45000$ kJ/kg; $\rho_L = 664$ kg/m

Given: $D_0 = 200\ \mu m$, $P = 0.1$ MPa, $T_\infty = 1300$ K, $T_{ad} = 2200$ K.

To find: t_D = droplet life time

Solution: We know that $t_d = \frac{D_0^2}{K}$; —① $K = \frac{8 k_g}{\rho_L C_{pg}} \ln(B_c + 1)$ —②

We will be evaluating properties at $\bar{T} = 0.5(T_f + T_s) = 0.5(2200 + 342) = \underline{1271}$ K

k_g of fuel-air mixture at $\bar{T}$ is to be evaluated.

$\boxed{k_g = 0.4 k_F + 0.6 k_{ox}} = 0.4 \times 0.18 + 0.6 \times 0.08 = 0.12$ W/m-K.

$C_{p,F} = 1.19$ kJ/kg K. at $\bar{T} = 1271$ K

$\boxed{C_p = 0.4 C_{p,F} + 0.6 C_{p,ox}}$ at $\bar{T} = 1271$ K

$$B_{ox,T} = \frac{C_p(T_\infty - T_s) + \Delta H_c f Y_{ox,\infty}}{\Delta H_v} = \frac{1.19(1300 - 342) + 45000 \times 0.0658 \times 0.232}{316} = 5.78$$

The stoichiometric oxidizer/fuel ratio ν as:

$$\nu = \left(x + \frac{y}{4}\right)\frac{4.76\ MW_{air}}{MW_F} = 15.16 \Rightarrow f = \frac{1}{\nu} = 0.0658$$

By using Eq (2), we can get $K = \frac{8 k_g}{\rho_L C_p}\ln(B_c + 1) = \frac{8 \times 0.12}{664 \times 1190}\ln(5.78 + 1) = 2.62 \times 10^{-6}\ m^2/s$

$$\boxed{t_d = \frac{D_0^2}{K} = 0.017\ s}$$

So, we will take an example how to evaluate right this thing. And keep in mind that this is as I told earlier that it is very important to understand how one can do that right. Because of fact that the transfer number although we have used by complex relationship for finding estimating the transfer number, but that will not be affecting much the droplet burning rate so, also the here droplet lifetime right, time lifetime of the droplet.

So, and for that the properties which need to be looked at it, so, let us look at an example to illustrate that point and hexane C 16 H 14 fuel droplet of diameter 200-micron meter is burnt in a combustors like let say one atmosphere pressure and T infinity 1300 Kelvin. And determine the lifetime of the single droplet right. And keep in mind that of course, the T adiabatic in this examples should be given; is something around 2200 Kelvin right. And we will have to find out to find basically droplet lifetime, or with some people that droplet burning time at the burning deviation time deviation.

So, we know that t d is basically D naught square by K and D naught is already given, right. So, given is D naught is given is 200-micron meter; pressure is given 0.1 mega Pascal, T infinity is 1300 Kelvin and T adiabatic is given ok. And what is K [noise? K this is burning rate constant, right t is equal to 8 kg by rho L C p ln B c plus 1. See I am using for combustion.

Now, question arises how we will evaluate the properties of kg and C p, because as I told the B c you can evaluate you know using this formula. But here this properties has to be done properly, because these effect the burning constant kg and C p right, rho L anyway it will not be because it is a almost constant density of the liquid fuel. The generally if you look at the temperature will be varying let say there is a droplet here right, the flame will be flame will be little away from that right. Let us say this is your flame surface.

Now, the temperature if I plot this temperature here right what will be this will be something temperature will be going up here right and then it will be receding. So, here it will be 2200 Kelvin, and ambient temperature here is something 1300 Kelvin. So, we will have to find out the properties. Now properties how I will have to find it out? I will have to use some approximation right because; we are doing calculation, in numerical simulation the properties will be changing along with the radial direction right. It will be changing, but in our case we cannot afford to do that we will use some average properties right.

We will be evaluating properties right at average temperature and that is what will be doing 0.5 T F lambda temperature or adiabatic temperature, I can write adiabatic temperature rather I should put flame temperature. Because in some cases if it is given the flame temperature, you can take in this example we are using the adiabatic temperature. Plus, the T B right we will be taking the T B and T B is you can find out from the this thing because we are using boiling temperature. The T B is basically 342 Kelvin variable. So, plus 342 I will be getting 1271 Kelvin at this temperature I will be evaluating the thing right.

Now, question arises, you know, you will be evaluating here, but there is oxygen also having different properties, fuel is having different properties, what you will have to do? You will have to let say gas thermal kg right is to kg of the mixture right, you can say of the fuel air mixture at T average is to be evaluated right. How we will do? We will be doing kg is equal to 0.4 K F right, plus 0.6 K oxidizer. Keep in mind that these relationship is not very sacrosanct, you can use some different thing, but people lot of people have used this approximation we are doing instead of 0.4, I can say look I will be using maybe 0.3 and then 0.7 the other one it can be done, but people have used so, therefore, we are using this.

Now, each one of them K F and K ox will be evaluated at what at this temperature 1271 Kelvin. Now, these values is not given, but you can get in the some table, right I will be using directly now 4 into 0.18 for K F plus 0.6 into 0.08 oxidizer here basically air. And that happens to be 0.12 watt per meter Kelvin right. And similarly, C p you will have to also evaluate, but C p what we are taking is basically the fuel right. C p F we are considering of course, F we are considering as 1.19 kilo joule per kg and Kelvin at T average 1271 Kelvin.

Otherwise similar formula can be done, but in this example I am just taking that way right. What I am saying? There is another way of doing it, like; in the similar way I can write down C p is basically 0.4 C p F plus 0.6 C p of oxidizer. I can do that, but I have I have just taken for the simplicity right, this can also be done, are you getting? And at T average is equal to 1271 Kelvin in this example I am using this thing,

So, now delta Hv is also should be given, or you will have to find out from the data table. That is your 316 kilo joule per kg and the heat of combustion has to be given, delta H c is equal to 45000 that is kilo joule per kg right.

So, what we will have to do? We will have to basically evaluate the v the transport number, this we can use any one of them, but we are using ox T is equal to C p T infinity minus T s plus delta H c f Y oxidizer infinity, divided by delta H v, right. So, T infinity is given, this is given, you have already evaluated C p so, you can find out this thing right. You can find out delta H c is given and y oxygen infinity is known to you right and delta Hv you can find it out right.

So, but you need to find out f right because, this is you can find out from oxygen air what will be; so, then f has to be found out, f for that you will have to use stoichiometric oxidizer, this is fuel right, ratio it is f is fuel oxidizer ratio right ok. Oxidizer fuel ratio you can write down nu as; nu is equal to x plus y by 4 4.76 molecular weight of air you can do other way around also. This is a direct formula I am using and if you substitute these values you will get, 15.16; that means, f will be 1 by nu is nothing but your 0.06588.

And what I will do? I will substitute all these values here and when I do the substitute values right 1.19, this is 1300 minus 342 plus 45000 into 0.0658 into 0.232, that is your oxidizer right divided by the 316, you will get something 5.78 right. And then from these you can find out basically K, right by using this equation, I can say this is equation 1 and this is 2, by using equation 2, we can get basically K as 8 kg rho L C p ln B c plus 1 right. So, that will be 8 into 0.12 rho L is density is not given, but you know we will have to find out; that is 640 kg per meter cube right, 640 into 1190 right C p this is kilo joule I am putting in joule. So, right this will be ln 5.78 plus 1, you will get a very small value right 10 power to minus 6 meter square per second is a meter square. So, it will be very, very small you can look at this burning rate constant.

Then after that you will find out t d is basically D naught square by K and this you will get is 0.017, right. So, when you substitute these values you will find it is a very, very small time and this is second ok. And if it is the droplet size is bigger then it will take more time, keep in mind that, this is quiescent atmosphere, there might be turbulence, there might be a flow which will be affecting the things. And also the droplet might be interacting so, the time will be different you know, but however, as I told it can be used as a design tool, right.

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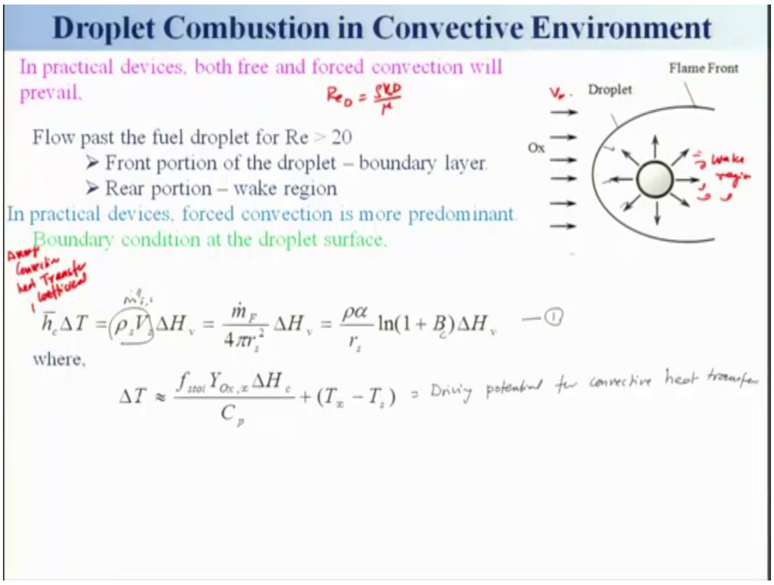

Droplet Combustion in Convective Environment

In practical devices, both free and forced convection will prevail.

$Re_D = \frac{\rho V D}{\mu}$

Flow past the fuel droplet for Re > 20

- Front portion of the droplet – boundary layer.
- Rear portion – wake region

In practical devices, forced convection is more predominant.

Boundary condition at the droplet surface.

Avg Convection heat Transfer coefficient

$$\bar{h}_c \Delta T = \underbrace{(\rho_s V_s)}_{\dot{m}''_{F,s}} \Delta H_v = \frac{\dot{m}_F}{4\pi r_s^2} \Delta H_v = \frac{\rho\alpha}{r_s} \ln(1+B_c) \Delta H_v \quad \text{—①}$$

where,

$$\Delta T \approx \frac{f_{stoi} Y_{Ox,\infty} \Delta H_c}{C_p} + (T_\infty - T_s) = \text{Driving potential for convective heat transfer}$$

So, till now we have discussed about basically under the quiescent atmosphere. Now we will see how we can utilize those relationship and look at to estimate the mass burning rate of the droplet when subjected to convective environment, right. So, that will be looking at because it is very important to do that from the practical point of view, right. So, because both the free and force convection will be prevailing in the practical devices.

So, for flow positive fuel when you look at a droplet, flow past the fuel droplet when the it is Reynold's number is greater than 20 this effect will be more predominant. And this Reynold's number is basically based on the diameter of the droplet, right.

If I say this is basically Re D rho V D by mu right and this you can say v infinity, this will be V infinity like which is having right that one can think of. So, if it is less than that, it will be not having much effect that people have found out. So, then the what will happen? Once it is greater than these the front portion of the droplet will be like that you know it will be coming and there is a boundary layer this is of course, your flame front the temperature you can say thermal boundary layer.

But there will be also aerodynamic boundary layer; which I have not shown and there is a wake region here. You know, again there will be some kind of vortices will be formed particularly downstream of here right, the droplet ok, not that place at least here not that far ok. So, this region is very important, but we will be not considering this wake region because this will be inherently 3 dimensional in nature.

So, therefore, that will be little difficult, what will be looking at the basically looking at the practical devices where the force convection is more predominant. And what will be doing? We will be looking at basically boundary condition right. And then trying to look at how we can estimate those things, by just tweaking or rather applying the boundary condition at the droplet surface.

So, if you look at what will be so, this is the expression if you look at h c, h c is basically if I look at the at the boundary condition, what will happening? This you know like heat transfer this is the heat transfer coefficient, right h c into delta T there will be some temperature. Let us look at this balance at the droplet surface, we are looking at where, there will be some heat will be convicted right because of the temperature gradient which will be there and that is been convicting that is affecting. That is h c, h c is basically convective heat transfer coefficient. Keep in mind that this I have taken average, because this will be changing right. And delta T is the temperature gradient with which the heat will be convicted right that is the thing.

And we will see how we can get the delta T, if you look at this term, then is equal to the mass flux at the surface. And this is nothing but you m dot F at surface; into delta H v that is the heat of vaporization right; is equal to basically same thing, the mass of the mass congestion of the fuel at the surface into the surface area right. That is nothing but that and that we have already seen, that is nothing but this expression is rho alpha l r s ln 1 plus B c, I can say this is for combustion right into delta H v.

See this is we are using quiescent atmosphere and trying to connect to the heat convicted due to the force convection that is the thing what we are trying to do. And this delta T if you look at basically, delta T is nothing but your f stoichiometry Y oxygen at the infinity delta Hc divided by C p right. This is the amount of heat which will be generated due to what due to flame front. And this is the heat which will be coming from this T s to this thing that will be also convicted.

So, therefore, you can this is the driving potential, because this is the driving potential for convective heat transfer because, without temperature gradient you cannot have convective heat transfer. So, therefore, that is and you keep in mind that by increasing the T right due to the chemical reactions, you can really enhance the mass consumption of the droplet.

Like, temperature of the flame is higher, then naturally what will happen? More heat will be transfer, and then because of convection it will be reaching and then it will be affecting the droplet burning rate, right. So, what we will do? We will basically look at this is equation 1. Now we will substitute this delta T in this equation 1, right.

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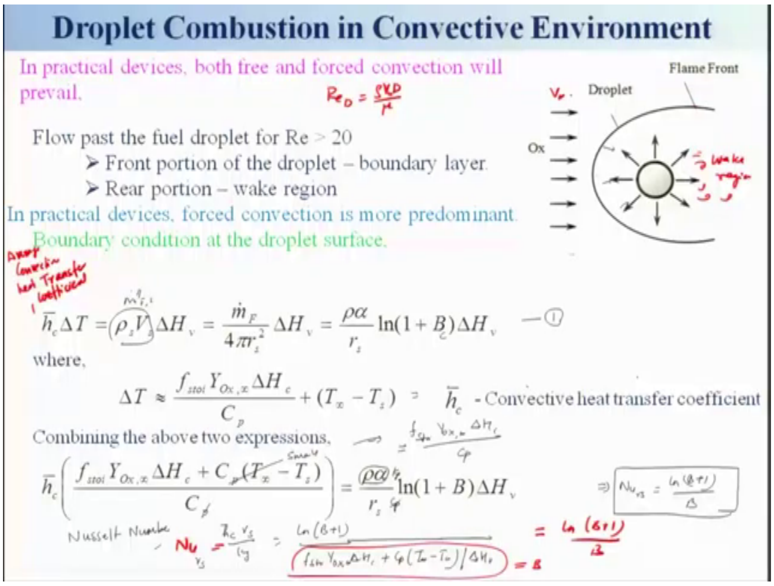

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where,

$$\Delta T \approx \frac{f_{stoi} Y_{Ox,\infty} \Delta H_c}{C_p} + (T_\infty - T_s)$$

$\bar{h}_c$ - Convective heat transfer coefficient

Combining the above two expressions, $\rightarrow = \frac{f_{st} Y_{Ox,\infty} \Delta H_c}{C_p}$

$$\bar{h}_c \left(\frac{f_{stoi} Y_{Ox,\infty} \Delta H_c + C_p (\overset{small}{T_\infty - T_s})}{C_p} \right) = \frac{\rho\alpha\, k}{r_s\, C_p} \ln(1+B)\Delta H_v \Rightarrow \boxed{Nu_{r_s} = \frac{\ln(B+1)}{B}}$$

Nusselt Number

$$Nu_{r_s} = \frac{\bar{h}_c r_s}{k_g} = \frac{\ln(B+1)}{\underbrace{\left(f_{st} Y_{Ox,\infty} \Delta H_c + C_p (T_\infty - T_s)\right) / \Delta H_v}_{= B}} = \frac{\ln(B+1)}{B}$$

And combined these 2 expression, what I will get? I am basically writing down these things right, s stoichiometry Y ox delta Hc into C p and then divided by C p is equal to this. And if you look at, I can write down this term like as a little different way, but before doing that let me tell you that this is basically a smaller term right. It is quite small as compared to this term, the left hand this is due to the combustion the heat release is very, very high.

So, I can write down that basically this as this will be kind of a also the B right. So, this will be if I look at this what I can write down this as, basically f stoichiometric y oxidizer infinity delta H c if that is 0 then divided by C p right. This is the term what it would be and right so, of course, I can do if it is approximately, but; however, we can write down this expression as little different form; that is, h c average r s by this is kg right, if I take this I will be getting, because this alpha this one is nothing but your kg by C p right. So, this rho alpha is kg by C p so, it will cancel it out.

So, kg will come, and then I will get is equal to ln B plus 1 right, divided by this is a term right f stoichiometric I am retaining the whole term delta H c plus C p T infinity minus T this thing. And keep in mind that divided by delta H v this you will find it is basically similar to the B right. This term you can see is basically B number so, I can write down this is ln B plus 1 divided by B. And keep in mind that this can you say what is that? This is basically Nusselt number right based on what? Based on r s, if I say this is based on r s this is nothing but your Nusselt number.

So, I can write down this expression as Nu r s is equal to basically ln B plus 1 divided by B is a very simple expression, what we are just deriving for by balancing. So, we will stop over here, in the next lecture we will be discussing little bit about this thing, and then we will be moving some other topic ok.

Thank you very much.